6) $\int \frac{x}{x^4+1}\,dx = \int \frac{x}{(x^2)^2+1}\,dx$

$= \frac{1}{2}\int \frac{1}{(x^2)^2+1}\cdot(2)x\,dx$

$\begin{bmatrix} u = x^2 \\ du = 2x\,dx \end{bmatrix}$

$= \frac{1}{2}\int \frac{1}{u^2+1}\,du$

only x' in the numerator

$= \frac{1}{2}\tan^{-1}(u) + c$

$= \frac{1}{2}\tan^{-1}(x^2) + c$

7) $\int \cos^2 x\,dx = \frac{1}{2}\int \cos(2x)+1\,dx$

$= \frac{1}{2}\cdot\frac{1}{2}\int \cos(2x)+1\cdot(2)\,dx$

$\begin{bmatrix} u = 2x \\ du = 2\,dx \end{bmatrix}$

$= \frac{1}{4}\int \cos u + 1\,du$

$= \frac{1}{4}(\sin u + u) + c$

$= \frac{\sin(2x) + 2x}{4} + c$

$*\ \sin(2x) = 2\sin x\cos x$

$\cos(2x) = \cos^2 x - \sin^2 x$

$\sin^2 x = 1 - \cos^2 x \Rightarrow \cos(2x) = 2\cos^2 x - 1$

$\cos^2 x = 1 - \sin^2 x \Rightarrow \cos(2x) = 1 - 2\sin^2 x$

$\cos^2 x = \frac{\cos(2x)+1}{2}$

$\sin^2 x = \frac{1-\cos(2x)}{2}$

8) $\int \ln x\,dx = \int (1)\cdot \ln x\,dx$

$= x\ln x - \int \frac{1}{x}\cdot x\,dx$

$\begin{bmatrix} f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x} \\ g'(x) = 1 \Rightarrow g(x) = x \end{bmatrix}$

$= x\ln x - x + c$