or $\vec{\tau} = P \times E$

NOTE :

1.Direction of torque is normal to the plane containing dipole moment P and electric field E and is governed by right hand screw rule.

2. If Dipole is parallel to E the torque is **Zero**.

3. Torque is **maximum** when Dipole is perpendicular to E and that torque is PE

4. This equation gives the definition of dipole moment. If E is 1 N/C then P=T.

Therefore; **Dipole Moment of a dipole is equal to the Torque experience by that dipole when placed in an electric field of strength 1 N/C at right angle to it.**

5. If a dipole experiencing a torque in electric field is allowed to rotate, then it will rotate to align itself to the Electric field. But when it reach along the direction of E the torque become zero. But due to inertia it overshoots this equilibrium condition and then starts oscillating about this mean position.

6.*Dipole in Non-Uniform Electric field* :

In case Electric field is non-uniform, magnitude of force on +q and –q will be different, hence a net force will be acting on centre of mass of dipole and it will make a linear motion. At the same time due to couple of forces acting, a torque will also be acting on it.

Work done in rotating a dipole in a uniform Electric field:

1.If a dipole is placed in a uniform electric field experience a torque. If it is rotated from its equilibrium position, work has to be done on it. If an Electric dipole with moment P is placed in electric field E making an angle α, then torque acting on it at that instant is

$\tau = PESin\alpha$

2. If it is rotated further by a small angle dα then work done dw = (PEsinα).dα

Then work done for rotating it through an angle θ from equilibrium position of angle 0 is :-

$W = \int_0^{\theta} (PEsin\alpha).d\alpha = PE[-Cos\,\alpha]_0^{\theta}$

Or, **W** = PE $[-cos\theta + \cos 0]$ = **pE** $[\mathbf{1 - Cos\,\theta}]$

3.If a dipole is **rotated through 90°** from the direction of the field, then work done will be

$$\boxed{W = pE\,[1 - \text{Cos } 90] = pE}$$

4. If the dipole is **rotated through 180°** from the direction of the field, then work done will be :

$$\boxed{W = pE\,[1 - \text{Cos } 180] = 2\,pE}$$

Potential Energy of a dipole kept in Electric field :

1. dipole in Equilibrium (P along E):-

A dipole is kept in Electric field in equilibrium condition, dipole moment P is along E

To calculate Potential Energy of dipole we calculate work done in bringing +q from zero potential i.e.∞ to location B, and add to the work done in bringing –q from ∞ to position A.

1.The work done on –q from ∞ up to A

= -(*Work done up to B* + Work done from B to A)

2. Work done on +q = +(*Work done up to B*)

Adding the two

Total work done = Work done on –q from B to A

= Force x displacement

= -qE x 2L = - 2qLE

=- P.E

This work done convert into Potential Energy of dipole

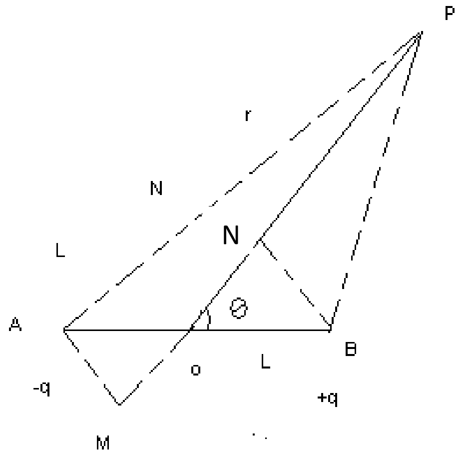
$$\mathbf{U = -\vec{P}\,.\,\vec{E}}$$

If P and E are inclined at angle θ to each other then magnitude of this Potential Energy is

U = - P E Cos θ

Electric – Potential

(1) Electric Potential is characteristic of a location in the electric field. If a unit charge is placed at that location it has potential energy (due to work done on its placement at that location). This potential energy or work done on unit charge in bringing it from infinity is called potential at that point.



Draw normal from A & B on PO

PB $\approx$ PN = PO – ON = r – L Cos θ ------------ (i)

PA $\approx$ PM = PO + OM = r + L Cos θ ----------- (ii)

$$V_{+q} = \frac{Q}{4\Pi \in_0 PB} = \frac{Q}{4\Pi \in_0 (r - L\cos\theta)}$$

$$V_{-q} = \frac{-Q}{4\Pi \in_0 PA} = \frac{-Q}{4\Pi \in_0 (r + L\cos\theta)}$$

Total V = V_{+q} + V_{-q} =

$$\frac{Q}{4\Pi \in_0}\left(\frac{1}{r - LCos\theta} - \frac{1}{r - LCos\theta}\right)$$

$$= \frac{Q}{4\Pi \in_0}\left(\frac{r + L\cos\theta - r + LCos\theta}{r^2 - L^2Cos^2\theta}\right)$$

