

Similarly 1 kg-mass on earth surface experiences a force of

$$F = \frac{6.673 \times 10^{-11} \times 1 \times 5.96504 \times 10^{24}}{(6371 \times 10^3)^2} = 9.80665 \text{ N}$$

Since, mass of earth = 5.96504×10^{24} kg

and radius of earth = 6371×10^3 m.

This force of attraction is always directed towards the centre of earth.

In common usage the force exerted by a earth on a body is known as weight of the body. Thus weight of 1 kg-mass on/near earth surface is 9.80665 N, which is approximated as 9.81 N for all practical problems. Compared to this force the force exerted by two bodies on each other is negligible. Thus in statics:

- Weight of a body = mg
- Its direction is towards the centre of the earth, in other words, vertically downward.
- The force of attraction between the other two objects on the earth is negligible.

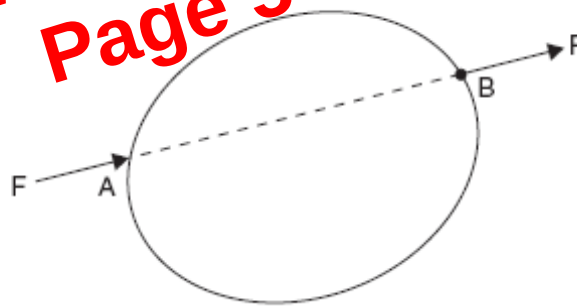
5. PRINCIPLE OF PHYSICAL INDEPENDENCE OF FORCES

It states, If a number of forces are simultaneously acting on a *particle, then the resultant of these forces will have the same effect as produced by all the forces. □

6. Law of Transmissibility

According to this law the state of rest or motion of the rigid body is unaltered if a force acting on the body is replaced by another force of the same magnitude and direction but acting anywhere on the body along the line of action of the replaced force.

Let F be the force acting on a rigid body at point A as shown in Fig. 2.4. According to this law, this force has the same effect on the state of body as the force F applied at point B, where AB is in the line of force F .



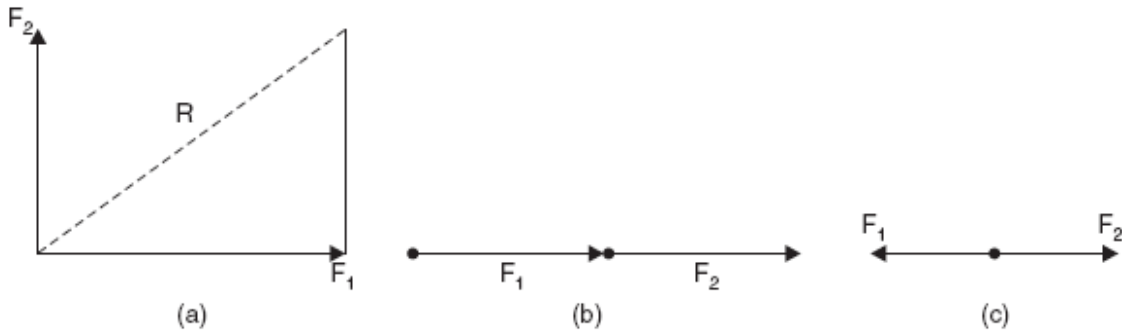
In using law of transmissibility it should be carefully noted that it is applicable only if the body can be treated as rigid. Hence if we are interested in the study of internal forces developed in a body, the deformation of body is to be considered and hence this law cannot be applied in such studies.

Q.6 Distinguish clearly between composition of forces and resolution of forces.

The process of finding a single Force which will have the same effect as a set of Forces acting on a body is known as composition of Forces. The resolution of Forces is exactly the opposite process of composition i.e., it is the process of finding two or more Forces which will have the same effect as that of a Force acting on the body.

Q.7 what are the various types of forces on a body?

