→ -ve f/b → poles shifted to left

+ve f/b → poles shifted to right

→ In closed loop systems if order of the system is very high, it is difficult to find roots of T/F. so we use RH, RL/BP/NP

* RH → char. eq to find CL. stability

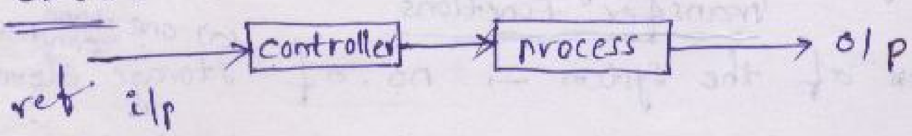
* RL/BP/NP → O/L

* Order → NP, RL, BP, RH.

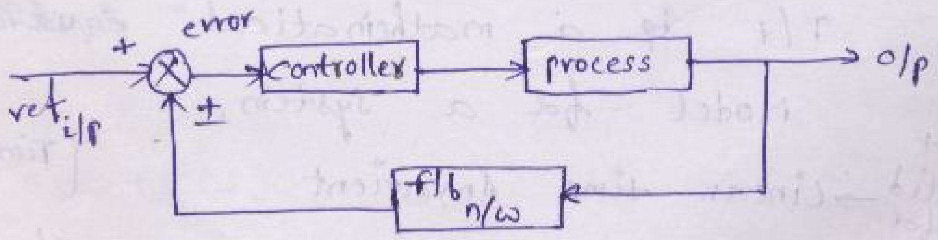
⇒ Control system: It is an arrangement of group of phy. components in such a way that it gives the desired o/p by means of controller. either direct method or indirect.

→ Based on the controller action, control systems O/L system C/L system.

OL CS :-



CLCS :-



OL CS :- A system in which the controller action is inde. of o/p. Eg:- fans, heater, normal iron box, traffic lights

Eg:- Any system which not sense the o/p.

CL CS :- The controller action is totally depends on o/p. Eg:- Any m/c with Automatic which sense the o/p. [Refrigerator, Iron box automatic]

Differential equations :- [D.E.]

1. find T/f.

$$\frac{d^2y}{dt^2} + 10\frac{dy}{dt} + 5y = 2\frac{dx}{dt}$$

where $y \rightarrow$ o/p & $x \rightarrow$ i/p

$$\frac{Y(s)}{X(s)} = \frac{\text{i/p related terms}}{\text{o/p related terms}}$$

$$= \frac{2s}{s^2 + 10s + 5}$$

2. $$\frac{d^3y}{dt^3} + 7.\frac{d^2y}{dt^2} + 10 = 5.\frac{d^2x}{dt^2}$$

$$T/F = \frac{5}{s+7}$$

* Here 10 is initial cond. So in T/f calculation initial condi-s are zero.

3. Obtain D.E for given T/f.

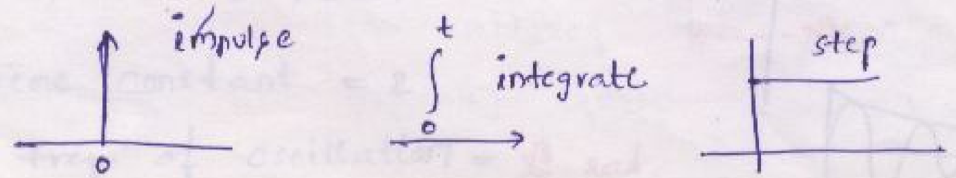
$$\frac{Y(s)}{X(s)} = \frac{10s}{s^2 + 7s + 6}$$

$$\frac{d^2y}{dt^2} + 7.\frac{dy}{dt} + 6y = 10.\frac{dx}{dt}$$

$\downarrow$ $+k$

Signals Response :-

TF = L[impulse response]

(1). 

$$T/F = \frac{\frac{5}{2s} - \frac{5}{2(s+2)} + \frac{5}{s^2}}{1/s}$$

$$=$$

3. A system described by,

$\frac{d^2y}{dt^2} + 3 \cdot \frac{dy}{dt} + 2y = x(t)$ is initially at rest, for the i/p $x(t) = 2\,u(t)$. the o/p $y(t)$ is — ?

for response / o/p :-

ⓐ. first find T/F.

ⓑ. substitute i/p

ⓒ. partial fractions.

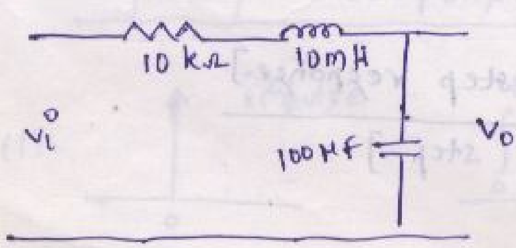
ⓓ. Apply L.T.

$$T/F = \frac{Y(s)}{X(s)} = \frac{1}{s^2+3s+2} \quad , \quad X(s) = \frac{2}{s}$$

$$\rightarrow \quad Y(s) = \frac{2}{s(s^2+3s+2)}$$

Ans. $(1-2e^{-t}+e^{-2t})\,u(t)$

4. for the ckt shown in fig. initial condi. are zero. its T/F is — ?

