FORMULA SHEET

Part 1 (Interest Theory)

Other Formulas and Principles

- 99% of the time, my mistake is a mistake of retranscription in one of the formulas, a mistake in what to calculate or a mistake in not using the good force of interest.

  NB. -Don't waste your time recalculating with the calculator.
- Calculator: Format AOS and 8 decimals
- Outlay = payment
- 60 through 72: [60; 72]
- One payment every other year = one payment every 2 years
- She borrows X that she intends to repay 5 years later: at $t = 5$, $X(1+i)^5$ not $\cancel{X}$
- A is greater than B: $A = 8 + B$ not $\cancel{A = 8B}$
- Write in M if there are 6 digits or more to simplify the writings. Furthermore, don't take into account decimals if there are 4 digits or more at the left. If not, take only 2 decimals.
- Each payment will be 3% less than the preceding payment: 0.97^{t-1} not $\cancel{(1.03)^{-(t-1)}}$
- At the end of 20 years, the total in the two funds is 10,000:

  $\sum funds = 10{,}000$ not $\cancel{fund_1 = fund_2 = 10{,}000}$
- Always put the force of interest in function of payment frequency to have this: $t = 0, 1, 2, \ldots$ For example, $I_t = iB_{t-1}$ with payments are every 2 years with: $t = 0, \frac{1}{1/2}, \frac{2}{1/2}, \ldots$ not $t = 0, \cancel{2, 4}, \ldots$

$\mathrm{B}_0 f_{\delta_i}(0;1)$ $\mathrm{B}_0 f_{\delta_i}(n-1;n)$

Separate funds: |-------|--- --|-------> δ_j

(interest) 0 1 ... n

$\mathrm{B}_0 f_{\delta_i}(1;2)$

|--------------|------------------------------------ --|-------> δ_j

0 2 ... n

...

... $\mathrm{B}_0 f_{\delta_i}(n-1;n)$

|---------------------------- --------------------------|-------> δ_j

0 ... n

$\mathrm{B}_0 f_{\delta_i}(0;1) f_{\delta_j}(1;2)$ ... $\sum_{t=1}^{n-1} \mathrm{B}_0 f_{\delta_i}(t-1;t) f_{\delta_j}(t;t+1)$

Side funds: |---------------|----------------------------------- --|-------> δ_k

(interest on interest) 0 2 ... n

$$\mathrm{B_n} = \mathrm{B_0} + \sum_{t=1}^{n} \mathrm{B}_0 f_{\delta_i}(t-1;t)$$

$$+ \sum_{p=1}^{n-1} \sum_{t=1}^{p} \mathrm{B}_0 f_{\delta_i}(t-1;t) f_{\delta_j}(t;t+1) \prod_{t=p+2}^{n} \left(1 + f_{\delta_k}(t-1;t)\right)$$

NB. -It doesn't change anything that the funds are separated (for the individual separate funds).

-With the force of interest of i, j and k for these 3 funds:

$\mathrm{B_n} = \mathrm{B_0} + \mathrm{n}(\mathrm{iB_0}) + (\mathrm{ijB_0})(Is)_{n-1|k}$.

-Interest reinvested in a different fund with R level payments (more simple case):

R R

|---|-------------------------- --|-------> i

0 1 n

b) Lender's point of view:

$L(1+r)^n = Lis_{n\rceil i=r} + L$

➔ Bond:

-Without reinvestment:

$\mathrm{P}\ (1+r)^n = coupon * s_{n\rceil r} + C,$

coupon = Fr

-With reinvestment:

$\mathrm{P}\ (1+r)^n = coupon * s_{n\rceil j} + C$

NB. –r, the overall yield, has nothing to do with r, the coupon rate.

➔ Short selling:

$Haircut(1+r)^n = FV(S_0)_{\delta\ or\ r_F} - S_T + Haircut(1+i_H)^n$

-For r_A and r_B from 0 to t and t to n.

➔ Callable bond with A who sells the bond to B at time t with reinvested coupons:

↓

|--------------------------------|-------> i (to calculate $P_{min}(0;t)$)

0 n

↓

|-------------|-----------------|-------> j_A = reinvestment rate

0 t n

$P_{min}(0;t)(1+r_A)^t = Frs_{t\rceil j_A} + \mathrm{C_t},$ (to isolate C_t)

where $P_{min}(0;t)$ = price of the callable bond = $Fra_{t\rceil i} + \mathrm{C_t v^t}$

C_t = call price of the bond = sold price of the bond$_t$

|------------------------------ --|-------> i

0 n

$C_t(1+r_B)^{n-t} = Frs_{\overline{n-t}|r_B} + C_n,$ (to find r_B)

where C_t = sold price of the bond_t

C_n = maturity value

NB. –“P” is not necessary equal to $P_{min}(0;t)$

-“r”, the coupon rate, has nothing to do with r_A and r_B, the overall yields.

