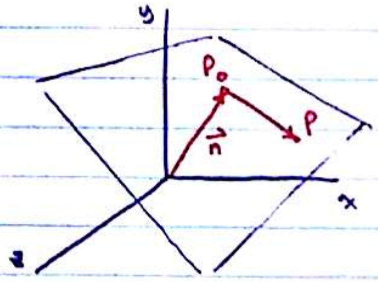
Plane in $\mathbb{R}^3$:



$\vec{n} \cdot (P - P_0) = 0$ (orthogonal vectors)

- Parametric Form: $\begin{bmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$
- $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

Observations:

- let $ax + by + cz = d$ be a plane
  - ↳ THEN $\vec{n} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ is orthogonal to the plane.
- 2 planes are parallel if the normal vectors are scalar multiples of each other ($\vec{n}_1 = c\vec{n}_2$)

ORTHOGONALITY:

- $\vec{v}$ and $\vec{w}$ are orthogonal if angle between them is $90°\,(\frac{\pi}{2})$. (A.K.A. the dot product is equal to 0, $\vec{v} \cdot \vec{w} = 0$)

i.e. $\begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} = 3 - 1 - 2 = 0$ ∴ Orthogonal

CROSS PRODUCT:

- let $\vec{v} = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$, $\vec{w} = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$
- $\vec{v} \times \vec{w} = \begin{bmatrix} y_1 z_2 - y_2 z_1 \\ x_1 z_2 - z_1 x_2 \\ x_1 y_2 - y_1 x_2 \end{bmatrix}$

PROPERTIES:

- $\vec{v} \times \vec{w} = 0$ IF $\vec{v} \parallel \vec{w}$
- $(\vec{v} \times \vec{w})$ is orthogonal to $\vec{v}$ & $\vec{w}$
- $\vec{v} \times (\vec{w} + \vec{u}) = \vec{v} \times \vec{w} + \vec{v} \times \vec{u}$
- $(\vec{v} + \vec{w}) \times \vec{u} = \vec{v} \times \vec{u} + \vec{w} \times \vec{u}$

Dimension

- $\dim(S)$ = # of vectors in a basis of S

Note: $\dim(\mathbb{R}^n) = n$ [standard basis $\{e_1, e_2 \ldots e_n\}$]

- $S \subseteq U$ and S, U are subspaces of $\mathbb{R}^n$

↳ $\dim(S) \leq \dim(U)$

- $\text{col}(A)$ = span of columns of matrix A

↳ $\dim(\text{col}(A)) = \text{rank}(A)$

* The statements are equivalent:

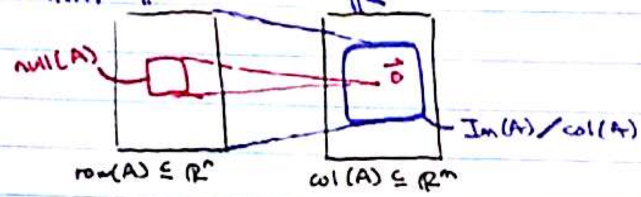
- $A_{n \times n}$ is invertible
- $\text{rank}(A) = n \iff \dim(\text{col}(A)) = \dim(\text{row}(A)) = n$
- All the columns are linearly independent
- $\text{span}\{c_1, c_2, c_3 \ldots c_n\} = \mathbb{R}^n$

- $\text{row}(A)$ = span of rows of matrix A in REF

↳ $\dim(\text{row}(A)) = \text{rank}(A)$

↳ $\text{row}(A) = \text{col}(A^T)$

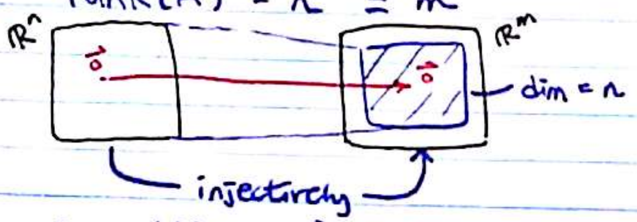
- $\dim(\text{null}(A)) = \dim(\text{null}(A\vec{V}))$ where $\vec{V}$ is $n \times n$ invertible & A is $m \times n$ matrix
- $A_{m \times n} : \mathbb{R}^n \longrightarrow \mathbb{R}^m$



- $\text{rank}(A) = \text{rank}(AV)$
  ↳ because V is invertible.
- $\text{nullity}(A) = \text{nullity}(AV)$

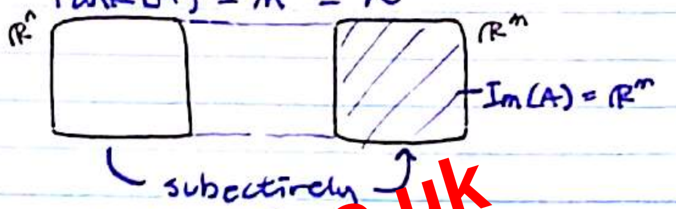
$$\dim(\text{null}(A)) + \dim(\text{col}(A)) = n$$

$\text{rank}(A) = n \leq m$



- $\text{null}(A) = \vec{0}$
- $\text{row}(A) = \mathbb{R}^n$
- col (A) lin indep.

$\text{rank}(A) = m \leq n$



- $\text{Im}(A) = \mathbb{R}^m$

- $\text{rank}(AB) = \text{rank}(B)$ where A is invertible.

$\downarrow$ $\dim(\text{col}(AB))$ $\quad$ $\downarrow$ $\dim(\text{col}(B))$

- let $B = \begin{bmatrix} c_1 & c_2 & \cdots & c_n \end{bmatrix}$, $AB = \begin{bmatrix} d_1 & d_2 & \cdots & d_n \end{bmatrix}$

↳ $d_1, d_2, \ldots d_n$ are linear combinations of $c_1, c_2 \ldots c_n$
↳ $\text{col}(AB) \subseteq \text{col}(B)$
↳ $\text{col}(A^{-1}AB) \subseteq \text{col}(AB) \subseteq \text{col}(B)$