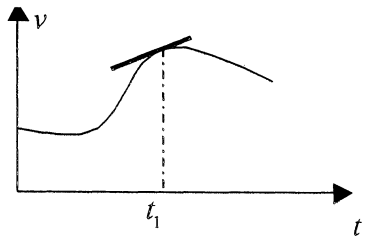
The **acceleration** of the particle is the rate of change of its velocity and is the vector $\boldsymbol{a} = \frac{dv}{dt}\boldsymbol{i}$.

If the velocity is increasing $\frac{dv}{dt}$ is positive and if the velocity is decreasing $\frac{dv}{dt}$ is negative.

If $|\,\boldsymbol{a}\,|$ is denoted by a then $a = \frac{dv}{dt}$ and $v = \int a\,dt$

Also $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$ and $\frac{d^2x}{dt^2}$ is sometimes denoted by $\ddot{x}$.



gradient of graph at $t = t_1$

$= \frac{dv}{dt}$ = **acceleration**

area under graph $t = 0$ to $t = t_1$

$= \int_0^{t_1} v\,dt$ = **displacement** at $t = t_1$

The **area under an acceleration/time graph** from $t = 0$ to $t = t_1$ is

$$\int_0^{t_1} a\,dt = \textbf{velocity at } t = t_1$$

Note

Students should be familiar with the dot notation for differentiating with respect to time. This is not used in all textbooks. Of the textbooks listed S&T, R&S and B&C use this notation while the others do not.

WORKED EXAMPLES

Example 1

A body moves along the x-axis with velocity, measured in ms^{-1}, given by

$$v = (3t^2 - 18t + 15)\boldsymbol{i},$$

where $\boldsymbol{i}$ is the unit vector in the positive direction of the x-axis and t is the time in seconds from the start of the motion.

At the start of the motion the displacement of the body from the origin is 30 m. Find:

a) the initial speed of the body;

b) the values of t for which the body is at rest;

c) the acceleration of the body when $t = 6$;

d) the displacement of the body from O when $t = 3$.

Solution

a) $v = 3t^2 - 18t + 15$. When $t = 0$, $v = 15$ ms^{-1}

b) $v = 3(t^2 - 6t + 5) = 3(t - 1)(t - 5) = 0$ when $t = 1, 5$

$\therefore$ body is at rest after 1 second and after 5 seconds from the start

c) $a = \dfrac{dv}{dt} = 6t - 18$ When $t = 6$, $a = 36 - 18 = 18$ ms^{-1}

d) $x = \int v\,dt = t^3 - 9t^2 + 15t + c$ Now $x = 30$ when $t = 0$ so $c = 30$

$x = t^3 - 9t^2 + 15t + 30$ When $t = 3$, $x = 27 - 81 + 45 + 30$ $\therefore$ displacement from O is 21 m in the positive direction.

Example 2

A body moves along the x-axis from rest at the origin with acceleration, measured in ms^{-2}, given by

$$\boldsymbol{a} = (2 - \sqrt{t})\boldsymbol{i}$$

where $\boldsymbol{i}$ is the unit vector in the positive direction of the x-axis and t is the time in seconds from the start of the motion.

a) Show that its speed increases to a maximum value and then decreases.

b) Find

i) the time till the body is instantaneously at rest again

ii) the time before it again passes through its starting point

Solution

a) $\dfrac{dv}{dt} = a = (2 - \sqrt{t})$

for $0 < t < 4$, $a > 0$ and so v is increasing. When $t = 4$, v has a stationary value and for $t > 4$, v is decreasing. At $t = 4$, v has a maximum value

$v = \int a\,dt = \int (2 - \sqrt{t})dt = 2t - \dfrac{2}{3}t^{\frac{3}{2}} + c$ when $t = 0$, $v = 0$ thus $c = 0$

When $t = 4$, $\quad v_P = 4i + 12j \quad$ so speed $= |v_P| = \sqrt{4^2 + 12^2} = \sqrt{160} = 12.65\text{ms}^{-1}$

$$\dot{r}_P = 4i + \frac{3t^2}{4}j \qquad \Rightarrow a_P = \ddot{r} = \frac{3}{2}tj$$

EFFECTS OF WINDS AND CURRENTS

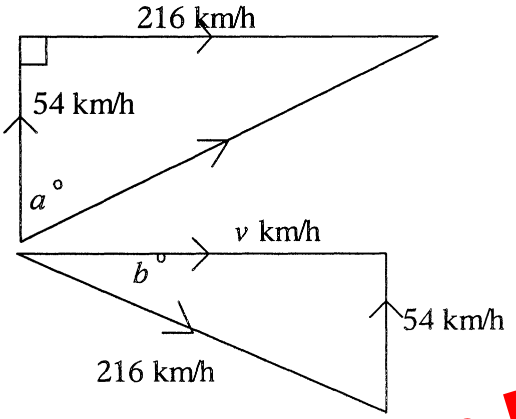
Worked Example

A wind is blowing from the north at 54 km/h. A plane can fly at 216 km/h.

a) If the pilot steers due east, on what bearing will the plane travel?

b) What course should the pilot set in order to fly due east? Calculate the actual speed of the plane.

Solution



Combine velocities of wind and plane using triangle law of vector addition.

