

- $E(\max(X,Y)) = (a + b)/2$  by symmetry (see graph)
- $X \sim U(0, 5)$

$$Y \sim U \quad -x^2/25 < y < -x^2/25$$

$$\Rightarrow Y|X=4 \sim U(-16/25, 16/25)$$

- $X$  and  $Y \sim U \quad 0 < x < \sqrt{y}$

$$\Rightarrow X|Y \sim U(0, \sqrt{y})$$

- $X$  has the uniform distribution on the unit interval:  $X \sim U(0,1)$

$$E(\max(X_1, \dots, X_n)) = \int_0^a 1 dx + \int_a^b s(x) dx$$

NB. -It's 1 because of the graph (as usual).

- $E(X|X \geq 3) = E(X) + 2$  for a Geometric
- $E(\min(X_1, \dots, X_n)) = \frac{1}{\sum \lambda_i}$  (shortcut for an exponential)

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Page 8 of 15

For an Exponential:

No memory:

$$-P(X > m+n | X > m) = P(X > n)$$

$$-\text{Var}(Y|X > 3 \text{ and } Y > 3) = \text{Var}(Y|Y > 3),$$

because of independence

$$= \text{Var}(Y),$$

because of no memory

$$-E[(X-1)^3 | X \geq 1] = E(X^3)$$

- For a Poisson:

$$X \sim P(\lambda^2) \Rightarrow X+Y \sim P(5\lambda^2)$$

$$Y \sim P(4\lambda^2)$$

- For a Poisson:

$$E(Y) = 0.2*1 + 0.4*2 = 1$$

- $Mode = \begin{cases} x \text{ in } f'(x) = 0, & \text{with discrete function} \\ x \text{ with the biggest probability,} & \text{with continuous function} \end{cases}$

NB. –For the discrete case, the value with the biggest probability is the probability before it decreases.

- $P(X \leq 3/4 | Y = 1/2) =$   $0 \leq y \leq x \leq 1$

$$\int_{1/2}^{3/4} f_{X|Y}(X|1/2) dx, \quad \text{with continuous function}$$

$$\text{where } f_{X|Y}\left(X\left|\frac{1}{2}\right.\right) = \frac{f_{X,Y}(X,1/2)}{f_Y(1/2)}$$

$$P(X \leq 3/4 | Y = 1/2) = \frac{P(X \leq 3/4, Y = 1/2)}{P(Y = 1/2)}, \quad \text{with discrete function}$$

- $P(X \leq 3/4 | Y \geq 1/2) = \frac{P(X \leq 3/4, Y \geq 1/2)}{P(Y \geq 1/2)},$  with discrete or continuous function

- $\text{Var}(H|L=0) = E[(H|L=0)^2] - (E[H|L=0])^2,$

where  $E[H|L=0]$  using  $P(H=0|L=0)$  and  $P(H=1|L=0)$  (for example)

$$P(H=0|L=0) = \frac{P(H=0, L=0)}{P(L=0)}$$

- $P\left(X \leq \frac{1}{2} \text{ or } Y \leq \frac{1}{2}\right) = P\left(X \leq \frac{1}{2}\right) + P\left(Y \leq \frac{1}{2}\right) - P\left(X \leq \frac{1}{2}, Y \leq 1/2\right)$

- $E(\max(X_1, \dots, X_n)) = \int y f_{\max}(y) dy,$

$$\text{where } f_{\max}(y) = \frac{d}{dy} P(\max(X_1, \dots, X_n) \leq y) = \frac{d}{dy} \prod_{i=1}^n P(X_i \leq y)$$

$$= \frac{d}{dy} (F_x(y))^n$$

- $E(\max(T, 2)) = \int_0^2 2f(x) dx + \int_2^\infty xf(x) dx$

- -Univariate distribution :

$$E(\min(X_1, \dots, X_n)) = \int xf_{\min}(x) dx,$$