Some results On Semiderivations Of Semiprime Semirings

Abstract- Let S be a semiprime semiring. An additive mapping $f : S \rightarrow S$ is called a semi derivation if there exists a function $g : S \rightarrow S$ such that (i) $f(xy) = f(x)g(y) + xf(y) = f(x)y + g(x)f(y)$,
(ii) $f(g(x)) = g(f(x))$ hold for all $x, y \in S$. In this paper we try to generalize some properties of prime rings with derivations to semiprime semirings with semiderivations.

Key words- Semirings, Semiprime semirings, Derivation, Semi derivation,Commuting mapping.

I. INTRODUCTION

Let S be a semiprime semiring with center $Z(S)$. For any $x, y \in S$, $[x, y], (x, y)$ represents $xy - yx, xy + yx$ respectively. Also we make use of basic commutator identities

$$[xy, z] = [x, z]y + x[y, z], \quad [x, yz] = y[x, z] + [x, y]z, \quad (xy, z) = (x, z)y + x[y, z] = [x, z]y + x(y, z)$$

J.C.Chang $[6]$ studied on semi derivations of prime rings. He obtained some results of derivations of prime rings into semiderivations. H.E.Bell and W.S.Martindale III $[7]$ investigated the commutativity property of a prime ring by means of semiderivations. C.L.Chuang $[8]$ studied on the structure of semiderivations in prime rings. He obtained some remarkable results in connection with the semiderivations. J.Bergen and P.Grezesczuk $[5]$ obtained the commutativity properties of semiprime rings with the help of semi derivations. A.Firat $[3]$ generalized some results of prime rings with derivations to the prime rings with semiderivations. In this paper we generalize some results of prime rings with semiderivations to the semiprime semirings with semiderivations.

II. PRELIMINARIES

Definition 2.1

A **semiring** S is a nonempty set S equipped with two binary operations $+$ and $\bullet$ such that

1. $(S, +)$ is a commutative monoid with identity element 0

2. $(S, \bullet)$ is a monoid with identity element 1

3.Multiplication left and right distributes over addition.

Definition 2.2

A semiring S is said to be **prime** if $xsy = 0$ implies $x = 0 \ or \ y = 0 \ for \, all \ x, y \in S.$

Definition 2.3

A semiring S is said to be **semiprime** if $xsx = 0$ implies $x = 0 \ \ for \, all \ x \in S.$

Definition 2.4