Before taking up equilibrium conditions of a body, it is necessary to identify the various forces acting on it. The



B) TRIANGLE METHOD OR TRIANGLE LAW OF FORCES

According to triangle law or method—If two forces acting simultaneously on a particle be represented (in magnitude and direction) by the two sides of a triangle taken in order their resultant is represented (in magnitude and direction) by the third side of triangle taken in opposite order.

OR

If two forces are acting on a body such that they can be represented by the two adjacent sides of a triangle taken in the same order, then their resultant will be equal to the third side (enclosing side) of that triangle taken in the opposite order.

The resultant force (vector) can be obtained graphically and analytically or trigonometry.

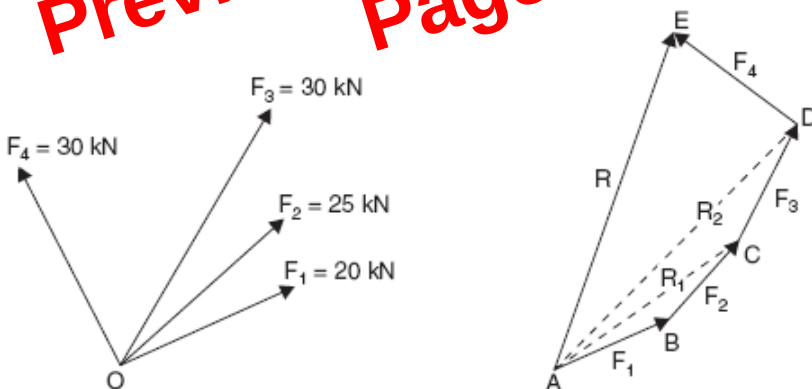
C) POLYGON METHOD

According to this method—if more than two forces acting on a particle be represented by the sides of a polygon taken in order their resultant will be represented by the closing side of the polygon in opposite direction—

OR

If more than two forces are acting on a body such that they can be represented by the sides of a polygon Taken in same order, then their resultant will be equal to that side of the polygon, which completes the polygon (closing side taken in opposite order).

The resultant of such forces can be determined by graphically and analytically.



2. METHOD OF RESOLUTION FOR THE RESULTANT FORCE

1. Resolve all the forces horizontally and find the algebraic sum of all the horizontal components (i.e., ΣH).
2. Resolve all the forces vertically and find the algebraic sum of all the vertical components (i.e., ΣV).
3. The resultant R of the given forces will be given by the equation :

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

4. The resultant force will be inclined at an angle θ , with the horizontal, such that

the given force F at A is replaced by a force F at B and a moment Fd .

Q. 19 Define Equilibrium. Enumerate principle of equilibrium.

if the resultant of a number of forces, acting on a particle is zero, the particle will be in equilibrium. Such a set of forces, whose resultant is zero, are called equilibrium forces.

The force, which brings the set of forces in equilibrium is called an equilibrant.

PRINCIPLES OF EQUILIBRIUM

Though there are many principles of equilibrium, yet the following three are important from the subject point of view :

1. Two force principle. As per this principle, if a body in equilibrium is acted upon by two forces, then they must be equal, opposite and collinear.
2. Three force principle. As per this principle, if a body in equilibrium is acted upon by three forces, then the resultant of any two forces must be equal, opposite and collinear with the third force.
3. Four force principle. As per this principle, if a body in equilibrium is acted upon by four forces, then the resultant of any two forces must be equal, opposite and collinear with the resultant of the other two forces.

Q. 20 Explain condition of equilibrium.

CONDITIONS OF EQUILIBRIUM

Consider a body acted upon by a number of coplaner non-concurrent forces. A little consideration will show, that as a result of these forces, the body may have any one of the following states:

1. The body may move in any one direction.
2. The body may rotate about itself without moving.
3. The body may move in any one direction and at the same time it may also rotate about itself.
4. The body may be completely at rest.

Now we shall study the above mentioned four states one by one.

