

$$(22) f(x) = \frac{\cot x}{1 - \sin x}$$

$$\text{let } u = \cot x$$

$$v = 1 - \sin x$$

$$du = -\csc^2 x$$

$$dv = -\cos x$$

$$f'(x) = \frac{v du - u dv}{v^2}$$

$$= \frac{(1 - \sin x)(-\csc^2 x) - (\cot x)(-\cos x)}{(1 - \sin x)^2}$$

$$= \frac{-\csc^2 x + \sin x \csc^2 x + \cot x \cos x}{(1 - \sin x)^2} \rightarrow \text{distribute } (-\csc^2 x) \text{ to } (1 - \sin x)$$

$$= \frac{-\csc^2 x + \sin x \left(\frac{1}{\sin^2 x}\right) + \cot x \cos x}{(1 - \sin x)^2} \rightarrow \csc x = \frac{1}{\sin x}$$

$$f'(x) = \frac{-\csc^2 x + \csc x + \cot x \cos x}{(1 - \sin x)^2}$$

$$(23) f(x) = \left(\frac{1 + \sin x}{1 - \sin x}\right)$$

$$\text{let } u = 1 + \sin x$$

$$v = 1 - \sin x$$

$$du = \cos x$$

$$dv = -\cos x$$

$$f'(x) = \frac{(1 - \sin x)(\cos x) - (1 + \sin x)(-\cos x)}{(1 - \sin x)^2} \rightarrow f'(x) = \frac{v du - u dv}{v^2}$$

$$= \frac{\cos x - \sin x \cos x + \cos x + \sin x \cos x}{(1 - \sin x)^2}$$

$$f'(x) = \frac{2 \cos x}{(1 - \sin x)^2}$$

$$(24) f(x) = \left(\frac{\sin x - 1}{\cos x + 1}\right)$$

$$\text{let } u = \sin x - 1$$

$$v = \cos x + 1$$

$$du = \cos x$$

$$dv = -\sin x$$

$$f'(x) = \frac{(\cos x + 1)(\cos x) - (\sin x - 1)(-\sin x)}{(\cos x + 1)^2} \rightarrow \frac{v du - u dv}{v^2}$$

$$= \frac{\cos^2 x + \cos x + \sin^2 x - \sin x}{(\cos x + 1)^2}$$

$$\text{but } \cos^2 x + \sin^2 x = 1$$

$$f'(x) = \frac{1 + \cos x - \sin x}{(\cos x + 1)^2}$$

$$(25) f(x) = (x - \sin x)(x + \cos x)$$

$$\text{let } u = x - \sin x \quad v = x + \cos x$$

$$du = -\cos x \quad dv = -\sin x$$

$$f'(x) = (x - \sin x)(-\sin x) + (x + \cos x)(-\cos x) \rightarrow u dv + v du$$

$$= -x \sin x + \sin^2 x - x \cos x + \cos^2 x$$

$$\text{but } \sin^2 x + \cos^2 x$$

$$= 1 - x \sin x - x \cos x$$

$$f'(x) = 1 - x(\sin x + \cos x)$$

$$(26) f(x) = (x^2 + \cos x)(2x - \sin x)$$

$$\text{let } u = x^2 + \cos x$$

$$v = 2x - \sin x$$

$$du = 2x - \sin x$$

$$dv = 2 - \cos x$$

$$f'(x) = u dv + v du$$

$$= (x^2 + \cos x)(2 - \cos x) + (2x - \sin x)(2 - \sin x)$$

$$f'(x) = (x^2 + \cos x)(2 - \cos x) + (2x - \sin x)(2 - \sin x)$$

$$(27) f(x) = \left(\frac{2 \csc x - 1}{\csc x + 2} \right)$$

$$\text{let } u = 2 \csc x - 1$$

$$v = \csc x + 2$$

$$du = -2 \csc x \cot x$$

$$dv = -\csc x \cot x$$

$$f'(x) = \frac{(2 \csc x - 1)(-\csc x \cot x) - (2 \csc x + 1)(-2 \csc x \cot x)}{(\csc x + 2)^2}$$

$$= \frac{-2 \csc^2 x \cot x - 4 \csc x \cot x + 2 \csc^2 x \cot x - \csc x \cot x}{(\csc x + 2)^2}$$

$$f'(x) = \frac{-5 \csc x \cot x}{(\csc x + 2)^2}$$

$$(28) f(y) = \left(\frac{\tan y + 1}{\tan y - 1} \right)$$

$$\text{let } u = \tan y + 1$$

$$v = \tan y - 1$$

$$du = \sec^2 y$$

$$dv = \sec^2 y$$

$$f'(y) = \frac{(\tan y - 1)(\sec^2 y) - (\tan y + 1)(\sec^2 y)}{(\tan y - 1)^2} \rightarrow \frac{v du - u dv}{v^2}$$

$$= \frac{\sec^2 y [(\tan y - 1) - (\tan y + 1)]}{(\tan y - 1)^2} \quad \text{factor out } \sec^2 y$$

$$f'(y) = -2 \sec^2 y$$