iii) The sum of the outcome is equal to one, (p + q = 1)

iv) The outcomes of the trials are statistically independent, i.e. the outcome, be it success/failure, do not affect the outcomes of the subsequent trials.

Binomial distribution describes a statistical probability of two possible alternatives to occur, i.e. success or failure.

It is mathematically expressed as:

$$P(x) = {}^{n}C_{x}P^{X}q^{n-x}$$

Where:

$$P(X) = probability\ of\ x\ to\ occur$$

$$x = expected\ value$$

$$p = probability\ of\ success$$

$$q = probability\ of\ failure$$

$$n = sample\ size\ or\ number\ of\ trials$$

Example 1 : A fisher man was able to catch 10 fishes, it was later found out that, 40% 0f the fishes caught by him, were caught alive, then what is the probability that:

i) More than 4 fishes will be caught alive
ii) Less than 4 fishes will be caught alive
iii) Exactly 5 fishes will be caught alive,

solution

i) for more than 4 will be:

$$P(x) = 1 - \left(P(0) + P(1) + P(2) + P(3) + P(4)\right)$$

$$P(0) = P(x) = {}^{n}C_{x}P^{X}q^{n-x}$$

Where:

$$x = 0,\ p = \frac{40}{100} = 0.4,\ \ q = 1 - p = 1 - 0.4 = 0.6.$$

$$P(o) = P(x) = {}^{10}\mathrm{C}_0 0.4^0 0.6^{10-0}$$

Therefore: $P(0) = 1 \times 1 \times 0.0060 = 0.0060$

$$P(1) = \ P(x) = {}^{n}\mathrm{C}_x P^X q^{n-x}$$

$$P(1) = P(x) = {}^{10}\mathrm{C}_1 0.4^1 0.6^{10-1}$$

$$P(1) = 10 \times 0.4 \times 0.0101 = 0.0404.$$

$$P(2) = {}^{10}\mathrm{C}_2 0.4^2 0.6^{10-2}$$

$$45 \times 0.16 \times 0.0168 = 0.12096$$

$$P(3) = {}^{10}\mathrm{C}_3 30.6^{10-3}$$

$$120 \times 0.064 \times 0.0280 = 0.21504$$

$$P(4) = {}^{10}\mathrm{C}_4 0.4^4 0.6^{10-4}$$

$$= 210 \times 0.0256 \times 0.0467 = 0.2511$$

Therefore:

$$P(x > 4) = 1 - (0.0060 + 0.0404 + 0.12096 + 0.21504 + 0.2511)$$

$$P(x > 4) = 1 - 0.6335 = 0.3665$$

Therefore, the probability of more than four fishes to be caught alive is, 0.3665

ii) for less than four fishes to be caught alive will be:

$$P(x < 4) = P(0) + P(1) + P(2) + P(3)$$

$$P(x > 3) = 1 - \left(\left(P(0) + P(1) + P(2)\right) + P(3)\right)$$

$$P(x > 3) = 1 - (0.00277 + 0.00757)$$

$$= 1 - 0.01034$$

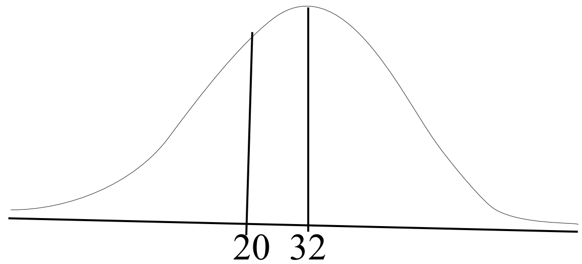
$$P(x > 3) = 0.98966$$

iii) for no error to be committed will be given as
$P(0) = 0.0000454$ *as earlier calculated*.

Probability cannot be conveniently taken if the statistical data is not normally distributed. Statistical probability unveils concepts like, Normal distribution, standard distribution, mathematical expectation and inferential statistics which may be off help to a statistician guiding him in studying data distribution and predicting possible event under normal circumstances. Let us closely discuss the above mentioned concepts.

NORMAL DISTRIBUTION

Normal distribution is a statistical distribution that describes a natural phenomenon. It state how a distribution is supposed to look like under normal circumstances. For a distribution to be normal, the mean must be equal to the median which is equal to the mode, therefore drawing to our notice what is called a symmetric distribution. In a normal distribution most values (data) falls around the mean, while those that fall far from the mean, either greater than the mean, or lower than the mean are regarded as extreme values. A normal distribution can be better explained with a diagram called the normal distribution curve. The normal distribution curve was drafted from the idea of frequency polygon. The height of the curve describe the level of central tendency while the width shows the rate of dispersion,



But due to the complexity of normal distribution values, representing it on the curve is an uphill task. Another easier way of doing this is by converting it into a standard distribution.

