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$f(x_0, y_0) = x + y^2 = 1.3465$

$k_1 = 0.1 \times 1 = 0.13465$

$k_2 = hf \cancel{(0.05, 0.1.05)} (0.15, 1.1838)$

$= \cancel{0.05 + (1.05)^2 \times} 0.1 = 0.1551$

$= \cancel{0.11525}$

$k_3 = hf(0.15, 1.1940)$

$= 0.1575$

$k_4 = hf(0.2, 1.274)$

$= 0.1823$

$\cancel{k_5} = k = \frac{1}{6}[\qquad]$

$k = 0.1544$

$y = y_0 + k$

$= 1.221$

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$y = y_0 + k = 1.11069$

Iteration. 2.

$x_0 = 1.2,\ y_0 = 1.11069\ ;\ h = 0.1$

$f(x,y) = xy$

$k_1 = hf(x_0, y_0) = 0.1332$

$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$

$= 0.1 f(1.25, 1.1772)$

$k_2 = 0.1471$

$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$

$= 0.1 f(1.25, 1.1842)$

$k_3 = 0.1480$

$k_4 = hf(x_0 + h, y_0 + k_3)$

$= 0.1(1.3, 1.2587)$

$= 0.1636$

$k = 0.1478$

$y = y_0 + k = 1.2584$

Type. IV

Taylor's Series Method.

$$y = y_0 + x y_0' + \frac{x^2 y_0''}{2!} + \frac{x^3 y_0'''}{3!} + \cdots$$

Pg 294) $\frac{dy}{dx} = x^2 y - 1$

$y' = x^2 y - 1$

$y(0) = 1$

$x_0 = 0,\ y_0 = 1$

$x = 0.1\ ?$

$x = 0.2 = ?$

$y' = x^2 y - 1,\ y_0' = x_0^2 y_0 - 1 = -1$

$y'' = [x^2 y' + y\,2x],\ y_0'' = [x_0^2 y_0' + 2x_0 y_0] = 0$

$y''' = [x^2 y'' + y' 2x] + 2(xy' + y),$

$y_0''' = x_0^2 y_0'' + y_0' 2x_0 + 2x_0 y_0' + 2y_0$
$= 2$

$y''' = x^2 y'' + 4xy' + 2y$

$y'''' = [x^2 y''' + y'' \times 2x] + 4[xy'' + y'] + 2y$

$y'''' = x^2 y'' + 6xy' + 6y',\ y_0'''' = -6$

$$\frac{dy}{dx} = -xy^2$$

$$\frac{dy}{y^2} = -x\,dx$$

$$-\frac{1}{y} = -\frac{x^2}{2} + c$$

Put $x = 0$, $y = 2$

$$c = -\frac{1}{2}$$

$$-\frac{1}{y} = -\frac{x^2}{2} - \frac{1}{2}$$

$$\frac{1}{y} = \frac{x^2}{2} + \frac{1}{2}$$

$$\frac{1}{y} = \frac{x^2+1}{2}$$

$$y = \frac{2}{x^2+1}$$

$$y = (x^2+1)^{-1} 2$$

$$y = 2[1 - x^2 + x^4 + \dots]$$

$$y = 2 - 2x^2 + 2x^4 \dots$$