MATH20802: Statistical Methods
Semester 2
Formulas to remember for the final exam

The moment generating function of a random variable X is $M_X(t) = E\left[\exp(tX)\right]$.

The fact that $E\left(X^n\right) = M_X^{('n)}(0)$.

The moment generating function of a $\Gamma(a, \lambda)$ random variable (where a is the shape parameter and λ is the scale parameter) is $M_X(t) = \left(\frac{\lambda}{\lambda - t}\right)^a$.

The moment generating function of an $Exp(\lambda)$ random variable is $M_X(t) = \frac{\lambda}{\lambda - t}$.

$\widehat{\theta}$ is an unbiased estimator of θ if $E\left(\widehat{\theta}\right) = \theta$.

$\widehat{\theta}$ is an asymptotically unbiased estimator of θ if $\lim_{n\to\infty} E\left(\widehat{\theta}\right) = \theta$.

The bias of $\widehat{\theta}$ is $E\left(\widehat{\theta}\right) - \theta$.

The mean squared error of $\widehat{\theta}$ is $E\left[\left(\widehat{\theta} - \theta\right)^2\right]$.

$\widehat{\theta}$ is a consistent estimator of θ if $\lim_{n\to\infty} E\left[\left(\widehat{\theta} - \theta\right)^2\right] = 0$.

The discrete uniform distribution on the set of integers $\{1, 2, \ldots, N\}$ has the probability mass function $p(x) = \frac{1}{N}$ for $x = 1, 2, \ldots, N$.

The gamma function is defined by $\Gamma(a) = \int_0^\infty t^{a-1} \exp(-t) dt$.

The fact that $\Gamma(n) = (n-1)!$.

The fact that $\Gamma(x+1) = x\Gamma(x)$.

The fact that $\Gamma(1/2) = \sqrt{\pi}$.

The beta function is defined by $B(a, b) = \int_0^1 t^{a-1}(1-t)^{b-1} dt$.

The fact that $B(a, b) = \dfrac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$.

The probability density function of $X \sim Exp(\lambda)$ is $f_X(x) = \lambda \exp(-\lambda x)$.

The cumulative distribution function of $X \sim Exp(\lambda)$ is $F_X(x) = 1 - \exp(-\lambda x)$.

The cumulative distribution function of $X \sim N(0, 1)$ is $\Phi(x)$.

The Type I error of $H_0 : \mu = \mu_0$ versus $H_0 : \mu \neq \mu_0$ occurs if H_0 is rejected when in fact $\mu = \mu_0$.

The Type II error of $H_0 : \mu = \mu_0$ versus $H_0 : \mu \neq \mu_0$ occurs if H_0 is accepted when in fact $\mu \neq \mu_0$.

The significance level of $H_0 : \mu = \mu_0$ versus $H_0 : \mu \neq \mu_0$ is the probability of type I error.

The power function of $H_0 : \mu = \mu_0$ versus $H_0 : \mu \neq \mu_0$ is $\Pi(\mu) = \Pr\left(\text{Reject} H_0 \mid \mu\right)$.

Let $X_1, X_2, \ldots, X_m$ be a random sample from a normal population with mean μ_X and variance σ_X^2 assumed known. Let $Y_1, Y_2, \ldots, Y_n$ be a random sample from a normal population with mean μ_Y and variance σ_Y^2 assumed known. Assume independence of