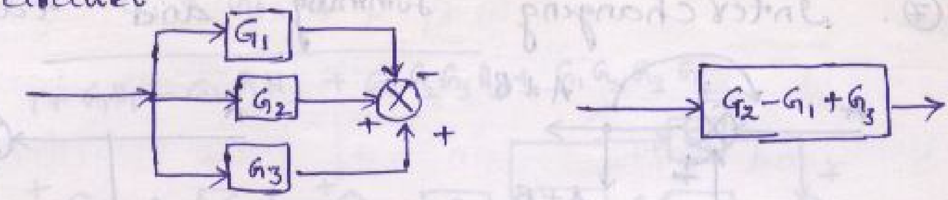


①. $\frac{1}{s^2+10^6 s+10^6}$

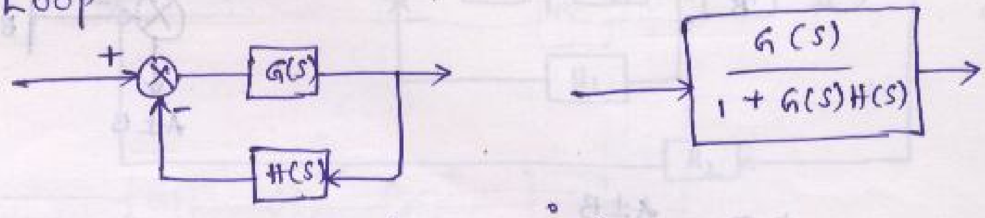
②. $\frac{10^6}{s^2+10^3 s+10^6}$

③. $\frac{10^3}{s^2+10^3 s+10^6}$

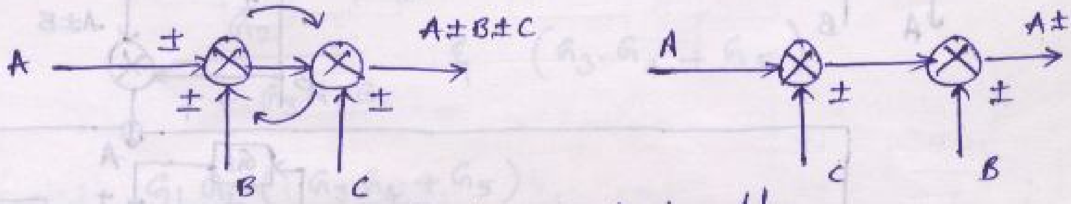
② . Parallel



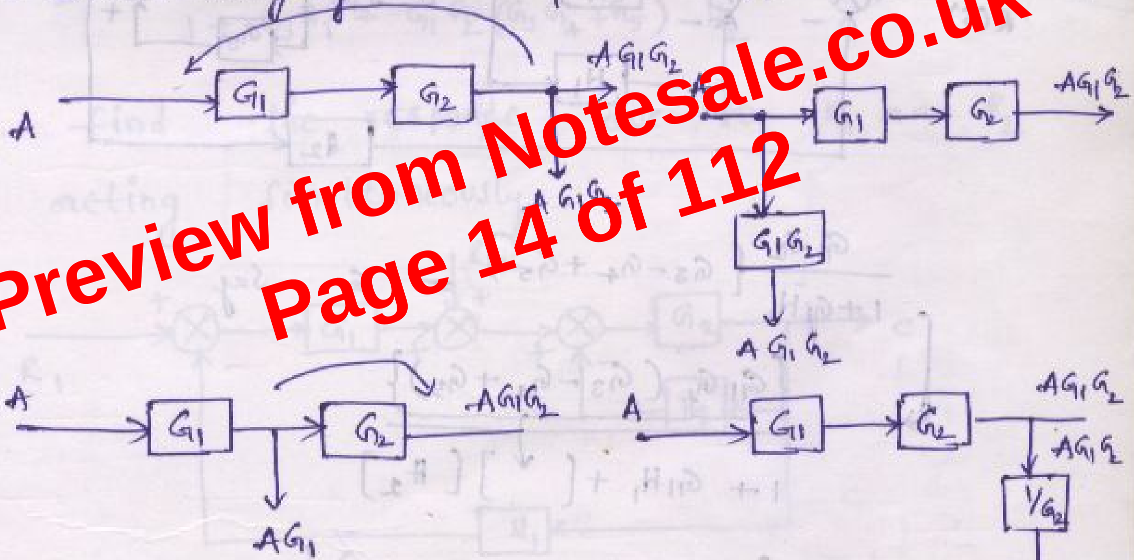
③ . Loop



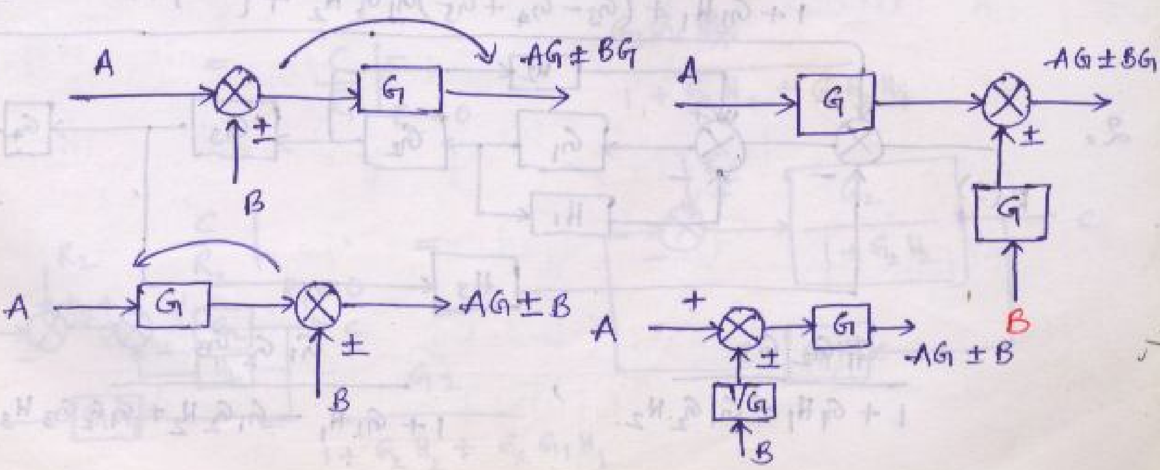
④ . Adjusting summing points



⑤ . Interchanging block & take off :-



⑥ . Interchanging block & summing point :-



10. find OL DC gain of a unity f/b system of CL T/f is $\frac{s+4}{s^2+7s+13}$

① 4/13
② 4/9
③ 4
④ 14

DC gain means $f=0$, ie sub. $s=0$, $s=j\omega = j2\pi f$.

$\rightarrow \frac{4}{13}$

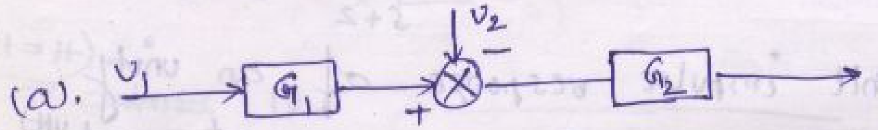
OL gain $/G$

$\frac{G}{1+G-G}$

11. In block diagram shown the o/p $Y(s) = ?$

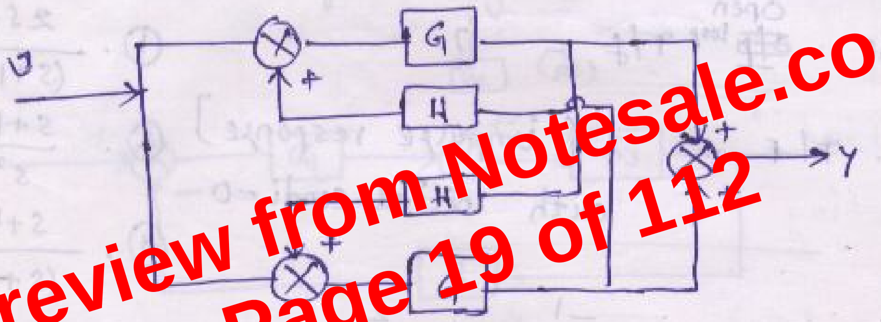
$(G_1U_1 - U_2)G_2$

(a).



