AP Calculus **Exam Study Guide**

Derivatives

Constant Rule $\frac{d}{dx}[c]=0$	Constant Multiple Rule $\frac{d}{dx}[cu]=cu'$
Power Rule $\frac{d}{dx}[x^n]=nx^{n-1}$	Sum and Difference Rules $\frac{d}{dx}[f\pm g]=f'\pm g'$
Product Rule $\frac{d}{dx}[fg]=gf'+fg'$	Quotient Rule $\frac{d}{dx}\left[\frac{f}{g}\right]=\frac{gf'-fg'}{g^2}$
Chain Rule $\frac{d}{dx}[f(u)]=f'(u)\cdot u'$	Inverse $\left(f^{-1}\right)'(x)=\frac{1}{f'(f^{-1}(x))}$

$\frac{d}{dx}[\sin x]=\cos x$	$\frac{d}{dx}[\sec x]=\sec x\tan x$	$\frac{d}{dx}[\tan x]=\sec^2 x$
$\frac{d}{dx}[\cos x]=-\sin x$	$\frac{d}{dx}[\csc x]=-\csc x\cot x$	$\frac{d}{dx}[\cot x]=-\csc^2 x$
$\frac{d}{dx}[e^u]=e^u\cdot u'$	$\frac{d}{dx}[\ln u]=\frac{1}{u}\cdot u'$	$\frac{d}{dx}[a^u]=(\ln a)a^u\cdot u'$
$\frac{d}{dx}[arc\sin u]=\frac{u'}{\sqrt{1-u^2}}$	$\frac{d}{dx}[\arccos u]=\frac{-u'}{\sqrt{1-u^2}}$	$\frac{d}{dx}[\arctan u]=\frac{u'}{1+u^2}$

Definition

If $y=f(x)$ then the derivative is defined to be

$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ or

$f'(x)=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$

The derivative at a point x=a where is defined as $f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$

Notation

If $y=f(x)$ then all of the following are equivalent notations for the derivative.

$$f'(x)=y'=\frac{df}{dx}=\frac{dy}{dx}=\frac{d}{dx}\left(f(x)\right)=Df(x)$$

If $y=f(x)$ all of the following are equivalent notations for derivative evaluated at $x=a$.

$$f'(a)=y'\Big|_{x=a}=\frac{df}{dx}\Big|_{x=a}=\frac{dy}{dx}\Big|_{x=a}=Df(a)$$

Interpretation of the Derivative

If $y=f(x)$ then,

1.) $m=f'(a)$ is the slope of the tangent line to $y=f(x)$ at $x=a$ and the equation of the tangent line at $x=a$ is given by $y=f(a)+m(x-a)$. This formula can also be written as $y=y_1+m(x-x_1)$ where $m=f'(x_1)$

2.) $f'(a)$ is the **instantaneous rate of change** of $f(x)$ at $x=a$. This is sometimes referred to as the slope of the curve f at $x=a$. **Average rate of change** can be found using the formula for the slope of a line $\left(\frac{\Delta y}{\Delta x}\right)$.

Differentiability

A function is not differentiable at a point x=a (meaning the derivative does not exist at x=a) if the function 1.) is not continuous at x=a (vertical asymptotes, POD's, etc.) 2.) has a sharp point at x=a (change in slope (+/-) with no horizontal tangent) 3.) has a vertical tangent has a vertical tangent at x=a (slope of the tangent is undefined). If a function is differentiable then the function is also continuous at x=a. If the function is continuous, it may be differentiable but that is not certain.