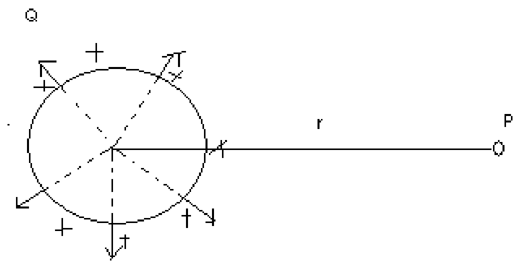
$$= \frac{QX\,2LCos\theta}{4\Pi \in_0 (r^2 - L^2Cos\theta)}$$

Or $V = \dfrac{PCos\theta}{4\Pi \in_0 (r^2 - L^2Cos\theta)}$

If r > > L

Then, Or, $V = \dfrac{PCos\theta}{4\Pi \in_0 r^2}$

Potential due to spherical shell



A spherical shell is given change Q. The electric field is directed normal to surface i.e., Radially outward. "Hence charge on the surface of a shell behaves as if all the charge is concentrated at centre.

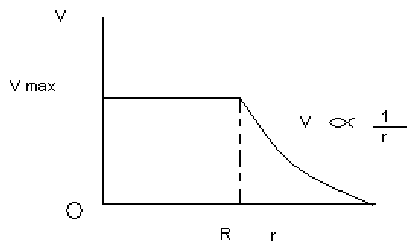
Hence potential at distance r is $V = \dfrac{Q}{4\Pi \in_0 r}$

Potential on the surface of shell $V = \dfrac{Q}{4\Pi \in_0 R}$

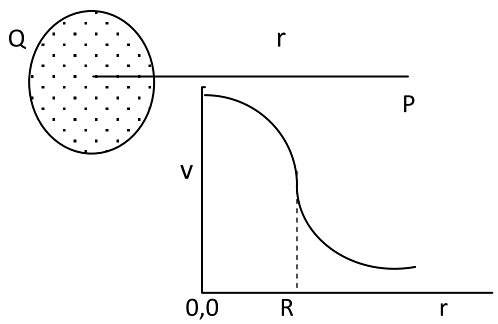
Inside shell Electric field is Zero.
Therefore change in potential dv = zero X dr = 0 i.e., No change in potential. Hence potential inside a spherical shell is same as on the surface and it is same at every point.

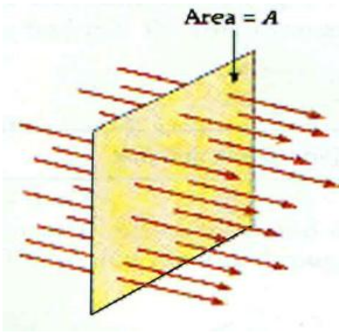
It is $V = \dfrac{Q}{4\Pi \in_0 R}$ Where R is radius of shell.

Relation of V & r for spherical shell

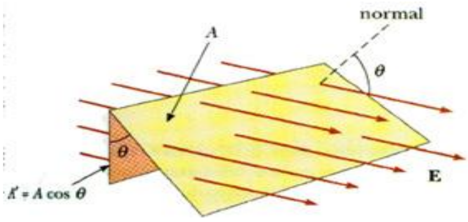


In case of non-conducting sphere of charge. potential keeps on increasing up to centre as per diagram.





If the electric field **E** is **not** perpendicular to the area, we will have to modify this to account for that.



Think about the "air flux" of air passing through a window **at an angle** θ. The "effective area" is A cos θ or the component of the velocity perpendicular to the window is v cos θ. With this in mind, we will make a general definition of the electric flux as

- Φ = E A cos θ
- Φ You can also think of the electric flux as the **number** of electric field lines that cross the surface.

Remembering the "dot product" or the "scalar product", we can also write this as

$$\Phi = \vec{\mathbf{E}} \cdot \vec{\mathbf{A}}$$

where **E** is the electric field and **A** is a vector equal to the area A and in a direction **perpendicular** to that area. Sometimes this same information is given as

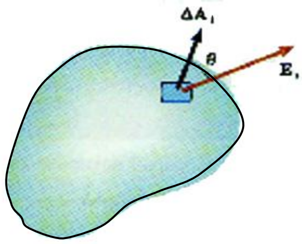
$$\mathbf{A} = A\,\mathbf{n}$$

where **n** is a **unit vector** pointing **perpendicular** to the area. In that case, we could also write the electric flux across an area as

$$\Phi = \mathbf{E} \cdot \mathbf{n}\, A$$

Both forms say the same thing. For this to make any sense, we must be talking about an area where the **direction** of **A** or **n** is constant.

For a curved surface, that will not be the case. For that case, we can apply this definition of the electric flux over a small area ΔA or Δ**A** or ΔA**n**.



