a medium in which sound propagates, the electric and magnetic waves propagates undergoes oscillatory change.

Simple Harmonic Motion:-

It is the simplest type of oscillatory motion.

A particle is said to be execute simple harmonic oscillation is the restoring force is directed towards the equilibrium position and its magnitude is directly proportional to the magnitude and displacement from the equilibrium position.

If F is the restoring force on the oscillator when its displacement from the equilibrium position is x, then

$$F \propto -x$$

Here, the negative sign implies that the direction of restoring force is opposite to that of displacement of body i.e towards equilibrium position.

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$$F= -kx \text{ .... } (1)$$

Where, k= proportionality constant called force constant.

$$Ma=-kx$$

$$M\frac{d2y}{dt2}=-kx$$

$$M\frac{d2y}{dt2}+kx=0$$

$$\frac{d2y}{dt2}+\frac{k}{M}x=0$$

$$\frac{d2y}{dt2}+\omega^2x=0 \text{ .... } (2)$$

Similarly, the solution of differential equation can be given as

x=Acos($\theta + \omega t$) ....(6)

Here A denotes amplitude of oscillatory system, ($\theta + \omega t$) is called phase and θ is called epoch/initial phase/phase constant/phase angel.

Equation (5) and (6) represents displacement of SHM.

<u>Velocity in SHM</u>:-

$$x\text{=A}\sin(\omega t + \theta)$$

$$\frac{dx}{dt}\text{=A}\omega\cos(\omega t + \theta)$$

$$\text{v=A}\omega\cos(\omega t + \theta) \ldots\ldots\ldots\ldots (7)$$

The minimum value of v is 0(as minimum value of Asin($\theta + \omega t$)=0 & maximum value is Aω. The maximum value of v is called velocity amplitude.

<u>Acceleration in SHM</u>:-

$$\text{a= -A}\omega^2\sin(\omega t + \theta) \ldots\ldots\ldots\ldots (8)$$

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The minimum value of 'a' is 0 & maximum value is Aω^2. The maximum valueof 'a' is called acceleration amplitude.

Also, a= ω^2x (from equation (5))

a $\propto$ –y

It is also the condition for SHM.

<u>Time period in SHM</u>:-

The time required for one complete oscillation is called the time period (T). It is related to the angular frequency(ω) by.

$$\text{T=}\frac{2\pi}{\omega} \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Frequency in SHM:-

The number of oscillation per time is called frequency or it is the reciprocal of time period.

$$\upsilon=\frac{1}{T}=\frac{\omega}{2\pi} \text{ ....(10)}$$

Potential energy in SHM:-

The potential energy of oscillator at any instant of time is,

$$U=-\int_0^x F dx$$

$$=-\int_0^x (-kx)\, dx$$

$$=\frac{1}{2} kx^2$$

$$=\frac{1}{2} kA^2 \sin^2(\theta + \omega t) \text{ .... (11)}$$

(By using equation (5)).

Kinetic energy in SHM:-

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The kinetic energy of oscillator at any instant of time is,

$$K=\frac{1}{2} M\left(\frac{dx}{dt}\right)^2$$

$$=\frac{1}{2} M v^2$$

$$K=\frac{1}{2} MA^2\omega^2 \cos^2(\theta + \omega t) \text{ .... (12)}$$

(By using equation (7))

Both kinetic and potential energy oscillate with time when the kinetic energy is maximum, the potential energy is minimum and vice versa. Both kinetic and potential energy attain their maximum value twice in one complete oscillation.

Total energy in SHM:-

$$\Rightarrow \theta = \frac{\pi}{2}$$

Using the value of θ & t=0 in the equation (vii) we have

v_0= -r ω_1

Where value of V_0 in ....

Calculation of Energy(instantaneous):

K.E = $\frac{1}{2}mv^2$

K.E = $\frac{1}{2}mv^2 e^{-2\beta t}[\beta^2\cos^2(\omega_1 t+ \theta)+ \omega_1{}^2\sin^2(\omega_1 t+ \theta)+ \beta\omega_1\sin^2(\omega_1 t+ \theta)]$

Potential Enegy:

P.E=$\frac{1}{2}kx^2$

$=\frac{1}{2}kr^2 e^{-2\beta t}\cos^2(\omega_1 t+\theta)$

Total Energy:

T.E=K.E+P.E

$=e^{-2\beta t}[(\frac{1}{2}mv^2+\frac{1}{2}kr^2)\cos^2(\omega_1 t+\theta)+\frac{1}{2}mr^2\omega_1{}^2\sin^2(\omega_1 t+\theta)$

$+ \ \frac{1}{2}mv^2 \ \beta\omega_1\sin2(\omega_1 t+ \theta)]$

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Total average energy:

$< E >=\frac{1}{2}mr^2 \ \omega_0{}^2 e^{-2\beta t}$

$=E_0 e^{-2\beta t}$

Where, E_0 =Total energy of free oscillation

The average energy decipated during one cycle

$< P(t) >$ =Rate of energy

$$= 2\beta m - \frac{im(\omega_0{}^2 - \omega^2)}{\omega}$$

$$z^* = 2\beta m + \frac{im(\omega_0{}^2 - \omega^2)}{\omega}$$

$$|z| = z^* z = m\left[4\beta^2 + \frac{1}{\omega^2}(\omega_0{}^2 - \omega^2)^2\right]^{1/2}$$

$$A = \frac{F}{\omega|z|}$$

For a particular ω, $\boxed{A \propto \frac{1}{|z|}}$

INTERFERENCE

Coherent Superposition:

