**LESS IMPORTANT FORMULAS AND PRINCIPLES**

## Part 1 (Interest Theory)

### Other Formulas and Principles

- Dates in the US :

  Month/Day/Year

- Rebate = remboursement ou rabais
- Triennial = triennal = every three days
- Stock index = indice boursier (en français)
- IRA = Individual Retirement Account
- Geometric mean < arithmetic mean
- To rationalize the denominator:

$$\frac{1}{\sqrt{a}-b} * \frac{\sqrt{a}+b}{\sqrt{a}+b} = \boxed{\frac{\sqrt{a}+b}{a-b^2}}$$



- $n!$ = the factorial of $n$ = $\prod_{t=1}^{n} t$
- 1 + ... + n = $\frac{n(n+1)}{2}$
- $1^2 + ... + n^2 = \frac{n(n+1)\,(2n+1)}{6}$
- Mathematically, when we do an integral, we must not have to same boundaries than the derived variable. For example, $\int_0^n f(t)\, e^{-\int_0^t \delta_r dr}\, dt$. But, this is not really important.
- $\frac{d}{dx} \mathrm{a}^{\mathrm{x}} = \mathrm{a}^{\mathrm{x}} \ln \mathrm{a}$

NB. $-a_n = \frac{f^{(n)}(0)}{n!}$ is fund with moment-generating function (MGF): $E(X^n) = f^{(n)}(0)$.

<u>Chap. 1 (The Measurement of Interest)</u>

- Rule of 72:

  Years required to double investment = 72 ÷ (compound annual interest rate*100)

  NB. –It's an approximation.

- $i_t = \frac{a(t) - a(t-1)}{a(t-1)} = f(t-1; t)$

- $d_t = \frac{a(t) - a(t-1)}{a(t)}$

- Amount function: $A(t) = A(0)*a(t)$ (fonction de capitalisation en français),

  where $A(0)$ = initial amount

- Accumulation function: $a(t)$

- Discount factor: $1/a(t) = 1/(1 + r)^t$ (fonction d'actualisation en français),

  where r = discount rate

- $d = d^{(1)} < d^{(2)} < \ldots < \delta < \ldots < i^{(2)} < i^{(1)} = i$

- $e^{\delta} = 1 + i = \frac{\delta^0}{0!} + \frac{\delta^1}{1!} + \frac{\delta^2}{2!} + \cdots = \sum_{t=0}^{\infty} \frac{\delta^t}{t!}$

- $\delta = \lim_{m\to\infty} d^{(m)} = \lim_{m\to\infty} i^{(m)} = \lim_{m\to\infty} m(1+i)^{\frac{1}{m}} - m = \frac{0}{0}$

$$= \frac{(1+i)^{\frac{1}{m}} - 1}{1/m} \overset{RH}{=} \lim_{m\to\infty} \frac{\frac{(1+i)^{\frac{1}{m}}}{\cancel{m^2}} \ln(1+i)}{\cancel{-\frac{1}{m^2}}} = \lim_{m\to\infty} \frac{(1+i)^{\frac{1}{m}}}{\ln(1+i)^{-1}} = \boxed{\ln(1+i)}$$

$$= \frac{i^1}{1} - \frac{i^2}{2} + \cdots = \sum_{t=0}^{\infty} \frac{i^t}{t} - 1 - 2\left(\sum_{t=0}^{\infty} \frac{i^{2t+2}}{2t+2}\right)$$

$$= -\ln(1-d) = \frac{d^1}{1} + \frac{d^2}{2} + \cdots = \sum_{t=1}^{\infty} \frac{d^t}{t}$$

$$= \frac{d^1+i^1}{2} + \frac{d^2-i^2}{4} + \cdots$$

$$= \frac{1}{2}\left(\sum_{t=0}^{\infty} \frac{d^t}{t} - 1 + \sum_{t=0}^{\infty} \frac{i^t}{t} - 1 - 2(\sum_{t=0}^{\infty} \frac{i^{2t+2}}{2t+2})\right)$$

- $d = 1 - e^{-\delta} = \frac{\delta^1}{1!} - \frac{\delta^2}{2!} + \cdots = \sum_{t=0}^{\infty} \frac{\delta^t}{t!} - 1 - 2\sum_{n=0}^{\infty} \frac{\delta^{2n+2}}{(2n+2)!}$

$$= 1 - \sum_{t=0}^{\infty} \frac{\delta^t}{t!}$$

- To find “IRR“ in a multivariate function:

1- Calculator method:

1) Click on CF, then enter CF at t = 0.
2) Click on Enter. Do the same thereafter for all CF with F01 meaning frequency at t = 1, for example, for how many years I have the same CF.
3) Click on CPT, and IRR.