-If there was no j_A, it would be i_A to isolate C_t.

-Because there are no reinvested coupons for B, we use i_B and ~~not~~ j_B.

- Internal rate of return (IRR) with NPV:

➔ If the investment comes from the company:

NPV = $-\text{Investment} + PV(revenues)_{IRR,j} - PV(expenses)_{IRR}$, (to find IRR)

= 0

where $PV(revenues)_{IRR,j} = \frac{\sum_{k=1}^{n} CF_{t_k}(1+j)^{t_n - t_k}}{(1+IRR)^{t_n}}$

n = number of cash inflows

t_k = time of the k^{th} moment

j = reinvestment rate

➔ If there is a loan as the investment:

NPV = L - $Li_{loan}a_{\overline{n}|IRR} - Lv_{IRR}^{n} - PV(expenses)_{IRR}$

$t = t^{th} moment$

$m = number\ of\ times/year\ the\ interest\ applies$

$B_{t_2} = B_{t_1}(1+i)^{t_2-t_1} \sum_{j=t_1+1}^{t_2} CF_j\,(1+i)^{t_2-j}, \qquad t_1 < t_2$

NB. -Same thing as bonds at premium.

- Amortization method:

-the accumulated amount is paid at the end as a lump sum:

|----------------------------- --|------->

0 n

$\sum$ I = L$(1+i)^n$- L

-level payments:

R R

|---|--->

0 1 ... n

$\sum$ I = nR – L,

where L = R$a_{n|i}$

-level principal payments:

X + f(0;1)B_0 X + f(n-1;n)B_{n-1}

|---|-------------------------- --|------->

0 1 ... n

$\sum$ I = $\sum_{t=1}^{n} f(t-1;t)(L-(t-1)X)$,

where R_t = X + I_t

Chap. 7 (Bonds)

- The annual coupon rate is 7% (with semi-annual coupons): $r^{(2)} = 7\%$
- \$5000 par bond: F = 5000
- A par coupon bond = \$1 par bond
- A 4% \$100 bond with semi-annual coupons: $r^{(2)} = 4\%$ and F = 100
- A \$10,000 5-year equity-linked CD: F = 10,000
- To purchase a bond at a price of 1700 (for a callable bond): $P_{min}(0; t) = 1700$

  not ~~P = 1700~~

  NB. -They don't say it's a bond price for a callable bond. They just say it's the price of the bond.

- The bond is sold at a price equal to its value = matures at par = redeemable at par = callable at par = a par value bond = a par bond = the bond is called at par value: C = F, not the bond is sold at par (P = F = C).

  NB. -These are expressions: "to call at", "to redeem at" and "to mature at".

- F = C and r = i => P = F = C,

  where C_t is not necessary equal to the maturity value (C at time n).

- By default, F = C if not mentioned (only seen in the spot rate numbers).
- $P_{min}(0; t)$ = minimum redemption value = minimum call price = call price

  NB. -It is not necessary equal to P or C_n.

- Accumulation bond = zero-coupon bond
- Bond with annual coupons of 6.75% <u>at par</u> = to buy a bond <u>at par</u>

  = bond A is priced <u>at par</u> = bond sells <u>at par</u>

  = to buy the bond with no premium or discount:

- ModD = vMacD = $-\frac{P'}{P}$

  NB. -ModD was developed by drawing a tangent line to the price curve: 

- Convexity$_L$ = $\frac{P_L''}{P_L}$ is not necessary equal, smaller or bigger than:

  $\sum_{j=1}^{n} w_j Convexity_{A_j} = \frac{P_A''}{P_A}$

- Change in the interest rate (in A or L):

  $P_{i\ after} = P_{i\ before} + \Delta P$,

  where $\Delta P \approx -P_{i\ before} * \Delta i * ModD$ (approximation with 1 term)

  $\Delta i = i_{after} - i_{before}$

- If there is a CF (A or L) at t = 0:

  $\begin{cases} \text{Defer all the CF (L and A) of one year, with exact matching} \\ \text{It doesn't change anything, with RI and NI} \end{cases}$ (to find w_j)

- Immunization:

  -Characteristics:

  ➔ Either MacD or ModD can be used to develop an immunization strategy

  ➔ The yield curve structure is not relevant

  ➔ Matching the PVs is not sufficient when the interest rates change

- To immunize = to exactly match = to exactly (absolutely) match = to match = to produce exact matching = method if not mentioned how to immunize

➔ The yield curve shifts in parallel when the interest rate changes

-if "i" is the same for A and L:

1) MacD = $\sum_{j=1}^{n} w_j MacD_{A_j}$

2) $P_L = \sum_{j=1}^{n} P_{A_j}$

where $P_{A_j} = w_j P_L$

NB. -Or we can just use the second equation for 2).

