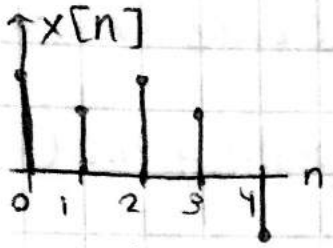
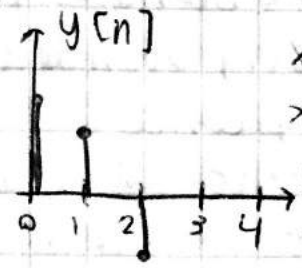
$y[n] = x[Mn]$ M is a positive constant

$y_1[n] = x[Mn - n_0] = x_1[Mn]$



if $M=2$ $\Longrightarrow$

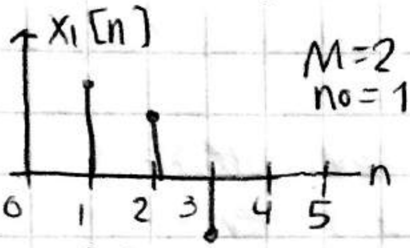


$x[Mn]$

$x[2 \cdot 0] = x[0]$

$x[2 \cdot 1] = x[2]$

$x[2 \cdot 2] = x[4]$



$M=2$

$n_0 = 1$

$$y_2[n-n_0] = x[M(n-n_0)]$$
$$= x[Mn - Mn_0] \neq y_1[n]$$

Not invariant system

- Causality -

For every choice of n_0 the output sequence value at the index $n = n_0$ depends only on the input sequence values for $n \leq n_0$, that is the system is nonanticipative.

$y[n] = x[n - n_d]$

Casual $n_d \geq 0$ $\Rightarrow$ $y[2] = x[-1]$ $n_d = 3$ no dependen de valores futuros

Non casual $n_d < 0$ $y[2] = [2+3] = x[5]$ $n_d = -3$

$y[n] = (x[n])^2$ casual

$w[n] = \log_{10}(|x[n]|)$ casual

$y[n] = x[Mn]$ if $M > 1$ the system is non casual,

$\Downarrow$ it depends of a future value.

$y[3] = x[6]$

$y[n] = x[n+1] - x[n]$ non casual

$y[n] = x[n] - x[n-1]$ casual.

- Memoryless -

Is memoryless if the output $y[n]$ at every value of n depends only on the input $x[n]$ at the same value of n.

$y[n] = x(x[n])^2$ memoryless you don't need to storage values

$y[n] = x[n - n_d]$ not memoryless

right ·

Difference Equations

17/03/16

$y(n) = 0.9y(n-1) - 0.81y(n-2) + x(n)$ $x(n)$ step function, causal.

$y(-2) = y(-1) = 1$

$y(n) - 0.9y(n-1) + 0.81y(n-2) = x(n)$

$z^{+}\{y(n) - 0.9y(n-1) + 0.81y(n-2)\} = z^{+}\{x(n)\}$

$$x^{+}(z) = y^{+}(z) - 0.9z^{-1}[y^{+}(z) + \underbrace{y(-1)z}_{\text{sumatoria}}] + 0.81z^{-2}[y^{+}(z) + \underbrace{(-1)z + (-2)z^{2}}_{\text{sumatoria}}]$$

Causality & Stability.

28/mar/16

$$H(z) = \sum_{n=-\infty}^{\infty} h(n)z^{-n} \Rightarrow |H(z)| = \left|\sum_{-\infty}^{\infty} h(n)z^{-n}\right| \leq \sum_{-\infty}^{\infty} |h(n)||z^{-n}|$$

If $|z| = 1 \Rightarrow |H(z)| \leq \sum_{-\infty}^{\infty} |h(n)|$

>> LTI stable:

$$|H(z)| \leq \sum_{-\infty}^{\infty} |h(n)| < \infty$$

Example

1. $H(z) = \dfrac{3-4z^{-1}}{1-3.5z^{-1}+1.5z^{-2}}$ a) Stable system b) Causal c) Anticausal

↳ debe contener el círculo unitario

b) ↑ ROC $|z|>3$

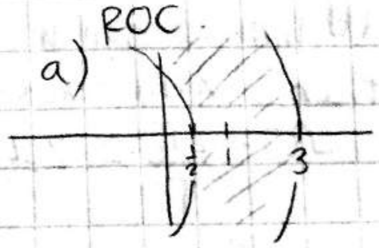
c) ↑ ROC $|z|<1/2$

a) $H(z) = \dfrac{A}{(1-1/2z^{-1})} + \dfrac{B}{(1-3z^{-1})}$

$A = \dfrac{3-4z^{-1}}{1-3z^{-1}}\Big|_{z=1/2} = 1$ $B = \dfrac{3-4z^{-1}}{1-1/2z^{-1}}\Big|_{z=3} = 2$

