MATHEMATICS 4 SURVEYING

A plane area is bounded by the curve $y^2 = 4x$ & the line $y = x$.

a.) Find the area bounded by the curve.

Points of intersection:

$y^2 = 4x$ & $y = x$

$y^2 = 4y$

$y^2 - 4y = 0$

$y(y-4) = 0$

$y = 0$ & $y = 4$

$x = 0$ & $x = 4$

$dA = (X_R - X_L)dy$

$X_R$ = line

$X_L = y^2/4$ → parabola

$dA = (y - y^2/4)dy$

$A = \int_0^4 \left[y - y^2/4\right] dy$

$= \left[y^2 - \frac{y^3}{12}\right]_0^4$

$= \left[4^2 - \frac{4^3}{12}\right] = 2.667$ sq. units.

OR: $A$ = A parabolic segment − A triangle

$= \left(\frac{2}{3}\right)(4)(4) - \frac{1}{2}(4)(4) = 2.667$

b.) How far from the x-axis is the centroid of the curve?

Centroid:

$A\bar{y} = \int_{y_1}^{y_2} y\,dA$

$2.667\bar{y} = \int_0^4 y\left(y - \frac{y^2}{4}\right)dy$

$= \int_0^4 \left(y^2 - \frac{y^3}{4}\right)dy$

$= \left[\frac{y^3}{3} - \frac{y^4}{16}\right]_0^4$

$= \left[\frac{4^3}{3} - \frac{4^4}{16}\right]$

$2.667\bar{y} = 5.333$

$\bar{y} = 2$ units