$Y(s) = G_1G_2U_1 - G_2U_2$

(b).



$Y(s) = \frac{G}{1-GH} + \frac{G}{1-GH} = \frac{2G}{1-GH}$

$\Rightarrow$ Signal flow Graph:-

It is a graphical representation of the system b/w the set of linear algebraic eq.s.

DE $\xrightarrow{LT}$ LAE ↑ linear Algebraic equation

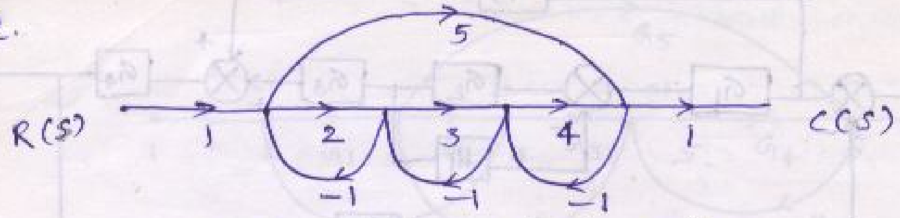
LAE → TF, SFG → TF

Q. construct signal flow graph for the following LAE s.

a). $x_2 = Ax_1$ b) $x_5 = x_1 - 2x_2 + 3x_3 + 10x_4$

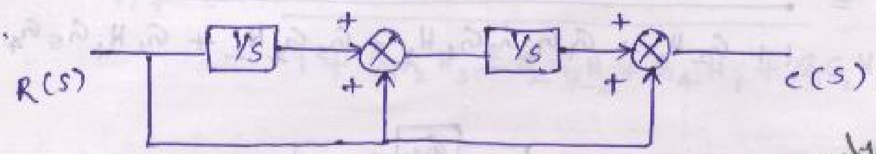
c). $x_2 = 7x_1$, $x_3 = 5x_1$, $x_4 = 3x_1$

Q.



$$\frac{C(s)}{R(s)} = \frac{24 + 5(1+3)}{1+2+3+4+5+8} = \frac{44}{23}$$

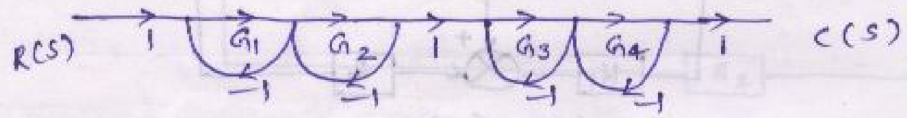
Q.



only forward paths

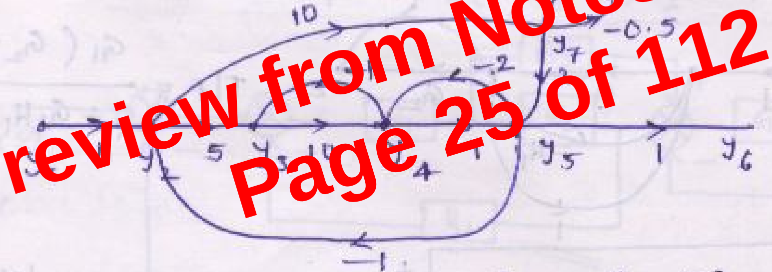
$$\frac{C(s)}{R(s)} = 1/s^2 + 1/s + 1$$

Q.



$$T/F = \frac{G_1 G_2 G_3 G_4}{1 + G_1 + G_2 + G_3 + G_4 + G_1G_3 + G_1G_4 + G_2G_3 + G_2G_4}$$

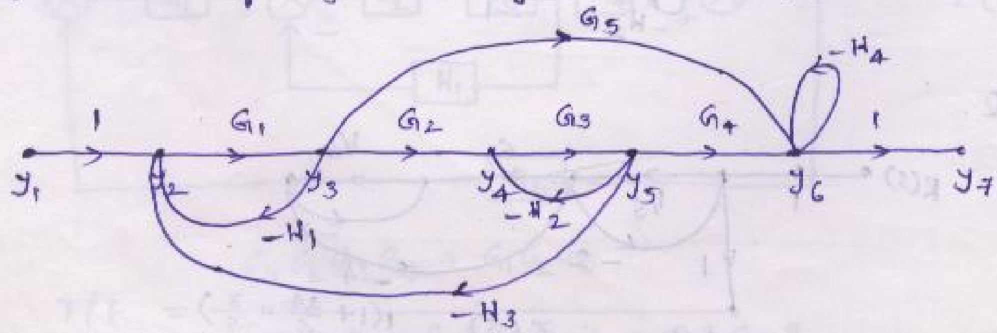
Q.



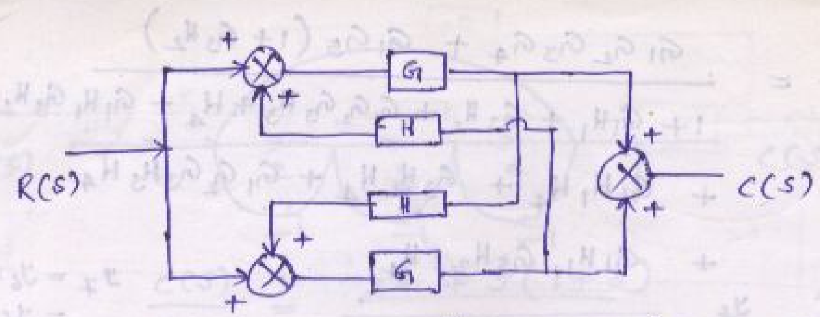
$$T/F = \frac{10\times5\times1[1+0.5] + 20[1+10]}{1+10+2+20+0.5+50+10\times20+10\times0.5+2\times0.5+50\times0.5} = 0.9$$

Q. Find y_6/y_1, y_7/y_1, y_2/y_1, y_4/y_1, y_7/y_2, y_5/y_3, y_4/y_3

[Ratio of given any two nodes].



Q.