$H(z) = \dfrac{1}{(1-1/2z^{-1})} + \dfrac{2}{(1-3z^{-1})}$

a) ROC



$h(n) = z^{-1}\{H(z)\}$

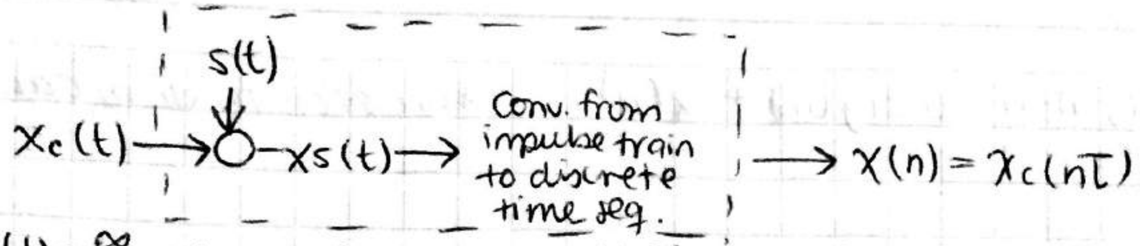
$= \left(\frac{1}{2}\right)^{n} u(n) - 2(3)^{n} u(-n-1)$

b) $h(n) = (1/2)^{n}u(n) + 2(3)^{n}u(n)$

c) $h(n) = -(1/2)^{n}u(-n-1) - 2(3)^{n}u(-n-1)$

Sampling

28/03/16



→ C/D converter

$$s(t)=\sum_{n=-\infty}^{\infty}\delta(t-nT)$$

$$x_s(t)=x_c(t)\cdot s(t)=x_c(t)\cdot\sum\delta(t-nT)=\sum x_c(t)\delta(t-nT)$$
$$=\sum x_c(nT)\delta(t-nT)$$

*Fourier Transform *para señal s(t) que es continua.*

- $S(j\Omega)=\frac{2\pi}{T}\sum_{n=-\infty}^{\infty}\delta(\Omega-k\Omega_s)$

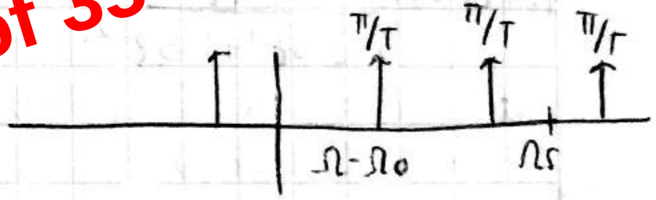
$\Omega_s=\frac{2\pi}{T}$ (rad/s)

- $X_s(j\Omega)=\frac{1}{2\pi}X_c(j\Omega)*S(j\Omega)$

$X_s(j\Omega)=\frac{1}{T}\sum_{k=-\infty}^{\infty}X_c(j\Omega-k\Omega_s)$ $\{\Omega_s\geq2\Omega_N\}$

Example

1. $X_c(t)=\cos(\Omega_0 t)$
   $X_c(j\Omega)=\pi\delta(\Omega-\Omega_0)+\pi\delta(\Omega+\Omega_0)$



NYQVIST-SHANNON

31/03/16

$x_c(t)$ limited signal, $X_c(j\Omega)=0$; $|\Omega|\geq\Omega_N$ ← *señal original*

then: $x(n)=x_c(nT)$ $n=0,\pm1,\pm2\ldots$

$\Omega_s=\frac{2\pi}{T}\geq2\Omega_N$ Ω_N *Nyquist frequency.* $2\Omega_N$ *Nyquist rate.*

$$X_s(j\Omega)=\sum_{n=-\infty}^{\infty}x_c(nT)e^{-j\Omega Tn}$$ since $x(n)=x_c(nT)$

and $X(e^{jw})=\sum_{-\infty}^{\infty}x(n)e^{-jwn}$

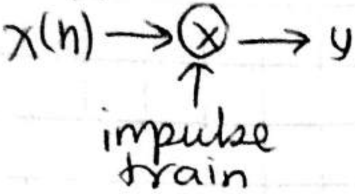
$X_s(j\Omega)=X(e^{jw})\big|_{w=\Omega T}$

» $w=\Omega T$ «

$$\left\{\begin{array}{l}X(e^{j\Omega T})=\frac{1}{T}\sum_{n=-\infty}^{\infty}X_c(j(\Omega-k\Omega_s))\\X(e^{jw})=1/T\sum_{-\infty}^{\infty}X_c(j(w/T-k2\pi/T))\end{array}\right\}$$

>> Reconstrucción de la señal <<

* LPF



$$xs(t) = \sum_{-\infty}^{\infty} x(n)\,\delta(t-nT)$$

then the signal goes to a LPF

$$xr(t) = \sum_{-\infty}^{\infty} x(n)\,hr(t-nT)$$

$$hr(t) = \frac{sen(\pi t/T)}{\pi t/T}$$ Fiter (LPF)

$$Xr(t) = \sum_{-\infty}^{\infty} x(n)\,\frac{sen(\pi(t-nT)/T)}{\pi(t-nT)/T}$$

$hr(0) = 1$

$hr(nT) = 0$ por $n = \pm 1, \pm 2 \ldots$ $\quad Xr(mT) = \chi c(mT)$

* zero-order hold.