2- Bisection method: (if we have opposite signs)

1) Evaluate the function with two different “i”s.
2) Calculate the middle point and, then evaluate the function at this point.
3) Do the same thing until having f(i) = 0.

3- Newton-Raphson method (recursion method):

1) $F(i) = a + b(1+i) + \ldots z(1+i)^y = 0$
2) Find f’(i).
3) Find “i” in this: $x_{s+1} = x_s - \frac{f(x_s)}{f(x_{s+1})}$ .

- Accumulation methods :

  - Compound interest:

| | | | | |
|---|---|---|---|---|
| **Total** | **$5,000** | **$670.52** | **$4,329.48** | |

NB. -i = 5% in this example.

- Sinking fund = fonds d'amortissement (en français)

Chap. 7 (Bonds)

- Price of a bond = book value:

-with linear method:

$B_t = Fr - t*P_t$,

where $P_t$ = adjustment = $\frac{P-C}{n}$,

P = purchase price or selling price

-with actuarial method:

$B_{t_2} = B_{t_1}(1+i)^{t_2-t_1} - \sum_{j=t_1+1}^{t_2} CF_j(1+i)^{t-j}$ (with retrospective formula)

$= Fra_{\overline{n-t_2}|i} + Cv^{n-t_2}$ (with prospective formula)

NB. –The method we use for the FM Exam is the 2nd one (the actuarial method).

- Price between 2 coupons:

$B^f_{t+k} = B^m_{t+k} + Fr_t$, $0 < k < 1$

= flat price = book value between two coupons (prix uniforme en français)

= actual price = full price = price-plus-accrued

= theoretical dirty price = $\begin{cases} B_t(1+i)^k, \; with\ assumption\ 1 \\ B_t(1+ik), \; with\ assumption\ 2 \\ B_t(1+i)^k, \; with\ assumption\ 3 \end{cases}$

- Arbitrage opportunity:

  1) $F \neq F(S)$: if $D\uparrow$ or $i\downarrow$ => $\text{Profit}_T$ (stock) > $\text{Profit}_T$ (forward)

     NB. –Buy low and sell high.

  2) observed $P_K$ or $C_K \neq$ (theoretical) $P_K$ or $C_K$ with Put/call parity:

     $PV(F) + P_k = PV(K) + C_k$

     NB. –Buy one side and sell the other$_0$ and do the opposite$_T$.

<u>Chap. 10 (Introduction to Derivatives)</u>

- Financial engineering = construction of a financial product from other products
- Go-between = intermediary
- Broker as a go-between = back-to-back = matched book
- Short selling (vente à découvert en français) = selling of a stock that the seller doesn't own



- Underlying assets: stocks, futures, commodities, currency index, bonds, etc.
- Number of shares of a stock at time t = $e^{-\delta(T-t)}$
- -A $(\text{option})_t$ = Ask price $(\text{option})_t$ (prix du vendeur $(\text{option})_t$ = cours vendeur $(\text{option})_t$ – en français)

  -B $(\text{option})_t$ = Bid price $(\text{option})_t$ (prix de l'acheteur $(\text{option})_t$ = cours acheteur $(\text{option})_t$ – en français)

  NB. -Same thing for a stock.

<u>Chap. 11 (Forward Contracts)</u>

- Forward contract (contrat à livrer en français) = agreement to enter into a transaction at a pre-specified time and price

$Profit_T$ = payoff – FV(premium),

where payoff = $-S_T - \max(0; K - S_T) = -\max(0; S_T - K) - FV(Z)$

premium = $-S_0 - P_k = -C_k - Z$

$$-Z = \begin{cases} borrowing\ or\ selling\ a\ zero - coupon\ bond_0 \\ lending\ or\ buying\ a\ zero - coupon\ bond_T \end{cases} = -PV(K)$$

<u>Chap. 16 (Put/Call Parity; Combining Options)</u>

- American, European, Bermudan options or the greeks, for example, have nothing to do with the geographic location
- European option = option which may be exercised only at the expiration date of the option, i.e. at a single pre-defined point in time.
- American option = option which may be exercised at any time before the expiration date.
- Bermudan option = a type of exotic option that can be exercised only on predetermined dates, typically every month.



| **Strategy** | **Reasons for Using Strategy** | **Comments** |
|---|---|---|
| Synthetic Long Forward | Expect stock price to rise; don't want insurance | - |
| Bull spread | Expect stock price to rise; want insurance if it declines | - |
| Box Spread | To borrow or lend money | Definite return (no stock market risk) |