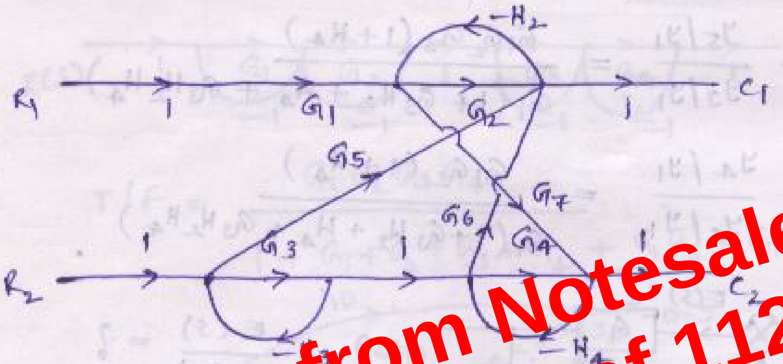


$$= \frac{2G}{1-GH}$$

$$T/f = \frac{G + G^2H_2 + G + G^2H}{1 - G^2H^2}$$

$$= \frac{2G\,[1+GH]}{(1+GH)(1-GH)} = \frac{2G}{1-GH}$$

Q. find C_1/R_1, C_1/R_2, C_2/R_1, C_2/R_2. [Multi i/p Multi o/p]



(i). $$\frac{C_1}{R_1}\bigg|_{C_2=R_2=0} = \frac{G_1G_2\,[1+G_3H_3+G_4H_4+G_3H_3\,G_4H_4] - G_1G_7H_4G_6\,[1+G_3H_3]}{\Delta}$$

(ii). $$\frac{C_1}{R_2}\bigg|_{C_2=R_1=0} = \frac{G_5\,[1+G_4H_4] + G_3G_6}{\Delta}$$

(iii). $$\frac{C_2}{R_1}\bigg|_{C_1=R_2=0} = \frac{G_1G_7\,[1+G_3H_3]}{\Delta}$$

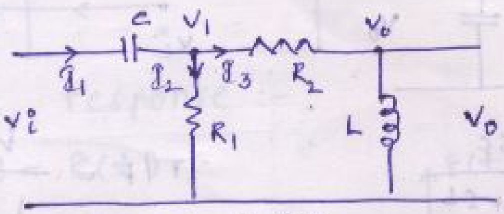
(iv). $$\frac{C_2}{R_2}\bigg|_{C_1=R_1=0} = \frac{G_3G_4\,[1+G_2H_2] - G_5H_2G_7 - G_3G_6H_2G_7}{\Delta}$$

SFG's for Electrical N/ω :- Ref: 1. Ogota 2. B.C. kuo

Steps :-

1. Select Branch 'current' or node voltage
2. Apply L.T. to all the var's & system components.
3. write the eq's of v/I
4. construct SFG.

Eg :-



$T/F = \frac{V_0}{V_i}$

$V_0 = \frac{R_1 sL}{\frac{1}{sC}[R_1+R_2+sL]+R_1[R_2+sL]}$

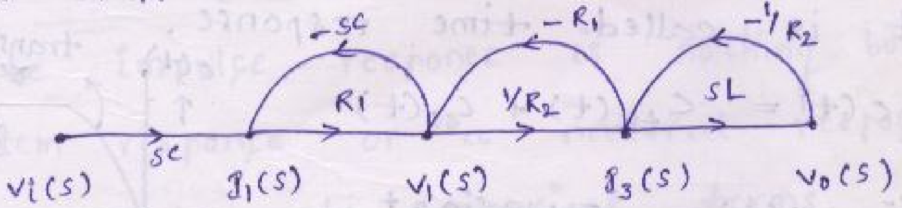
$I_1(s) = \frac{V_i(s) - V_1(s)}{1/sC} = sC[V_i(s) - V_1(s)] \rightarrow ①$

$V_1(s) = I_2(s) \cdot R_1 = R_1[I_1(s) - I_3(s)] \rightarrow ②$

$I_3(s) = \frac{V_1(s) - V_0(s)}{R_2} \rightarrow ③$

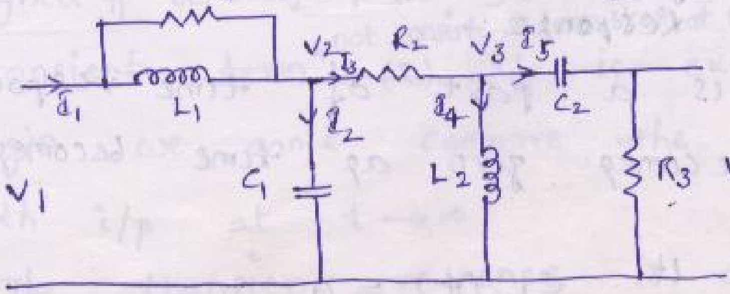
$V_0(s) = sL \cdot I_3(s) \rightarrow ④$

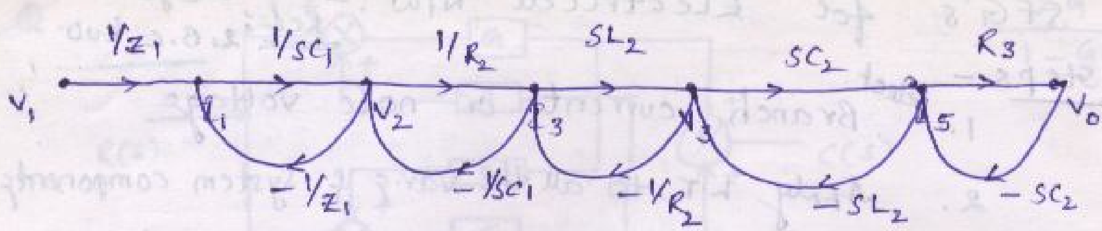
→ whatever the variables along the series branch they are taken as nodes in SFG.



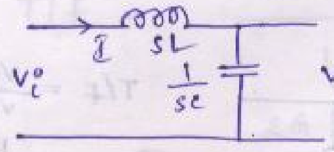
$\frac{V_0(s)}{V_i(s)} = T/F = \frac{sC R_1 sL \cdot 1/R_2}{1 + R_1 sC + \frac{R_1}{R_2} + \frac{sL}{R_2} + \frac{R_1 sC \cdot sL}{R_2}} =$

Q.