Standard distribution: A distribution is said to be a standard distribution if the standard deviation of the distribution is equal to one(1) and the mean of that distribution is equal to zero(0). Just like normal distribution, standard distribution has its curve, known as the standard curve. Area under the standard curve is equated to a unity(1) while the mean is rounded up to zero(0). Just like in the case of probability, the total area under a standard curve is equals to one (1). Converting a normal distribution to a standard distribution can be done using the expression below

$$\boldsymbol{z = \frac{x - \mu}{\sigma}}$$

Where;

$$\boldsymbol{z = the\ standard\ value}$$

$$\boldsymbol{\mu = the\ mean\ value}$$

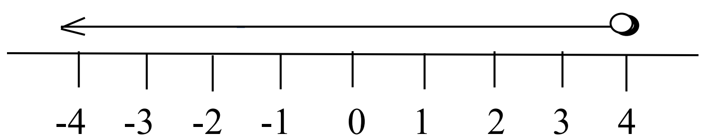
$$\boldsymbol{x = the\ normal\ value}$$

Example 1: convert a distribution of 55 having a mean of 30, with a standard deviation of 20, to a standard distribution and represent it on a curve

$$z = \frac{x - \mu}{\sigma}$$

We might encounter certain situation whereby we will be asked to determine a distribution, that falls under stipulated distribution, or we will be asked to supply probability of a distribution being greater or less than the given distribution, and also represent it on a curve. Lets draw back to the number line in mathematics, because the idea of number line is important in solving similar questions. supposing we are asked to represent the following on a number line.

i. $x < 4$



now lets apply same principle to standard curve. Before we proceed let us note the following

$P(x < z)$ *interprete the probability distribution being less than the given value of z*

$P(x > z)$ *interprete the probability of the distribution being greater than z*

$P(x \leq z)$ *interprete the probability being less than or equal to z*

$P(x \geq z)$ *interprete the probability being greater than or equal to z*

The following are the rules of Probability of a standard curve are

Rules 1: to calculate the probability of a distribution being less than Z (P(x < -z), first take the table value of z, and subtract the corresponding value from 0.5, or 1 – $(t_2 + 0.5)$ where t_z is table value of z.

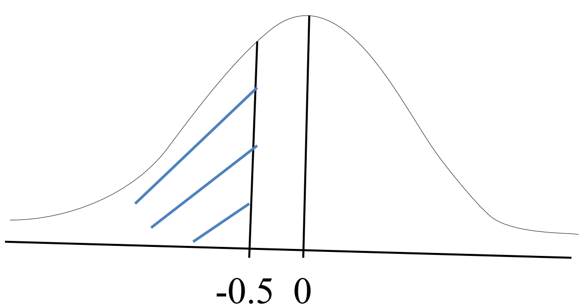
Example: 20,000 students sat for General Examination. Those that pass have a mean age of 21, with a standard deviation of 12 years, calculate the probability of those that will pass at

(i) Below fifteen years
(ii) Above twenty two years
(iii) Above twenty years
(iv) Greater than twenty but less than twenty four years
(v) Between twenty three and twenty five years
(vi) Less than twenty years and greater than twenty four

Solution

i) $x = 15$ $\mu = 21$ $\sigma = 12$

$$z = \frac{x-\mu}{\sigma} = \frac{15-21}{12} = \frac{-6}{12} = -0.5$$



The probability of those passing the exam at below 15 years is

$P(x < -z) = 0.5 - t_z = 0.5 - 0.1915 = 0.3085 \times 20{,}000 = 6{,}170.$

Therefore 6,170 students below the age of 15 are expected to pass the examination.

ii) Above twenty two years.

Whereas Profit

$$P = TR - TC$$

Let TR be the weighted Profits $\in (x)$ =$P_P X_P$

For A$\in (x) = 5{,}000 \times 0.6 = 3{,}000$

and TC be the weighted loss $\in (x) = P_l X_l$

$$= 0.4 \times 2000$$

$$= 800$$

$$\therefore\ P = TR - TC = 3{,}000 - 800 = \$2{,}200$$

For business B

$$P = TR - TC$$

$TR = \in (xp) = P_P X_P = 15{,}000 \times 0.3 = 4{,}500$

$TC = \in (x1) = P_L X_L = 6{,}000 \times 0.5 = 3{,}000$

$\therefore\ P = 4{,}500 - 3{,}000 = \$1{,}500$

For Business C

$P = TR\ - TC$

$TR = \in (Xp)p = Pp \times Xp = 4{,}800 \times 0.7$

=3,360

$TC = \in (Xl) = Pl \times l = 1{,}500 \times 0.2 = 300$

$\therefore \cap = 3{,}360 - 300 = 3{,}060$