§11.4 The Cross Product

1. Definition of $\bar{a} \times \bar{b}$.
2. Basic properties:
   (a) $\bar{a} \times \bar{a} = \bar{0}$
   (b) $\bar{a} \times \bar{b} = -(\bar{b} \times \bar{a})$
   (c) $\bar{a} \times (\bar{b} + \bar{c}) = \bar{a} \times \bar{b} + \bar{a} \times \bar{c}$
   (d) $(\bar{b} + \bar{c}) \times \bar{a} = \bar{b} \times \bar{a} + \bar{c} \times \bar{a}$
   (e) $c(\bar{a} \times \bar{b}) = (c\bar{a}) \times \bar{b} = \bar{a} \times (c\bar{b})$
3. Additional properties:
   (a) $\bar{a} \times \bar{b}$ is perpendicular to both $\bar{a}$ and $\bar{b}$. This is extremely useful.
   (b) $||\bar{a} \times \bar{b}|| = ||\bar{a}|| ||\bar{b}|| \sin\theta$
   (c) $\bar{a}$ and $\bar{b}$ are parallel iff $\bar{a} \times \bar{b} = \bar{0}$ but this is not a particularly good way to check.

§11.5 Lines in Space

1. Intro: What determines a line? What can we use for an equation? The fundamental way to define a line is to have a point on the line and a vector pointing along (parallel to) the line. If $\bar{a}\,\hat{\imath} + \bar{b}\,\hat{\jmath} + \bar{c}\,\hat{k}$ is parallel to the line and if the line contains the point $P = (x_0, y_0, z_0)$ then:
2. Parametric Equations: $x = x_0 + at$, $y = y_0 + bt$ and $z = z_0 + ct$. Each t gives a point on the line.
3. Vector Equation: $\bar{r} = (x_0 + at)\,\hat{\imath} + (y_0 + bt)\,\hat{\jmath} + (z_0 + ct)\,\hat{k}$. Each t gives a vector which points from the origin to a point on the line. This is far from unique since on any given line there are many points and many vectors pointing along the line.
4. Symmetric Equations: Solve for t in each of the parametric equations and set them all equal. If one doesn't have a t in it just leave it be. If two do not then leave those alone and don't even write the third because that variable could be anything.
5. If a line has point P and vector $\bar{L}$ then the distance from another point Q to the line is $\frac{||\bar{L} \times \overrightarrow{PQ}||}{||\bar{L}||}$.

§11.6 Planes in Space

1. A plane is determined by a point $P = (x_0, y_0, z_0)$ and a normal vector $\bar{N} = a\,\hat{\imath} + b\,\hat{\jmath} + c\,\hat{k}$ which is perpendicular to the plane. A point $Q = (x, y, z)$ is on the plane iff the vector from P to Q is perpendicular to $\bar{N}$, meaning $(a\,\hat{\imath} + b\,\hat{\jmath} + c\,\hat{k}) \cdot \overrightarrow{PQ} = 0$ which is $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$. This is often rearranged to get $ax + by + cz = d$.
2. If a plane has point P and normal vector $\bar{N}$ then the distance from another point Q to the plane is $\frac{|\bar{N} \cdot \overrightarrow{PQ}|}{||\bar{N}||}$.
3. Sketching planes:
   - Those like $ax + by + cz = d$, draw a little triangle using the intercepts.
   - Those like $z = 0$ or $x = 2$ or $y = -3$, parallel to the coordinate planes.
   - Those like $2x + y = 10$, draw a line and extend in the direction of the missing variable.