Interval Estimation & Hypothesis testing

Confidence Intervals

• Confidence intervals for the population mean, μ
– when population standard deviation σ is known

– when population standard deviation σ is unknown

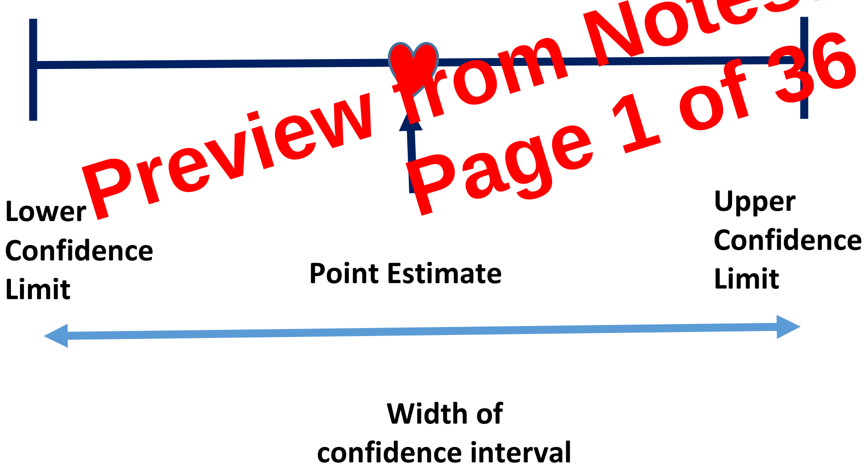
• Confidence intervals for the population proportion, p

• Determining the required sample size

Point and Interval Estimates

• A point estimate is a single number

• A confidence interval provides additional information about variability



Point Estimates

We can estimate a Population Parameter …		with a Sample Statistic (a Point Estimate)
Mean	μ	$\overline{X}$ **Error! Bookmark not defined.**
	π	

Confidence Interval for μ (σ Known)

• Assumptions
– Population standard deviation σ is known
– Population is normally distributed
– If population is not normal, use large sample
• **Confidence interval estimate**

$$\bar{X} \pm Z \frac{\sigma}{\sqrt{n}}$$

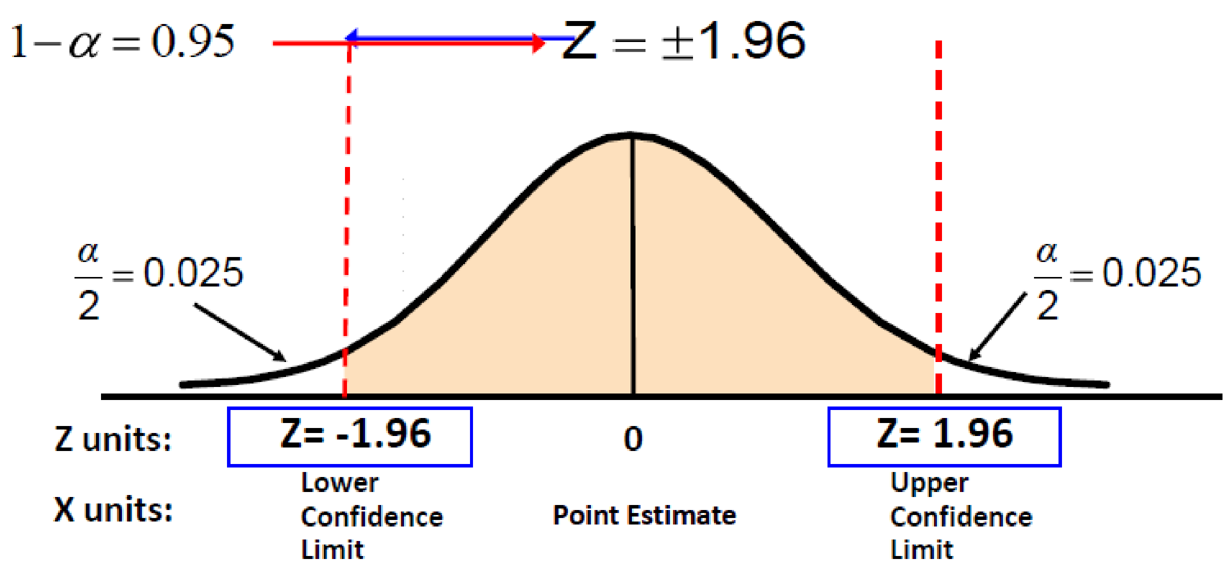
where $\bar{X}$ is the point estimate

Z is the normal distribution critical value for a probability of $\alpha/2$ in each tail

$\sigma/\sqrt{n}$ is the standard error

Finding the Critical Value, Z

• Consider a 95% confidence interval



Common Levels of Confidence

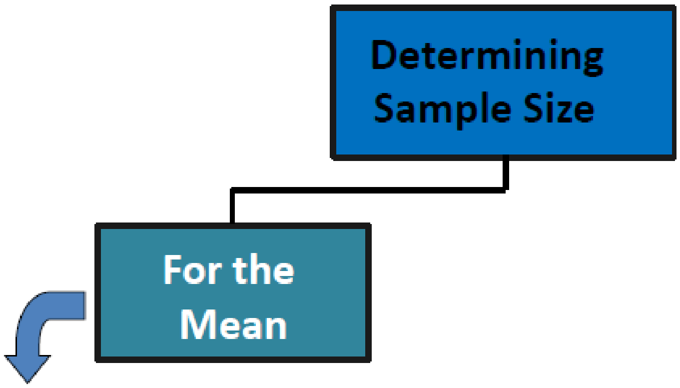
• Commonly used confidence levels are 90%, 95% and 99%

