Differentiating from first principles

Gradient of chord:

$$m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\Rightarrow \frac{f(x+h) - f(x)}{(x-h) - x}$$

$$f'(x) = \lim_{h \to 0} \Rightarrow \frac{f(x+h) - f(x)}{h}$$

Simplify, then let $h = 0$ to give the derivative of $f(x)$

Example:

$$f(x) = \sqrt{x}$$

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$\Rightarrow \frac{\sqrt{x+h} - \sqrt{x}}{h} = \frac{\sqrt{x+h} - \sqrt{x}}{h} \times \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

- we have multiplied the numerator to get a factor of h → to prevent the denominator tending to zero
- remove square roots by multiplying top and bottom of the fraction by $\sqrt{x+h} + \sqrt{x}$ & using the difference of two squares

$$\Rightarrow \frac{(x+h) - (x)}{h(\sqrt{x+h} + \sqrt{x})}$$

$$\Rightarrow \frac{h}{h(\sqrt{x+h} + \sqrt{x})}$$

$$\Rightarrow \frac{1}{\sqrt{h+x} + \sqrt{x}}$$

divided top and bottom by h

let $h \to 0$...

$$\therefore f'(x) = \lim_{h \to 0} \frac{1}{\sqrt{h+x} + \sqrt{x}}$$

$$\Rightarrow \frac{1}{\sqrt{x} + \sqrt{x}}$$

$$\Rightarrow \frac{1}{2\sqrt{x}}$$