$\tan a° = 216/54 \Rightarrow a = 76$

$\sin b° = 1/4 \Rightarrow b = 14{\cdot}5$

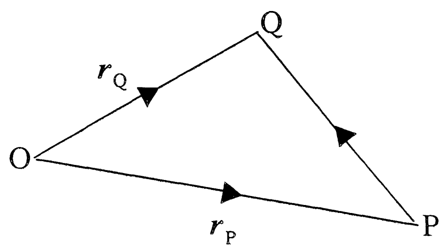
So set course on bearing of 104.5°

$v^2 = 216^2 - 45^2$

$\Rightarrow v = 209{\cdot}1$

RELATIVE POSITION, VELOCITY AND ACCELERATION

The diagram on the right shows the positions of two bodies P and Q and a fixed origin O. With respect to O, P and Q have instantaneous position vectors r_P and r_Q. The instantaneous position vector of Q relative to P is $\overrightarrow{PQ}$.



Now $\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ}$ and

So $\boxed{\overrightarrow{PQ} = r_Q - r_P}$

RESOURCES/EXAMPLES

S&T Chapter 1 (Vectors)
Pages 9 –12; Chapter 10 Page 210
Chapter 2 (Distance, Velocity and Acceleration)
Pages 21 – 23; Chapter 10 Pages 219 – 229
Chapter 16 (Use of Calculus)
Pages 393-399; Ex 16.4, N^{os} 15 – 20, 33 – 38
Chapter 10 (Resultant Velocity and Relative Velocity)
Pages 223 – 247;Ex 10A (omit N^{o} 2) EXs10B-G, 10H(**A/B**)

RCS Chapter 4 (Kinematics in one dimension)
Pages 59, 60, 64 – 66 (using forces)
Chapter 2 (Kinematics)
Pages 38, 39: Ex 2J, Page 40, Ex 2K, N^{os} 1 – 6
Chapter 15 (Relative Motion)
Page 269 Ex15A;Pages 286 – 294, Ex 15B, 15C, 15D

TG Chapter 3 (Vectors and Forces)
Pages 29 – 31 (using forces);
Chapter 5 (Motion and Vectors)
Pages 62 - 66 Ex5.1B; Ex 5.2A N^{os} 1, 6, 7
Chapter 21 (Relative Motion)
Pages 368 – 374;Ex 21.1A N^{o} 1 – 10;Ex 21.1B, N^{o} 1–5, 7-10
Page 374 Consolidation Ex (**A/B**)
Chapter 7 (Motion with Variable Forces and Acceleration)
Pages 95 – 98; Ex 7.1A, N^{os} 1,2,5,8;
Pages 99,100; Ex 7.1B, N^{os} 1,5,6,7,8,10
Page 103;Ex 7.2A, N^{os} 1,5; Pages 104,105; Ex 7.2B, N^{os} 1,5,6

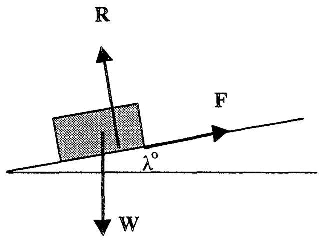
B&C Chapter 2 (Vectors. Components and Resultants….)
Pages 19 – 23; Chapter 13 Pages 420 – 422
Chapter 13 (Resultant Motion. Relative Motion)
Pages 420 – 431; Ex 13a; Page 424, Ex 13b N^{os} 1,2; Ex 13c
Pages 440 – 444, Ex 13e; Pages 448,449 Ex 13(**A/B**)
Chapter 4 (Velocity and Acceleration)
Pages 134,136

OG Chapter 6 (Vectors,……)
Page 292, Page 295, questions 6, 18
Chapter 7 (Kinematics of a Particle)
Pages 394 – 396
Pages 386-390; Ex 7.2:1, N^{os} 1,2,3,13,21,22,26
Pages 391,393; Ex 7.2:2, N^{os} 1 – 6,8,9,10,19,21
Pages 398 - 400, Ex 7.2.3, N^{os} 2, 4–7, 10–40, 42–52, 53–64

ANGLE OF FRICTION

A block of mass *m* kilograms rests on a plane, which is gradually tilted until the block is on the point of moving down the plane. Suppose the angle of the plane to the horizontal is λ when the block is on the point of slipping and the coefficient of friction between the block and the plane is μ.

Since the block is on the point of moving down the plane, friction is acting up the plane.



On the point of slipping: $F = \mu R$

No motion:

Resolving perpendicular to the plane

$$R = W\cos\lambda = mg\cos\lambda$$

Resolving parallel to the plane

$$F = mg\sin\lambda$$

Thus $\mu = \dfrac{F}{R} = \dfrac{mg\sin\lambda}{mg\cos\lambda} = \dfrac{\sin\lambda}{\cos\lambda} = \tan\lambda$

λ is known as the angle of friction.

Now, $\mu = \tan\lambda$, so $T = \dfrac{\mu mg}{\cos\theta + \mu\sin\theta} = \dfrac{\dfrac{\sin\lambda}{\cos\lambda}mg}{\cos\theta + \dfrac{\sin\lambda}{\cos\lambda}\sin\theta} = \dfrac{\mu mg\sin\lambda}{\cos\theta\cos\lambda + \sin\theta\sin\lambda}$

Thus $T = \dfrac{\mu mg\sin\lambda}{\cos(\theta - \lambda)}$ and so T has a minimum value of $\mu mg\sin\lambda$ when $\theta = \lambda$