Q. Draw SFG for,



$$T/F = \frac{1/sC}{1/sC + sL} = \frac{1}{1+s^2LC}$$

$$T/F = \frac{1/s^2LC}{1+1/s^2LC} = \frac{1}{1+s^2LC}$$

→ TIME DOMAIN ANALYSIS :-

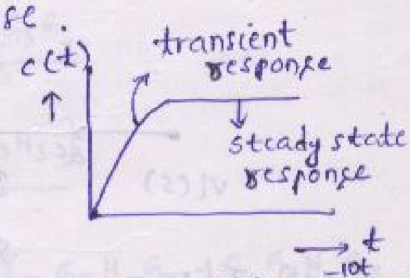
Ref: 1. Nagrath / Gopal

- time domain specifications
- e_{ss}
- Responses

→ Time Response :-

If the response of the system varies w.r.t time then it is called time response.

Time Response $c(t) = c_{tr}(t) + c_{ss}(t)$



Ex:- $c(t) = 5 + 10 \sin 2t + e^{-10t} \cos 5t + \dots$

Identify $c_{tr}(t)$ and $c_{ss}(t)$.

$c_{tr}(t) = e^{-10t} \cos 5t$

$c_{ss}(t) = 5 + 10 \sin 2t$

→ Transient Response :-

It is a part of time response that becomes zero as time becomes very large.

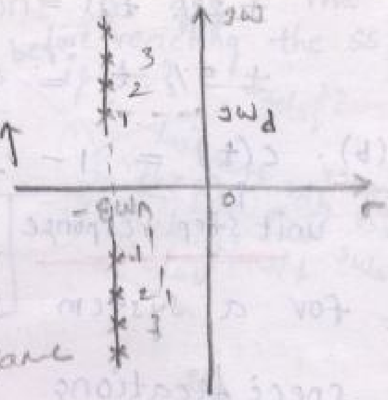
$$\lim_{t\to\infty} c_{tr}(t) = 0$$

$\omega_d \uparrow\uparrow$ $\quad \xi\omega_n$ const.

$t_d, t_r, t_f \downarrow\downarrow$ $\quad \omega_n\sqrt{1-\xi^2} \uparrow$

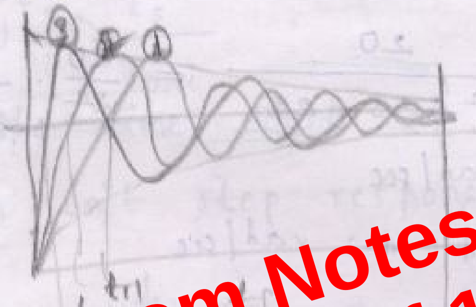
so $\xi\omega_n$ – const $\Rightarrow \sqrt{\omega_n^2 - \xi^2\omega_n^2} \uparrow$

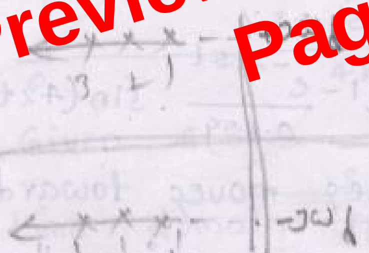
ξ must be decreased $\quad \omega_n \uparrow \quad \xi^2\omega_n^2$ const



$\xi \downarrow \Rightarrow \% M_p \uparrow$ $\quad t_s$ – same

→ * As poles moves vertically $\parallel^{el}$ to $j\omega$ axis, the damping factor is decreased and hence $\% M_p$ increases.





ω_d const ;

$t_p = \frac{\pi}{\omega_d}$ → const.

$\simeq t_r, t_d$

$\omega_d = \sqrt{\omega_n^2 - \omega_n^2\xi^2}$, ξ increases as well as ω_n increases

$t_d = \frac{1 + 0.7\xi \uparrow}{\omega_n \uparrow}$ (approximately const) slight variation

$t_r \downarrow = \frac{\pi - \cos^{-1}(\xi\uparrow) \downarrow}{\omega_d \uparrow}$ slightly

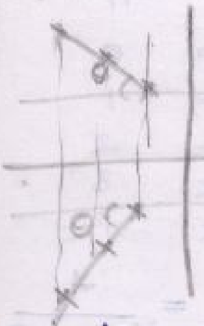
$\tau \downarrow\downarrow \rightarrow t_s \downarrow\downarrow$

$\xi \uparrow\uparrow \rightarrow \% M_p \downarrow\downarrow$

* As ω_d is constant, the t_p is same. Even t_r, t_d are approximately constant.

As the poles moves towards L.H.S., the time constant decreases hence ts decreases.

As ξ increases, the % μ_p decreases.



$\omega_d \uparrow\uparrow \rightarrow t_d, t_r, t_p \downarrow\downarrow$

$\tau \downarrow \rightarrow t_s \downarrow\downarrow$

$\xi = \cos\theta \rightarrow$ % $\mu_p \rightarrow$ const

* As the inclination of the poles is constant, the ξ is constant. hence the % μ_p is constant.

Q. find the time domain specification for unit step i/p. for the given system.

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 8y = 8x$$

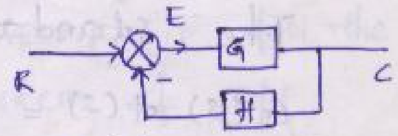
Ans:- $\frac{Y(s)}{X(s)} = \frac{8}{s^2+4s+8}$

steady state errors:-

It is the deviation of o/p from the reference i/p at the steady state [$t \to \infty$]

* $e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$

$= \lim_{s\to 0} \frac{s \cdot R(s)}{1+G(s) \cdot H(s)}$.



$\frac{E(s)}{R(s)} = \frac{1}{1+GH}$

* The SSE are depends on

(1). type of i/p (ie) R(s) (2). type of system ie G(s)H(s) char. eq. $\Rightarrow s^3 + ps^2 + 3s + 1 + k(s+1) = 0$

$\Rightarrow s^3 + ps^2 + s(3+k) + 1 + k = 0$

$s^3 \quad 1 \quad 3+k$

$s^2 \quad p \quad k+1 \leftarrow AE$

$s^1 \quad \frac{p(3+k)-(k+1)}{p} > 0 \ (M.S) \rightarrow = 0 \Rightarrow p = \frac{k+1}{k+3}$

$s^0 \quad k+1$

AE : $ps^2 + (k+1) = 0$

$s = j\omega = j2 \ ; \Rightarrow p = \frac{k+1}{4} \quad k = 1$

$\Rightarrow s^2 = -4 ; \Rightarrow -4p + (k+1) = 0 \quad p = 0.5$

Q. A unity f/b control system has an OL T/F

$G(s) = \frac{k(s+13)}{s(s+3)(s+7)}$. Find the value of k for system stability (b). Determine the value of ξ, $>, <$ $\alpha = 1$ when $k = 1$.