$$H(jw) = e^{-jwT/2}\left(\frac{2\,sen(wT/2)}{w}\right)$$

Reduction in sampling

11 / 04/2016

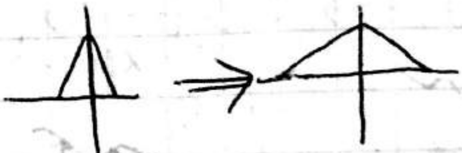
$Xb[n] = x[nN]$

$x[nN] = Xb[n]$

$xb[n] = x[nN]$

$$Xb(e^{jw}) = \sum_{K=-\infty}^{\infty} Xb[K]\,e^{-jwK} = \sum_{K=-\infty}^{\infty} x[KN]\,e^{-jwK} \qquad K = n/N$$

$$\gg Xb(e^{jw}) = \sum_{n=-\infty}^{\infty} x(n)\,e^{-jw\frac{n}{N}} = \underset{\sim\sim}{X(e^{j\frac{w}{N}})} \ll$$



Increasing sampling frequency

$$Xe(n) = \begin{cases} x(n/L) & n = 0, \pm 1, \pm 2 \ldots \\ 0 & \text{otherwise} \end{cases}$$

$$Xe(e^{jw}) = \sum_{n=-\infty}^{\infty} Xe(n)\,e^{-jwn} = \underset{\sim\sim}{X(e^{-jwL})}$$

Zero location of Linear Phase FIR.

18/04/16

$$H(z) = \sum_{n=0}^{N} h[N-n] z^{-n} \quad \text{making } m=N-n \Rightarrow H(z) = z^{-N} \sum_{m=0}^{N} h[m] z^{m}$$

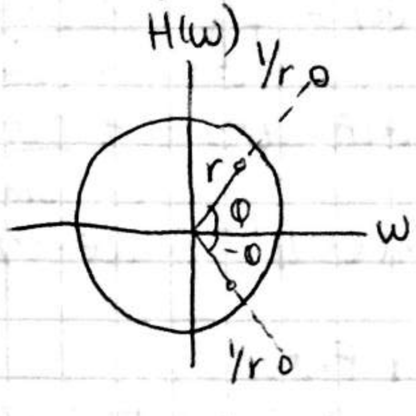
- symmetric $\curvearrowright$ $= z^{-N} H(z^{-1})$

- Asymmetric impulse response

$$H(z) = -\sum_{n=0}^{N} h[N-n] z^{-n} \Rightarrow H(z) = -z^{-N} H(z^{-1})$$

If $z_1 = re^{j\theta}$ is a zero of $H(z)$:

$\frac{1}{z_1} = \frac{1}{r} e^{-j\theta}$ is a zero of $H(z)$



>> Systems with real coefficients:

if $z_1 = re^{j\theta}$ $\quad z = re^{\pm j\theta}$

$z_2 = re^{-j\theta}$ $\quad z = \frac{1}{r} e^{\pm j\theta}$

a) Type 1 FIR filter

Either an even number or no zeros at $z=1$ & $z=-1$

b) Type 2 FIR filter

Either an even number or no zeros at $z=1$ and odd number of zeros at $z=-1$

$H(z) = (-1)^N H(z^{-1})$ *even length $h(n) = h(N-1)$*

$H(-1) = (-1)^N H((-1)^{-1})$

c) Type 3 FIR filter

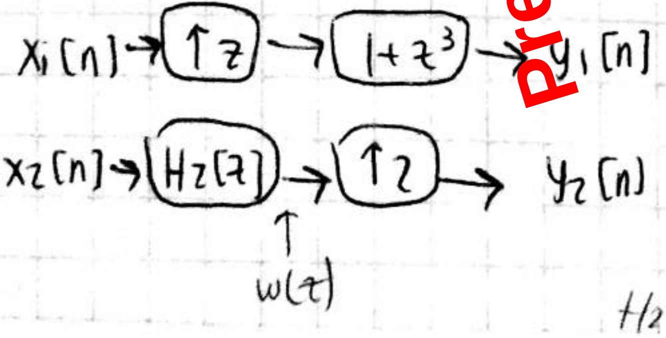
An odd number of zeros at $z=1$ and $z=-1$

$H(1) = -(-1)^N H(1)$

$H(-1) = -(-1)^N H(-1)$ *because N is even*

d) Type 4 FIR filter

Odd number of zeros at $z=1$ and either an even number or no zeros at $z=-1$



$y_1(z) = X_1(z^2) + z^3 X_1(z^2$

$y_2(z) = X_1(z^2) + z^3 X_1(z^2)$

$w(z) = X_1(z)(1 + z^{3/2})$

$H_2(z) = 1 + z^{\textcircled{3/2}}$

$H_2(z)$ no existe porque el índice debe ser entero