Maths Class 11 Chapter 5 Part -1 Quadratic equations

1. **Real Polynomial:** Let $a_0, a_1, a_2, \ldots, a_n$ be real numbers and x is a real variable. Then, $f(x) = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$ is called a real polynomial of real variable x with real coefficients.

2. **Complex Polynomial:** If $a_0, a_1, a_2, \ldots, a_n$ be complex numbers and x is a varying complex number, then $f(x) = a_0 + a_1x + a_2x^2 + \ldots + a_{n-1}x^{n-1} + a_nx^n$ is called a complex polynomial or a polynomial of complex variable with complex coefficients.

3. **Degree of a Polynomial:** A polynomial $f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots + a_nx^n$, real or complex is a polynomial of degree n , if $a_n \neq 0$.

4. **Polynomial Equation:** If f(x) is a polynomial, real or complex, then f(x) = 0 is called a polynomial equation. If f(x) is a polynomial of second degree, then f(x) = 0 is called a quadratic equation .

Quadratic Equation: A polynomial of second degree is called a quadratic polynomial. Polynomials of degree three and four are known as cubic and biquadratic polynomials respectively. A quadratic polynomial f(x) when equated to zero is called quadratic equation. i.e., $ax^2 + bx + c = 0$ where $a \neq 0$.

Roots of a Quadratic Equation: The values of variable x ,which satisfy the quadratic equation is called roots of quadratic equation.

Important Points to be Remembered

- An equation of degree n has n roots, real or imaginary .
- Surd and imaginary roots always occur in pairs of a polynomial equation with real coefficients i.e., if $(\sqrt{2} + \sqrt{3}i)$ is a root of an equation, then' $(\sqrt{2} - \sqrt{3}i)$ is also its root. .
- An odd degree equation has at least one real root whose sign is opposite to that of its last' term (constant term), provided that the coefficient of highest degree term is positive.
- Every equation of an even degree whose constant term is negative and the coefficient of highest degree term is positive has at least two real roots, one positive and one negative.
- If an equation has only one change of sign it has one positive root.
- If all the terms of an equation are positive and the equation involves odd powers of x, then all its roots are complex.

Solution of Quadratic Equation

1.**Factorization Method:** Let $ax^2 + bx + c = \alpha(x - \alpha)(x - \beta) = O$. Then, $x = \alpha$ and $x = \beta$ will satisfy the given equation.

2. **Direct Formula:** Quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ has two roots, given by

If α and β are the roots of 'a quadratic equation, then the equation is $x^2 - S_1X + S_2 = 0$

i.e., $x^2 - (\alpha + \beta)\,x + \alpha\beta = 0$

2. **Cubic Equation**

If α, β and γ are the roots of cubic equation, then the equation is

$$x^3 - S_1x^2 + S_2x - S_3 = 0$$
$$x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma = 0$$

3. **Biquadratic Equation**

If α, β, γ and δ are the roots of a biquadratic equation, then the equation is

$$x^4 - S_1x^3 + S_2x^2 - S_3x + S_4 = 0$$
$$x^4 - (\alpha + \beta + \gamma + \delta)x^3 + (\alpha\beta + \beta\gamma + \gamma\delta + \alpha\delta + \beta\delta + \alpha\gamma)x^2 - (\alpha\beta\gamma + \alpha\beta\delta + \beta\gamma\delta + \gamma\delta\alpha)x + \alpha\beta\gamma\delta = 0$$

Equation In Terms of the Roots of another Equation

If α, β are roots of the equation $ax^2 + bx + c = 0$, then the equation whose roots are.

(i) $-\alpha, -\beta \Rightarrow ax^2 - bx + c = 0$ (replace x by $-x$)

(ii) $\alpha^n, \beta^n; n \in N \Rightarrow a(x^{1/n})^2 + b(x^{1/n}) + c = 0$ (replace x by $x^{1/n}$)

(iii) $k\alpha, k\beta \Rightarrow ax^2 + kbx + k^2c = 0$ (replace x by x/k)

(iv) $k + \alpha, k + \beta \Rightarrow a(x - k)^2 + b(x - k) + c = 0$ (replace x by $(x - k)$)

(v) $\frac{\alpha}{k}, \frac{\beta}{k} \Rightarrow k^2ax^2 + kbx + c = 0$ (replace x by kx)

(vi) $\alpha^{1/n}, \beta^{1/n}; n \in N \Rightarrow a(x^n)^2 + b(x^n) + c = 0$ (replace x by x^n)

The quadratic function $f(x) = ax^2 + 2hxy + by^2 + 2gx + 2fy + c$ is always resolvable into linear factor, iff

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

Condition for Common Roots in a Quadratic Equation

1. **Only One Root is Common**