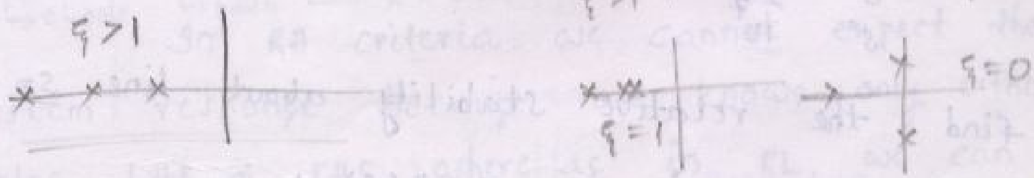
Sol. $s^3 + 10s^2 + (21+k)s + 13k = 0 \quad 210 + 10k > 13k$

$210 > 3k$

when $k = 1$, $\Rightarrow 0 < k < 70$.

$s^3 + 10s^2 + 22s + 13 = 0$

$\Rightarrow s = -1, -1.7, -7.2$



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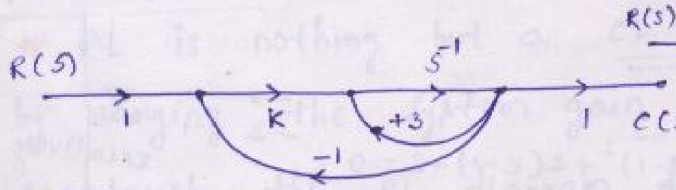
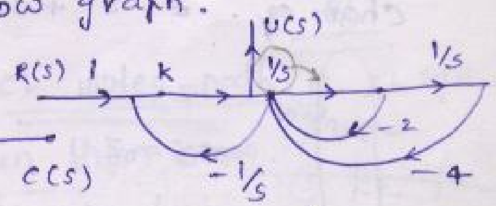
Q. Find the range of k-value for system stability for the given signal flow graph.





$\frac{C(s)}{R(s)} = \frac{k/s}{1 - 3/s + k/s} = \frac{k}{s-3+k}$

$s^1 \quad 1$

$s^0 \quad k-3 > 0 \Rightarrow k > 3.$

$T/F = \frac{k}{1 + \frac{k}{s} + \frac{2}{s} + \frac{4}{s^2}}$

$= \frac{ks}{s^2 + ks + 2s + 4}$

$s^2 \quad 1 \quad 4$

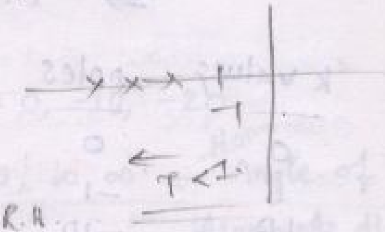
$s^1 \quad k+2$

$s^0 \quad 4 \quad \Rightarrow k > -2.$

Q. check whether the T is greater or lesser or equal to 1 sec. for $s^3 + 7s^2 + 25s + 39 = 0$.

$s = -\frac{1}{T}$ (or)

$s = -1$



$s = z - 1$ Sub. and then solve using R.H

Q. If the RH criteria applicable, is applicable for

* sine & cosine terms - ?

The RH criteria not applicable for trigonometric terms and exponential terms b'coz gives infinite series. but approximate soln. can be obtained for exponential terms. transportation delay system

Q. find the system stability for $G(s) = \frac{e^{-sT}}{s(s+1)}$, $H(s) = 1$

Transpotation delay system not effect the magnitude it effects

$$e^{-sT} \simeq (1 - sT)$$

$$G(s) = \frac{(1-sT)}{s(s+1)}$$

ch. eq $= s^2 + s + 1 - sT = 0$

s^2	1	1	
s^1	$1-T$		$\rightarrow > 0$
s^0	1		$\Rightarrow T < 1$ sec

Root Locus:-

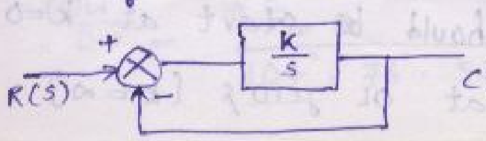
CL pole path $\rightarrow k = 0$ to ∞

In RH criteria we cannot expect the system response because we knows only either poles LHS or RHS where as in RL, we can find the system response by observing the CL poles path.

RL

* RL is nothing but a CL poles path by varying the system gain from 0 to ∞.

Q. construct the RL diagram for the following block diagrams.



① CL sys
② <k<
③ k mar / ω mar
④ k undamped / crit...
⑤ RS
⑥ θ = inclination

Q. find where the RL diagram starts and ends.

$$G(s).H(s) = \frac{k(s+5)}{s(s+10)(s+20)}$$

starts : OL poles $k=0$, $s = 0, -10, -20$

Ends : OL zero's $k=\infty$, $s=-5, \infty, \infty$ ← Along Angle of Asymptote dire.

⇒ Angle & Magnitude condition :-

* The CL system stability is given by char. eq. $1+GH=0$.

The construction rules of RL are obtained from angle & magnitude condition.

$\Rightarrow G(s).H(s) = -1+j0$.

But the RL diagram drawn for O/L T/f ie $GH = -1+j0$ (± 180) $-1+j0$

Angle condition : $\angle G(s).H(s) = \angle -1+j0$

$= \pm(2q+1)180°$, $q = 0,1,2..$

$= \text{odd multiples}(\pm 180)$

(± 360) $2(360)$ $4(360)$

purpose :-

To check any point existing on RL or not that means all the points on RL must satisfy the angle condition.

Q. Check whether the following points lies on root locus plot for $GH = \frac{K}{s(s+2)(s+4)}$

① $s = -0.75$ ② $s = -1+j4$

$$\angle GH = \frac{\angle K}{\angle s \ \angle s+2 \ \angle s+4}\Big|_{s=-0.75}$$

$\tan\theta = 0$, $\tan 180 = 0$

$$= \frac{\angle K}{\angle -0.75 \ \angle 1.25 \ \angle 3.24} = \frac{0°}{\pm 180° . 0° . 0°}$$

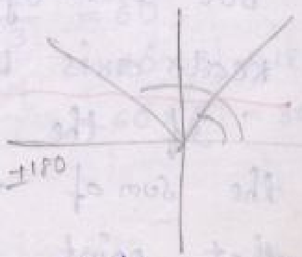
bcoz of -ve sign

$= \pm 180°$ satisfies angle condi. so the given point on RL.

for $s = -1+j4$

$$\angle GH = \frac{\angle K}{\angle(-1+j4) \ \angle(1+j4) \ \angle(3+j4)}$$

± 180



$$= \frac{0}{104° . 76° . 53°}$$ not satisfies, so the given point not on RL.

