Electric field intensity at any point due to a short electric dipole

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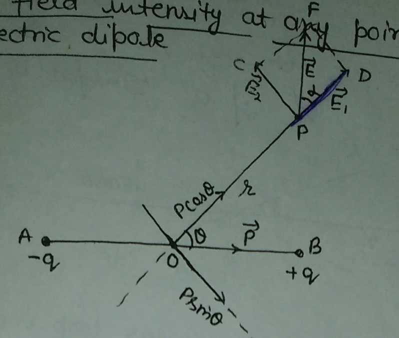

Let a short dipole AB have dipole moment $\vec{P}$ and P be the point at the distance 'r' from centre O of the dipole at which we have to calculate electric field intensity. Let OP makes an angle θ with $\vec{P}$.

Resolve $\vec{P}$ into two components

(i) $P\cos\theta$ (Along OP)

(ii) $P\sin\theta$ (Perpendicular to OP)

Electric field intensity at P w.r.to $P\cos\theta$ is

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{2P\cos\theta}{r^3} \quad \text{Along PD}$$

Electric field intensity at P w.r.to $P\sin\theta$ is

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{P\sin\theta}{r^3} \quad \text{Along PC}$$

Resultant electric field intensity at P is-

$$E = \sqrt{E_1^2 + E_2^2}$$

$$= \frac{P}{4\pi\epsilon_0 r^3}\sqrt{4\cos^2\theta + \sin^2\theta} = \frac{P}{4\pi\epsilon_0 r^3}\sqrt{3\cos^2\theta + \cos^2\theta + \sin^2\theta}$$

$$\therefore \boxed{E = \frac{P}{4\pi\epsilon_0 r^3}\sqrt{3\cos^2\theta + 1}}$$

Direction of E :- $\tan\alpha = \frac{FD}{PD} = \frac{E_2}{E_1} = \frac{1}{2}\tan\theta$

Case-(i) If $\theta = 0°$ (P is on the axial line)

$$E = \frac{P}{4\pi\epsilon_0 r^3}\sqrt{3+1} = \frac{1}{4\pi\epsilon_0}\frac{2P}{r^3}$$

Case (ii) If $\theta = 90°$ (P is on equatorial axis)

$$E = \frac{P}{4\pi\epsilon_0 r^3}\sqrt{0+1} = \frac{1}{4\pi\epsilon_0}\frac{P}{r^3}$$