1. If the body moves in any direction, it means that there is a resultant force acting on it. A little consideration will show, that if the body is to be at rest or in equilibrium, the resultant force causing movement must be zero. Or in other words, the horizontal component of all the forces (ΣH) and vertical component of all the forces (ΣV) must be zero. Mathematically,

$$\Sigma H = 0 \text{ and } \Sigma V = 0$$

2. If the body rotates about itself, without moving, it means that there is a single resultant couple acting on it with no resultant force. A little consideration will show, that if the body is to be at rest or in equilibrium, the moment of the couple causing rotation must be zero. Or in other words, the resultant moment of all the forces (ΣM) must be zero. Mathematically,

$$\Sigma M = 0$$

3. If the body moves in any direction and at the same time it rotates about itself, it means that there is a resultant force and also a resultant couple acting on it. A little consideration will show, that if the body is to be at rest or in equilibrium, the resultant force causing movements and the resultant moment of the couple causing rotation must be zero. Or in other words, horizontal component of all the forces (ΣH), vertical component of all the forces (ΣV) and resultant moment of all the forces (ΣM) must be zero. Mathematically,

$$\Sigma H = 0 \text{ } \Sigma V = 0 \text{ and } \Sigma M = 0$$

4. If the body is completely at rest, it necessarily means that there is neither a resultant force nor a couple acting on it. A little consideration will show, that in this case the following conditions are already satisfied :

$$\Sigma H = 0 \text{ } \Sigma V = 0 \text{ and } \Sigma M = 0$$

The above mentioned three equations are known as the conditions of equilibrium.

Q.21 state and prove LAMI's Theorem.

It states, "If three coplanar forces acting at a point be in equilibrium, then each force is proportional to the sine of the angle between the other two." Mathematically,

Where,

I_{AB} = moment of inertia about axis AB

I_{GG} = moment of inertia about centroidal axis GG

parallel to AB.

A = the area of the plane figure given and

y_c = the distance between the axis AB and the parallel centroidal axis GG.

Proof: Consider an elemental parallel strip dA at a distance y from the centroidal axis

Then,

$$\begin{aligned} I_{AB} &= \Sigma(y + y_c)^2 dA \\ &= \Sigma(y^2 + 2y y_c + y_c^2) dA \\ &= \Sigma y^2 dA + \Sigma 2y y_c dA + \Sigma y_c^2 dA \end{aligned}$$

Now,

$$\begin{aligned} \Sigma y^2 dA &= \text{Moment of inertia about the axis } GG \\ &= I_{GG} \end{aligned}$$

$$\begin{aligned} \Sigma 2y y_c dA &= 2y_c \Sigma y dA \\ &= 2y_c A \frac{\Sigma y dA}{A} \end{aligned}$$

In the above term $2y_c A$ is constant and $\frac{\Sigma y dA}{A}$ is the distance of centroid from the reference axis GG . Since GG is passing through the centroid itself $\frac{\Sigma y dA}{A}$ is zero and hence the term $\Sigma 2y y_c dA$ is zero.

Now, the third term,

$$\Sigma y_c^2 dA = y_c^2 \Sigma dA$$

$$= A y_c^2$$

$$\therefore I_{AB} = I_{GG} + A y_c^2$$

Note: The above equation cannot be applied to any two parallel axis. One of the axis (GG) must be centroidal axis only.

Q.33 Derive expression for Moment of Inertia of different sections.

$$\begin{aligned}
 &= \int_0^R \int_0^{2\pi} \frac{(1 - \cos 2\theta)}{2} d\theta dr \\
 &= \int_0^R \frac{r^3}{2} \left[\theta - \frac{\sin 2\theta}{2} \right]_0^{2\pi} dr \\
 &= \left[\frac{r^4}{8} \right]_0^R [2\pi - 0 + 0 - 0] = \frac{2\pi}{8} R^4 \\
 I_{xx} &= \frac{\pi R^4}{4}
 \end{aligned}$$

If d is the diameter of the circle, then

$$R = \frac{d}{2}$$

$$\therefore I_{xx} = \frac{\pi}{4} \left(\frac{d}{2} \right)^4$$

$$I_{xx} = \frac{\pi d^4}{64}$$

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