Confidence Level	Confidence Coefficient, $1-\alpha$	Z value
80%	0.80	1.28
90%	0.90	1.645
95%	0.95	1.96
98%	0.98	2.33
99%	0.99	2.58
99.8%	0.998	[illegible]
99.9%	0.999	3.27

Intervals and Level of Confidence

Sampling Distribution of the Mean

Determining Sample Size



$$e = Z\frac{\sigma}{\sqrt{n}}$$

Now solve for n to get

$$n = \frac{Z^2\sigma^2}{e^2}$$

Determining Sample Size

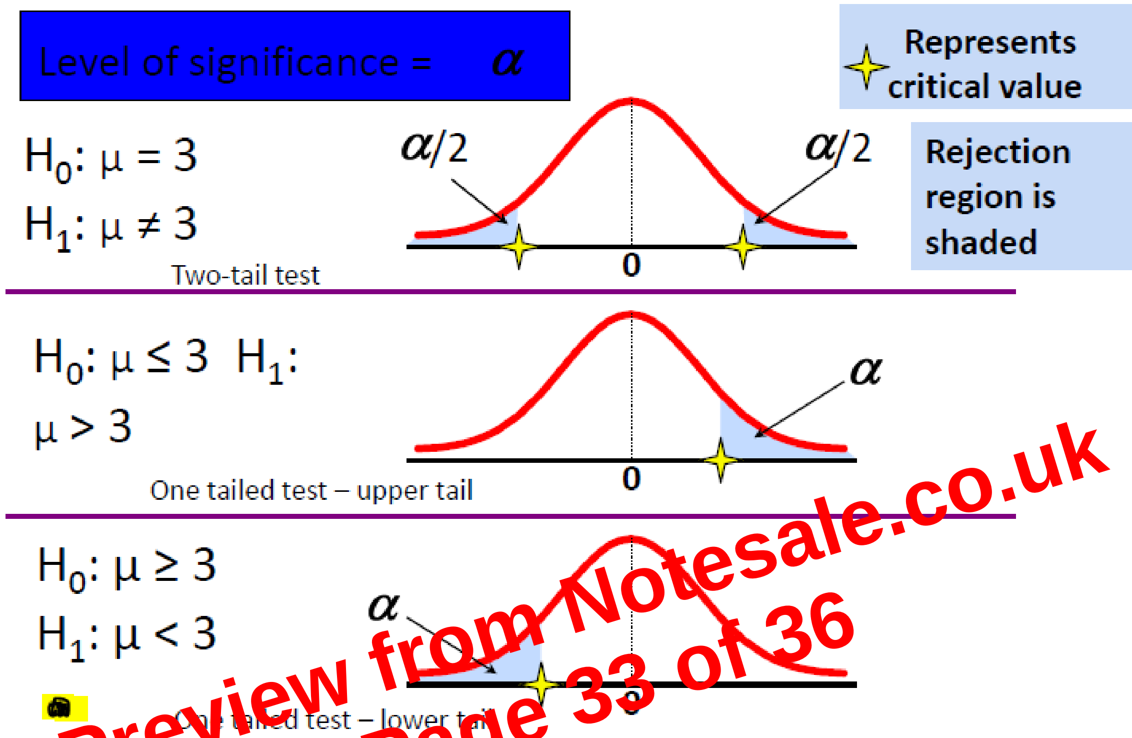
- To determine the required sample size for the mean, you must know

– The desired level of confidence (1 -), which determines the critical Z value

– The acceptable sampling error, e

– The standard deviation, σ

Level of Significance and the Rejection Region



Formal 5-step procedure (reduced from text 6 steps)

Step 1: State null and alternative hypotheses
Step 2: State the decision rule
Step 3: Calculate the test statistic
Step 4: Compare and make a decision
Step 5: State a conclusion
There are two approaches: critical values & p-value

Test 2: t test on one population mean

- A serious problem facing strawberry growers is the control of nematodes. After fumigation the yield currently averages 3kg of fruit per plot. A new fumigant is applied to 8 plots which yield an average of 3.3 kg with a standard deviation of 0.6 kg.
- Is there sufficient evidence to conclude the new fumigant results in a higher yield? Assume the yields are approx N and use 5% significance level.
- Given: $\mu = 3$, $n = 8$, $X = 3.3$, $s = 0.6$, $\alpha = 5\% = 0.1$
- Test if new fumigant results in $\mu > 3$

Test 2: solution

- H_0: $\mu = 3$ (yield is 3kg) or $\mu \leq 3$

  H_a: $\mu > 3$ (yield is higher 3kg)
- Decision rule: reject H_0 if $t_{calc} > t_{0.05,7} = 1.895$
- Test statistics

$$t_{calc} = \frac{\overline{X} - \mu}{S/\sqrt{n}} = \frac{3.3 - 3}{0.6/\sqrt{8}} = 1.414$$

- Decision: since t_{cal} not greater than 1.895 → not reject H_0
- Conclusion: Insufficient evidence at the 10% level of significance, to conclude that the new fumigant results in better yield.