(b) $\log 10^2$ lies between $\frac{1}{3}$ and $\frac{1}{4}$

Here,

$2^3 < 10 < 2^4$

$\Rightarrow \log 10^{2^3} < \log 10^{10} < \log 10^{2^4}$

$\Rightarrow 3\log 10^2 < 1 < 4\log 10^2$

$\Rightarrow 3\log 10^2 < 1$ & $4\log 10^2 > 1$

$\Rightarrow \log 10^2 < \frac{1}{3}$ $\Rightarrow \log 10^2 > \frac{1}{4}$

$\Rightarrow \log 10^2$ lies between $\frac{1}{3}$ and $\frac{1}{4}$

(c) $\frac{1}{3} < \log_{34} 5^5 < \frac{1}{2}$

Here,

$5^2 < 34 < 5^3$

$\Rightarrow \log_{34} 5^2 < 34 < \log_{34} 5^3$

$\Rightarrow 2\log_{34} 5 < \log_{34} 34 < 3\log_{34} 5$

$\Rightarrow 2\log_{34} 5 < 1 < 3\log_{34} 5$

$\Rightarrow \log_{34} 5 < \frac{1}{2}$ & $\log_{34} 5 > \frac{1}{3}$

$\Rightarrow \frac{1}{3} < \log_{34} 5 < \frac{1}{2}$

4)(a)

(i) $\log \frac{a^3 b^3}{c^3} + \log \frac{b^3 c^3}{d^3} + \log \frac{c^3 d^3}{a^3} - 3\log b^2 c$

$\Rightarrow \log \left(\frac{a^3 b^3 \times b^3 c^3 \times c^3 d^3}{c^3 \times d^3 \times a^3} \right) - 3\log b^2 c$

$\Rightarrow \log (b^6 c^3) - 3\log b^{2\times 3} c^{1\times 3}$

$= \log b^6 c^3 - \log b^6 c^3 = 0$