## Arden's Theorem

In order to find out a regular expression of a Finite Automaton, we use Arden's Theorem along with the properties of regular expressions.

***Statement*** −

Let **P** and **Q** be two regular expressions.

If **P** does not contain null string, then **R = Q + RP** has a unique solution that is **R = QP***

**Proof −**

R = Q + $Q + RP$P [After putting the value R = Q + RP]

= Q + QP + RPP

When we put the value of **R** recursively again and again, we get the following equation −

R = Q + $Q^P + QP^2 + QP^3$.....

R = Q (ε + P + $P^2 + P^3$ + .... )

R = $QP^*$ [As P* represents (ε + P + $P^2 + P^3$ + ....) ]

Hence, proved.



## Assumptions for Applying Arden's Theorem −

- The transition diagram must not have NULL transitions
- It must have only one initial state

## Method

**Step 1** − Create equations as the following form for all the states of the DFA having **n** states with initial state $q_1$.

$q_1 = q_1R_{11} + q_2R_{21} + \ldots + q_nR_{n1} + \varepsilon$

$q_2 = q_1R_{12} + q_2R_{22} + \ldots + q_nR_{n2}$

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$q_n = q_1R_{1n} + q_2R_{2n} + \ldots + q_nR_{nn}$

$\mathbf{R_{ij}}$ represents the set of labels of edges from $\mathbf{q_i}$ to $\mathbf{q_j}$**,** if no such edge exists, then $\mathbf{R_{ij} = \varnothing}$

**Step 2** − Solve these equations to get the equation for the final state in terms of $\mathbf{R_{ij}}$

## Problem

Construct a regular expression corresponding to the automata given below −