-otherwise (conditions for Redington immunization):

1) $P_A = P_L$

2) $P'_A = P'_L$

3) $P''_A > P''_L$

NB. -$P'_A = P'_L$ comes from -ModD$_A$ = $-\frac{P'_A}{P_A} = -\frac{P'_L}{P_L}$ = -ModD$_L$ and $P''_A > P''_L$ comes from Convexity$_A$ = $\frac{P''_A}{P_A} > \frac{P''_L}{P_L}$ = Convexity$_L$.

➔ To validate the RI conditions, we must find P_{A_1} and P_{A_2} and then validate with 3).

3- Full immunization:

-Characteristic:

➔ It protects against any change in the interest rate

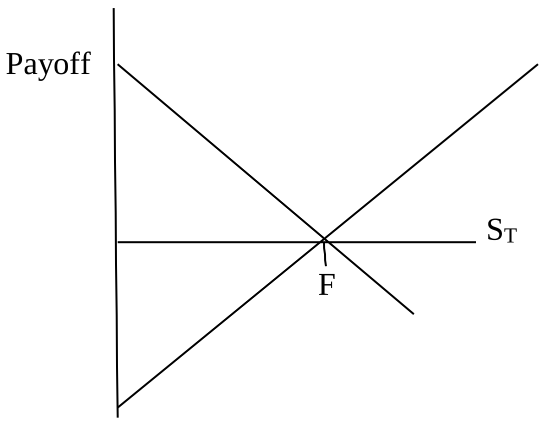
-if "i" is the same for A and L:

1) MacD = $\sum_{j=1}^{n} w_j MacD_{A_j}$

2) $P_L = \sum_{j=1}^{n} P_{A_j}$ or $P_{A_j} = w_j P_L$

We buy a forward to lock in the purchase price (we want S_T = 1,025). He wants to sell an asset at t = T.

- F or F(S):



NB. -Slope = 1 or -1.

- Forward:

Profit_T = npayoff – FV(premium),

where n = number of options

$$\text{payoff} = \begin{cases} S_T - F, & \textit{with a long forward} \\ F - S_T, & \textit{with a short forward} \end{cases}$$

F = forward price = observed forward price = current forward price

$$\text{premium} = \begin{cases} PV(F) - PV(K) = C_k - P_k, & \textit{with a long forward} \\ PV(K) - PV(F) = P_k - C_k, & \textit{with a short forward} \end{cases}$$

$$= \begin{cases} 0, & \textit{usually} \\ \neq 0, & \textit{with an off}-\textit{market forward} \end{cases}$$

$$PV(F) = F^P = \begin{cases} S_t - PV(\text{dividends}), & \text{with discrete dividends} \\ S_t e^{\delta_D (T-t)}, & \text{with continuous dividends} \\ S_t, & \text{with no dividends} \end{cases}$$

= prepaid forward

NB. –There is one contract.

-> This strategy locks in the selling price (for example, wheat) between K_1 and K_2.

NB. -Strategy = collar, but with a stock.

-> zero-cost collar:

a) the put <u>can</u> be at-the-money: $K_1 - S_0 = 0 \Rightarrow K_1 = S_0$

b) the call <u>cannot</u> be at-the-money: $S_0 - K_2 = 0 \Rightarrow S_0 = K_2$

NB. –We expect: $K_1 > S_0$: we expect $Profit_0 = K_1 - S_0 > 0$.

-But, $K_1 < K_2 = S_0$ if the call is at-the-money.

-We don't mind it's a zero-cost collar.

c) $K_1 \leq F \leq K_2$

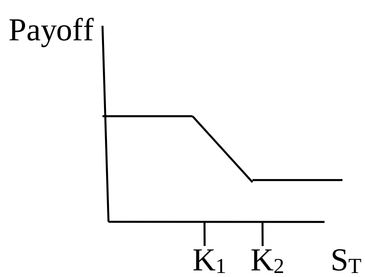
NB: -Proof of $K_1 \leq F$

$0 = P_{k_1} - C_{k_2} = C_{k_2} - P_{k_1} \leq F = C_{k_1} - P_{k_1} = S_0 - PV(K_1) = PV(F) - PV(K_1)$

$0 \leq PV(F) - PV(K_1)$

$0 \leq F - K_1 \Rightarrow \boxed{K_1 \leq F}$

6) Bear spread:



Premium = $-P_{k_1} + P_{k_2}$, $k_1 < k_2$

$= -C_{k_1} + C_{k_2}$