§14.5 Triple Integrals in Cylindrical Coordinates

1. Cylindrical coordinates are just polar coordinates plus z. The thing to watch out for is how equations change. For example:
   (a) $r = 2$ is a cylinder, as are $r = 3\cos\theta$ and $r = 2\sin\theta$.
   (b) The sphere $x^2 + y^2 + z^2 = 9$ becomes $r^2 + z^2 = 9$.
   (c) The cone $z = \sqrt{x^2 + y^2}$ becomes $z = r$.
   (d) The plane $x = 2$ becomes $r\cos\theta = 2$ or $r = 2\sec\theta$.
2. If D is the solid between the graphs of $z = low(x, y)$ and $z = high(x, y)$ above the region R in the xy-plane and if R is polar then we have to convert low and $high$ to polar functions $z = low(r, \theta)$ and $z = high(r, \theta)$ in terms of r and/or θ and then
   $\iiint\limits_D f(x, y, z)\ dV = \int_\alpha^\beta \int_{near}^{far} \int_{low}^{high} f(r\cos\theta, r\sin\theta, z)\ r\ dz\ d\theta\ dr.$

§14.6 Triple Integrals in Spherical Coordinates

1. Describe how spherical coordinates work and make sure to mention the conversions:
   (a) $x = \rho\sin\phi\cos\theta$
   (b) $y = \rho\sin\phi\sin\theta$
   (c) $z = \rho\cos\phi$
   (d) $x^2 + y^2 + z^2 = \rho^2$
   (e) $x^2 + y^2 = \rho^2\sin^2\phi$
2. Equations can change here. For example:
   (a) $\rho = 2$ is a sphere.
   (b) The cylinder $x^2 + y^2 = 4$ becomes $\rho = 2\csc\phi$.
   (c) $\phi = \frac{\pi}{4}$ is a cone.
   (d) The plane $z = 3$ becomes $\rho = 3\sec\phi$.
3. To describe a solid in spherical we take a range $\alpha \leq \theta \leq \beta$ and $\gamma \leq \phi \leq \delta$ From the point of view of a person at the origin this describes a "window" looking out. In that window we have a near function $\rho = near(\phi, \theta)$ and a far function $\rho = far(\phi, \theta)$.
4. If D is described this way then
   $\iiint\limits_D f(x, y, z)\ dV = \int_\alpha^\beta \int_\gamma^\delta \int_{near}^{far} f(\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi)\ \rho^2\sin\phi\ d\rho\ d\phi\ d\theta.$
   Don't forget that $\rho^2\sin\phi$. It's the "Jacobian" again.