Intersection point with Real axis $\Rightarrow$ Ima. part $=0$

$\Rightarrow -j\omega^3 + 11 j\omega = 0$ Intersection point with

$\Rightarrow \omega = \sqrt{11}$ rad/sec -ve real axis = ω_{pc}.

$$M = \frac{1}{\sqrt{12 \times 15 \times 20}} = \frac{1}{60}$$

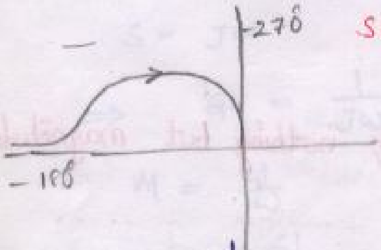
Q. $GH = \frac{1}{s^2(s+1)}$

$\omega = 0;\ \infty \angle -180^\circ$

$\omega = \infty;\ 0 \angle -270^\circ$

cw ED: cw

SD: $\downarrow$P $\rightarrow$ cw



Q. $GH = \frac{1}{s(s+1)}$

$\omega = 0;\ \infty \angle -90^\circ$

$\omega = \infty;\ 0 \angle -180^\circ$

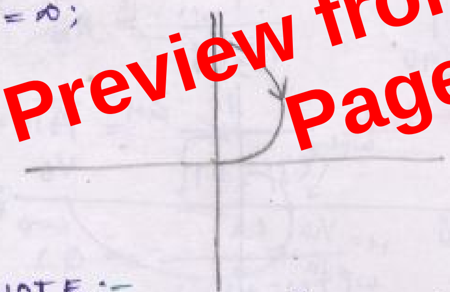
cw ED: +ve cw

SD: $\downarrow$P: cw

Q. $GH = \frac{1}{s^3(s+1)}$

$\omega = 0;$

$\omega = \infty;$



* The addition of pole at origin shift the total plot by -90° in the cw direction.

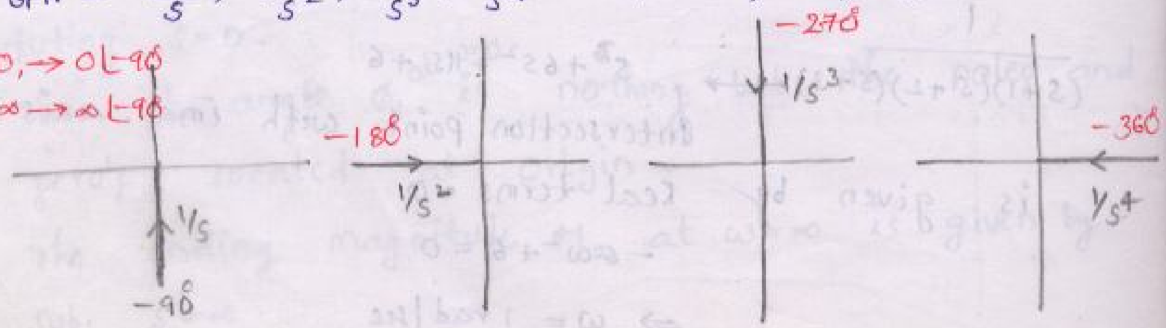
NOTE:- for the poles and z's at origin the polar plot is nothing but a angle lines.

[T/F should not consists any finite p's and z's].

$GH = \frac{1}{s}, \frac{1}{s^2}, \frac{1}{s^3}, \frac{1}{s^4}$ and s, s^2, s^3, s^4.

$\omega = 0, \rightarrow 0 \angle -90^\circ$

$\omega = \infty \rightarrow \infty \angle -90^\circ$



Magnitude condi. $\left| \frac{k}{\omega\sqrt{(4+\omega^2)(16+\omega^2)}} \right|_{\omega_{gc}=0.72} = 1$

$\Rightarrow k = 6.2$

$s(s+2)(s+4)$

s^3+6s^2+8s

$GM = -20 \log M|_{\omega=\omega_{pc}}$ $\Rightarrow \omega_{pc} = \sqrt{8}$ rad/sec

$20 = -20 \log \left| \frac{k}{\omega\sqrt{(4+\omega^2)(16+\omega^2)}} \right|_{\omega=\omega_{pc}=\sqrt{8}}$

$\Rightarrow k = 4.8$.

Q. The OL T/F of a unity f/b control system is given as $G(s) = \frac{as+1}{s^2}$, the value of 'a' to give PM = 45° is —

$45 = 180 - 180 + \tan^{-1}(a\omega)|_{\omega=\omega_{gc}}$

$\Rightarrow a\omega = 1$

$\omega_{gc} = 1/a$

$\left| \frac{\sqrt{(a\omega)^2+1}}{\omega^2} \right|_{\omega=1/a} = 1$

$\Rightarrow a = \frac{1}{\sqrt[4]{2}}$

Q. In the GH plane, the Nyquist plot of T/F

$GH = \frac{\pi e^{-0.25s}}{s}$ passes through the -ve real axis, at a point — ?

1). $(-0.25, j0)$
2). $(-0.5, j0)$
3). $(-1, j0)$
4). $(-2, j0)$

$-180 = -90 - 0.25\omega \times \frac{180}{\pi}\Big|_{\omega=\omega_{pc}}$

$\Rightarrow \omega_{pc} = 2\pi$ rad/sec

exp, m=1

$M = \frac{\pi}{\omega}\Big|_{\omega_{pc}=2\pi} = 0.5$ $(-0.5, j0)$ $e^{j\theta} = \cos\theta + j\sin\theta$

exponential demp never effect magnitude $= \sqrt{\cos^2\theta + \sin^2\theta}$

but effect phase angle

* whenever the T/F not gives the mag. of '1' at any freq. then consider $\omega_{gc} = 0$.

Q. $\dot{x}_1 = -2x_1 + u$, $\dot{x}_2 = 3x_1 - 5x_2$

$$A = \begin{bmatrix} -2 & 0 \\ 3 & -5 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} ; \quad Q_c = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix} \rightarrow \text{controllable}$$

Observability :-

A system is said to be observable, if it is possible to determine initial states of the system by observing the o/p's in a finite time interval.

