Solution

$$|A| = \begin{vmatrix} 6 & 2 & 8 \\ -3 & 4 & 1 \\ 4 & -4 & 5 \end{vmatrix} = 6\begin{vmatrix} 4 & 1 \\ -4 & 5 \end{vmatrix} - 2\begin{vmatrix} -3 & 1 \\ 4 & 5 \end{vmatrix} + 8\begin{vmatrix} -3 & 4 \\ 4 & -4 \end{vmatrix}$$
$$= 6(24) - 2(-19) + 8(-4)$$
$$= 150$$

(c) Show that $A(\operatorname{adj} A) = (\operatorname{adj} A)A = \det(A)I_3$.

Solution

$$A(\operatorname{adj} A) = \begin{bmatrix} 6 & 2 & 8 \\ -3 & 4 & 1 \\ 4 & -4 & 5 \end{bmatrix}\begin{bmatrix} 24 & -42 & -30 \\ 19 & -2 & -30 \\ -4 & 32 & 30 \end{bmatrix}$$
$$= \begin{bmatrix} 150 & 0 & 0 \\ 0 & 150 & 0 \\ 0 & 0 & 150 \end{bmatrix} = 150\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \det(A)I_3$$

$$(\operatorname{adj} A)A = \begin{bmatrix} 24 & -42 & -30 \\ 19 & -2 & -30 \\ -4 & 32 & 30 \end{bmatrix}\begin{bmatrix} 6 & 2 & 8 \\ -3 & 4 & 1 \\ 4 & -4 & 5 \end{bmatrix}$$
$$= \begin{bmatrix} 150 & 0 & 0 \\ 0 & 150 & 0 \\ 0 & 0 & 150 \end{bmatrix} = 150\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \det(A)I_3$$

9. Given
$$A^{-1} = \frac{1}{\det(A)}(\operatorname{adj} A).$$
Find the inverse, if it exists, of

(a) $\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 2 \\ 3 & 2 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & -1 & 2 \\ 1 & -1 & 2 & 1 \\ 1 & 3 & 3 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 2 \\ 1 & 2 & -1 & 1 \\ 5 & 9 & 1 & 6 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$

(e) $\begin{bmatrix} 1 & 2 & 2 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}$

Solution

The matrix is singular or noninvertible since the determinant is 0.

(d) Finding determinant:

$$\begin{vmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 1 & 1 & 2 \end{vmatrix} = 1\begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} + 1\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 1+1 = 2.$$

Finding cofactors:

$$\begin{aligned} A_{11} &= \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 1, A_{12} = -\begin{vmatrix} 0 & 3 \\ 1 & 2 \end{vmatrix} = 3, A_{13} = \begin{vmatrix} 0 & 2 \\ 1 & 1 \end{vmatrix} = -2, \\ A_{21} &= -\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = -1, A_{22} = \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 1, A_{23} = -\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0, \\ A_{31} &= \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 1, A_{32} = -\begin{vmatrix} 1 & 1 \\ 0 & 3 \end{vmatrix} = -3, A_{33} = \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} = 2. \end{aligned}$$

Finding adjoint:

$$\operatorname{adj} A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 3 & 1 & -3 \\ -2 & 0 & 2 \end{bmatrix}$$

Finding inverse:

$$A^{-1} = \frac{1}{\det(A)}(\operatorname{adj} A) = \frac{1}{2}\begin{bmatrix} 1 & -1 & 1 \\ 3 & 1 & -3 \\ -2 & 0 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{3}{2} & \frac{1}{2} & -\frac{3}{2} \\ -1 & 0 & 1 \end{bmatrix}.$$

(e) Finding determinant:

$$\begin{vmatrix} 1 & 2 & 2 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{vmatrix} = 1\begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} - 1\begin{vmatrix} 2 & 2 \\ 1 & 3 \end{vmatrix} + 1\begin{vmatrix} 2 & 2 \\ 3 & 1 \end{vmatrix} = 8-4-4 = 0.$$

The matrix is singular or noninvertible since the determinant is 0.

10. If possible, solve the following linear system by Cramer's rule:

$$\begin{aligned} 3x + 4y + 6z &= 4 \\ x + 2y \quad\quad &= 4 \\ x + 3y - z &= 8 \end{aligned}.$$

Solution