Then the electric flux through that small area is ΔΦand

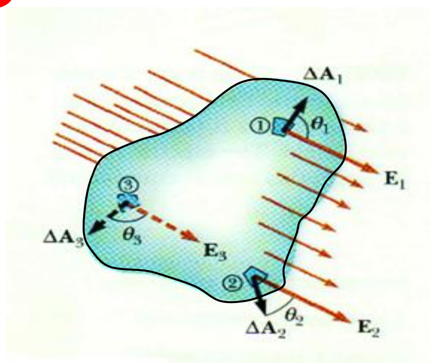
$\Delta\Phi = E\,\Delta A \cos\theta$ or

$$\Delta\Phi = \vec{\mathbf{E}} \cdot \Delta\vec{\mathbf{A}}$$

To find the flux through all of a closed surface, we need to sum up all these contributions of ΔΦ over the entire surface,

$$\Phi_c = \oint \mathbf{E} \cdot d\vec{\mathbf{A}} = \oint E_n\, dA \cos\theta$$

We will consider flux as **positive** if the electric field **E** goes from the inside to the outside of the surface and we will consider flux as **negative** if the electric field **E** goes from the outside to the inside of the surface. This is important for we will soon be interested in the **net** flux passing through a surface.



<u>Gauss's Law</u> : Total electric flux though a closed surface is $1/\varepsilon_0$ times the charge enclosed in the surface.

$$\Phi_E = q / \varepsilon_0$$

But we know that Electrical flux through a closed surface is $\oint \vec{E}.\vec{ds}$

$\therefore \oint \vec{E}.\vec{ds} = q / \varepsilon_0$

This is Gauss's theorem.

It is a device to store charge and in turn store the electrical energy.

Any conductor can store charge to some extent. But we cannot give infinite charge to a conductor. When charge is given to a conductor its potential increases. But charge cannot escape the conductor because air, or medium around conductor is di-electric.

When due to increasing charge the potential increase to such extent that air touching the conductor starts getting ionized and hence charge gets leaked. No more charge can be stored and no more potential increase. This is limit of charging a conductor.

The electric field which can ionize air is 3×10^{9} vm^{-1}.

CAPACITANCE OF A CONDUCTOR

Term capacitance of a conductor is the ratio of charge to it by rise in its Potential

$$C = \frac{q}{V}$$

In this relation if V=1 then C= q. Therefore ,

<u>Capacitance of a conductor is equal to the charge which can change its potential by one volt.</u>

Unit of capacitance : Unit of capacitance is farad, (symbol F).

<u>One farad is capacitance of such conductor whose potential increase by one volt when charge of one coulomb is given to it.</u>

One coulomb is a very large unit. The practical smaller units are
i. Micro farad (μF) = 10^{-6}F.(used in electrical circuits)
Ii Pieco farad (pF) = 10^{-12} used in electronics circuits

Expression for capacitance of a spherical conductor :
If charge q is given to a spherical conductor of radius r, its potential rise by $V = \frac{q}{4\pi\epsilon_0 r}$
Therefore capacitance $C = \frac{q}{V} = q/\frac{q}{4\pi\epsilon_0 r} = 4\pi\epsilon_0 r$

Or for a sphere $\boxed{C = 4\pi\epsilon_0 r}$

The capacitor depends only on the radius or size of the conductor.

The capacitance of earth (radius 6400 km) is calculated to be 711×10^{-6} coulomb.

PARALLEL PLATE CAPACITOR : -

Since single conductor capacitor do not have large capacitance , parallel plate capacitors are constructed.

Principle : Principle of a parallel plate capacitor is that an uncharged plate brought bear a charged plate decrease the potential of charged plate and hence its capacitance ($C = \frac{q}{V}$) increase. Now it can take more charge. Now if uncharged conductor is earthed, the potential of charged plate further decreases and capacitance further increases. This arrangement of two parallel plates is called parallel plate capacitor.

Expression for capacitance :

Charge q is given to a plate

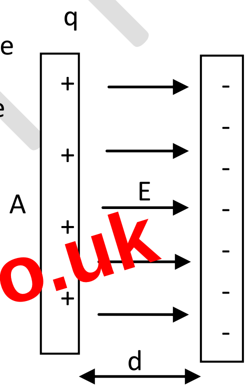
Of area 'A'. Another plate

is kept at a distance 'd'.

After induction an

Electric field E is set-up

Between the plates. Here



$q = \sigma A$ and $E = \frac{\sigma}{\varepsilon_0}$

The Potential difference between plates is given by

$V = Ed = \frac{\sigma}{\varepsilon_0} d$

Now $C = \frac{q}{V} = \frac{\sigma A}{\frac{\sigma}{\varepsilon_0} d} = \frac{\varepsilon_0 A}{d}$

$$\boxed{C = \frac{\varepsilon_0 A}{d}}$$

If a dielectric of dielectric constant K is inserted between the plates, then capacitance increase by factor K and become

$$\boxed{C = \frac{\varepsilon_0 K A}{d}}$$

Note : The capacitance depends only on its configuration i.e. plate area and distance, and on the medium between them.
The other examples of parallel plate capacitors is

Cylindrical capacitor $C = \frac{4\pi\varepsilon_0 KL}{\log\ r^2/r_1}$

and Spherical capacitor. $C = \frac{4\pi\varepsilon_0 K r_2 r_1}{\log\ r_2 - r_1}$