Kalman's Test for Observability :-

$$Q_o = [\, C^T \quad A^T C^T \quad (A^T)^2 C^T \; \ldots \; (A^T)^{n-1} C^T \,]$$

(or)

$$Q_o = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix} ; \quad \text{Rank of } Q_o = \text{Rank of } A$$

$|Q_o| \neq 0$

$\rightarrow$ Observable

Q. Check the controllability & observability,

$$\dot{x} = \begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u ; \quad y = [1 \quad 1]\, x$$

$$Q_c = \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}, \quad |Q_c| = 0 \longrightarrow \text{Not controllable}$$

$$Q_o = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad |Q_o| = 0 \longrightarrow \text{Not observable.}$$

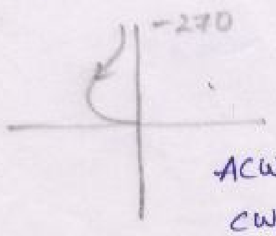
Q. $\dot{x}_1 = -2x_1 + x_2 + u$

$\dot{x}_2 = -x_2 + u$

$y = x_1 + x_2$

$$Q_c = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \xrightarrow{|Q_c| = 0} \text{Not controllable}$$

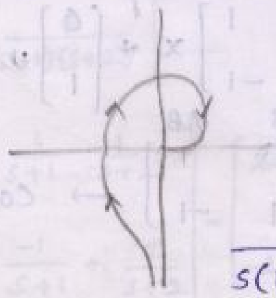
$$Q_o = \begin{bmatrix} 1 & 1 \\ -2 & 0 \end{bmatrix} \rightarrow |Q_o| \neq 0 \quad \text{observable.}$$



ACW → zero

CW → pole

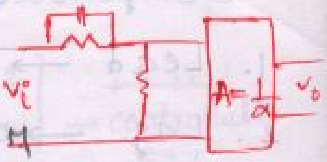
$$\frac{k(1+sT_1)}{s^3}$$



$$\frac{k}{s(1+sT_1)(1+sT_2)(1+sT_3)}$$

Let α – lead const. $= \frac{R_2}{R_1+R_2} < 1$

τ – lead time const. $= R_1 C$



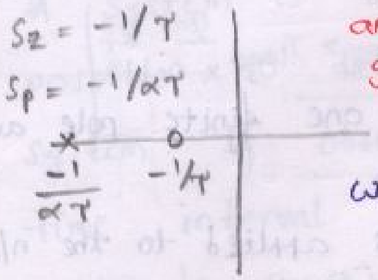
$$\frac{V_o(s)}{V_i(s)} = \frac{(\alpha)(1+\tau s)}{(1+\alpha \tau s)} \cdot \left(\frac{1}{\alpha}\right)$$

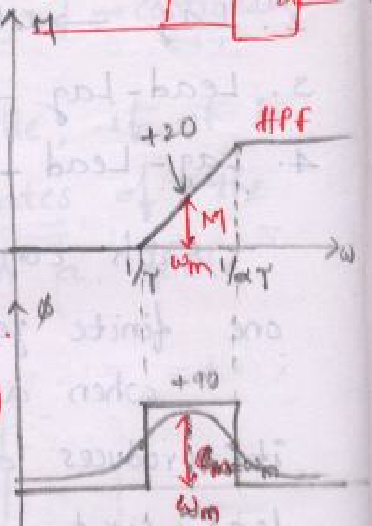
amplifier gain

high pass filter

$S_z = -1/\tau$

$S_p = -1/\alpha\tau$





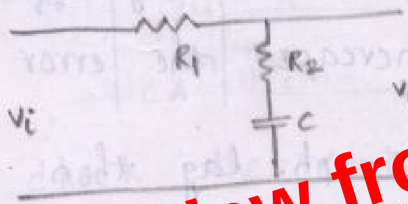
$\omega_m = \frac{1}{\tau\sqrt{\alpha}}$; $M = 10 \log 1/\alpha$

$\phi_m = \sin^{-1}\left(\frac{1-\alpha}{1+\alpha}\right)$

α – Attenuation factor

b'coz α is < 1. The main dis.adv in lead comp. is signal strength is attenuated. So elimi-nate attenuation we required to connect amplifier with gain of $1/\alpha$ in series to compen.

Lag compensator:-



$$\frac{V_o(s)}{V_i(s)} = \frac{R_2 + 1/sC}{R_1 + R_2 + 1/sC}$$

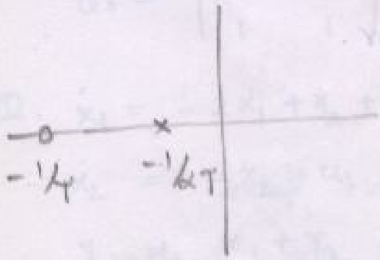
$$= \frac{1 + sCR_2}{1 + \frac{R_1+R_2}{R_2} \cdot sCR_2}$$

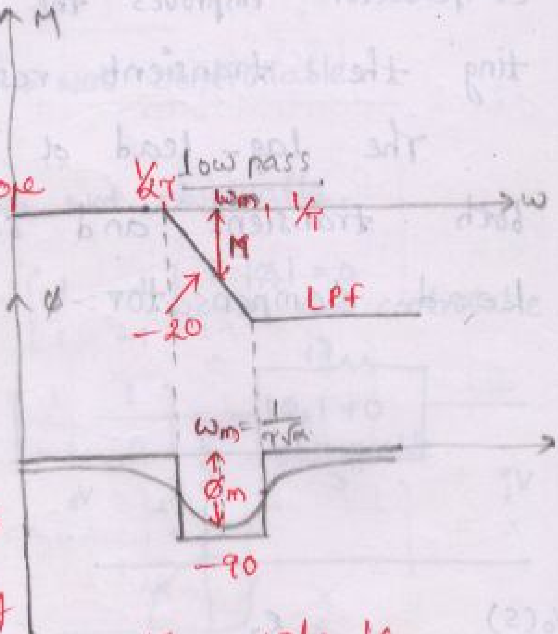
α – lag constant $= \frac{R_1+R_2}{R_2} > 1$

τ – lag time constant $= R_2 C$

$$\frac{V_o(s)}{V_i(s)} = \frac{1+\tau s}{1+\alpha\tau s}$$

Initial slope $= 0$





In compensators zero location is fixed, the change is only in poles location.

$M = 10 \log 1/\alpha$

$\omega_m = \frac{1}{\tau\sqrt{\alpha}}$

$\phi_m = \sin^{-1}\left(\frac{\alpha-1}{\alpha+1}\right)$