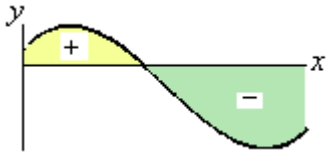
Applications of Integrals

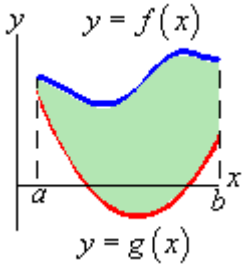
Net Area : $\int_a^b f(x)\,dx$ represents the net area between $f(x)$ and the x-axis with area above x-axis positive and area below x-axis negative.



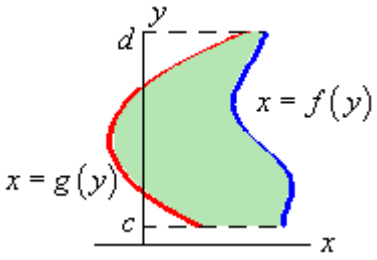
Area Between Curves : The general formulas for the two main cases for each are,

$y = f(x) \Rightarrow A = \int_a^b [\text{upper function}] - [\text{lower function}]\,dx$ & $x = f(y) \Rightarrow A = \int_c^d [\text{right function}] - [\text{left function}]\,dy$

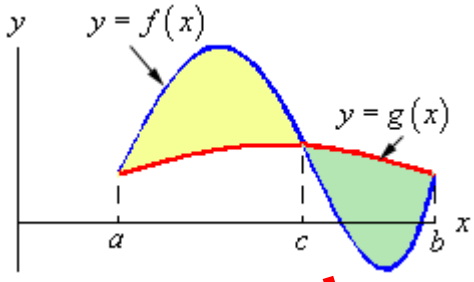
If the curves intersect then the area of each portion must be found individually. Here are some sketches of a couple possible situations and formulas for a couple of possible cases.



$A = \int_a^b f(x) - g(x)\,dx$



$A = \int_c^d f(y) - g(y)\,dy$



$A = \int_a^c f(x) - g(x)\,dx + \int_c^b g(x) - f(x)\,dx$

Volumes of Revolution : The two main formulas are $V = \int A(x)\,dx$ and $V = \int A(y)\,dy$. Here is some general information about each method of computing and some examples.

Rings

$A = \pi\left((\text{outer radius})^2 - (\text{inner radius})^2\right)$

Limits: x/y of right/bot ring to x/y of left/top ring

Horz. Axis use $f(x)$, $g(x)$, $A(x)$ and dx.

Vert. Axis use $f(y)$, $g(y)$, $A(y)$ and dy.

Cylinders

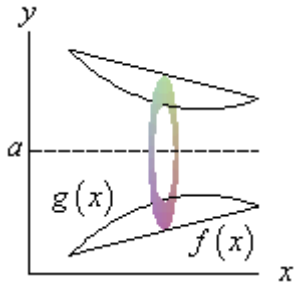
$A = 2\pi\,(\text{radius})(\text{width / height})$

Limits : x/y of inner cyl. to x/y of outer cyl.

Horz. Axis use $f(y)$, $g(y)$, $A(y)$ and dy.

Vert. Axis use $f(x)$, $g(x)$, $A(x)$ and dx.

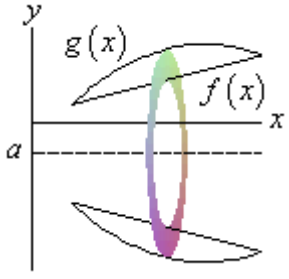
Ex. Axis : $y = a > 0$



outer radius : $a - f(x)$

inner radius : $a - g(x)$

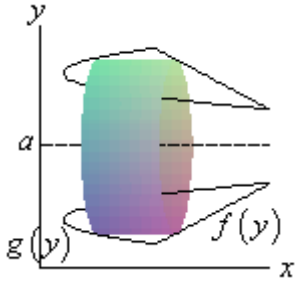
Ex. Axis : $y = a \le 0$



outer radius: $|a| + g(x)$

inner radius: $|a| + f(x)$

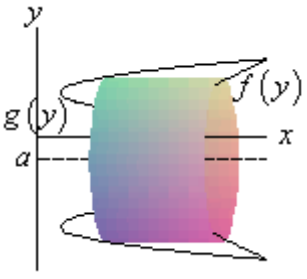
Ex. Axis : $y = a > 0$



radius : $a - y$

width : $f(y) - g(y)$

Ex. Axis : $y = a \le 0$



radius : $|a| + y$

width : $f(y) - g(y)$

These are only a few cases for horizontal axis of rotation. If axis of rotation is the x-axis use the $y = a \le 0$ case with $a = 0$. For vertical axis of rotation ($x = a > 0$ and $x = a \le 0$) interchange x and y to get appropriate formulas.