The superposition is said to be coherent if two waves having constant phase or zero phase difference.
In this case, the resultant intensity differs from the sum of intensities of individual waves due to interfering factor.
i.e. $I \neq I_1 + I_2$

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Incoherent Superposition:

The superposition is said to be incoherent if phase changes frequently or randomly.
In this case, the resultant intensity is equal to the sum of the intensities of the individual waves.
i.e. $I = I_1 + I_2$

Two Beam Superposition:

When two beam having same frequency, wavelength and different in amplitude and phase propagates in a medium, they undergo principle of superposition which is known as two beam superposition.

$$\sum_{i=1}^{N} A_i A_j \cos(\varphi_j - \varphi_i) = 0$$

$$A^2 = N \sum_{i=1}^{N} A_i^2$$

Now , $I_{incoherent} = KA^2$

$= KN \sum_{i=1}^{N} A_i^2$

$= KNA_1^2$

$\Rightarrow I_{incoherent} = NI_1 \qquad \Rightarrow N = \dfrac{I_{coherent}}{I_{icoherent}}$

Interference:

The phenomenon of modification in distribution of energy due to superposition of two or more number of waves is known as interference.

To explain the interference, let us consider a monochromatic source of light having wavelength λ and emitting light in all possible directions.

According to Huygens's principle, as each point of a given wavefront will act as centre of disturbance they will emit secondary wave front on reaching slit S_1 and S_2.

As a result of which, the secondary wave front emitted from slit S1 and S2 undergo the Principle of superposition.

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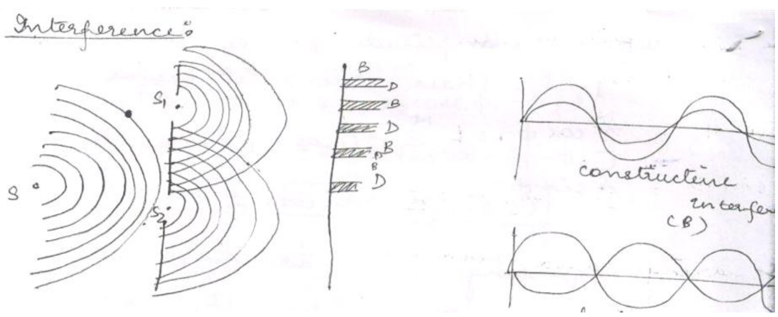

Component light waves are allowed to travel different optical path so that they will suffer a path difference and hence phase difference.

$$\left[phase\ difference = \frac{2\pi}{\lambda} \times path\ difference\right]$$

Methods for producing coherent sources/Types of interferences

Coherent sources can be produced by two methods

1) Division of wave front
2) Division of amplitude

Division of Wave front

The process of coherent source or interference by dividing the wave front of a given source of light is known as division of wave front.

This can be done by method of reflection or refraction. In this case a point source is used.

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Examples

1. YDSE
2. Lylord's single mirror method
3. Fresnel's bi-prism
4. Bilet splitting lens method

DIVISION OF AMPLITUDE

The process of obtaining a coherent source by splitting the amplitude of light waves is called division of amplitude which can be done by multiple reflections.

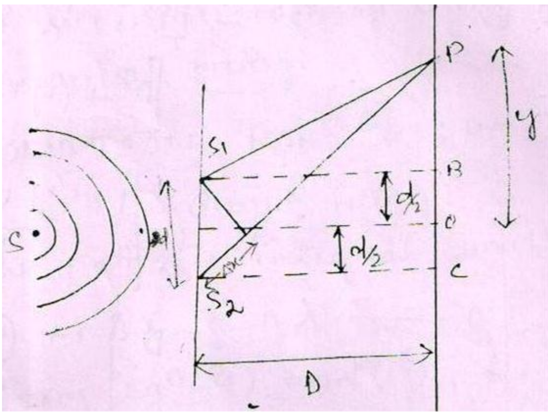
In this case, extended source of light is used.

1.Newton’s ring method

2.Thin film method

3. Michelson’s interferometer

Young’sDouble Slit Experiment:



In 1801 Thomas Young demonstrated the phenomenon of interference in the laboratory with a suitable arrangement. It is based on the principle of division of wavefront of interference. The experiential arrangement consists of two narrow slits S_1 and S_2 closely spaced, illuminated by a monochromatic source of light S. A screen is placed at a distance D from the slit to observe the interference pattern.

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In the figure,

d → Slit separation

D → Slit and screen separation

$\lambda \rightarrow$ Wavelength of light

Y → distance of interfering point from the centre of slit

$\Delta x \rightarrow$ Path difference coming from the light S_1 and S_2

Optical path difference between the rays coming through S_1 and S_2

$$\beta_w = \frac{\lambda_w D}{d}$$

When YDSE is performed with white light instead of monochromatic light we observed,

I. Fringe pattern remains unchanged
II. Fringe width decreases gradually
III. Central fringe is white and others are coloured fringes overlapping

When YDSE is performed with red, blue and green light

$$\lambda_R > \lambda_G > \lambda_B$$

So $\beta_R > \beta_G > \beta_B$

$$\mu = \frac{C}{V} = \frac{f\lambda_0}{f\lambda_m}$$

$$\mu = \frac{\lambda_0}{\lambda_m}$$

$$\Rightarrow \left[\lambda_m = \frac{\lambda_0}{\mu}\right]$$

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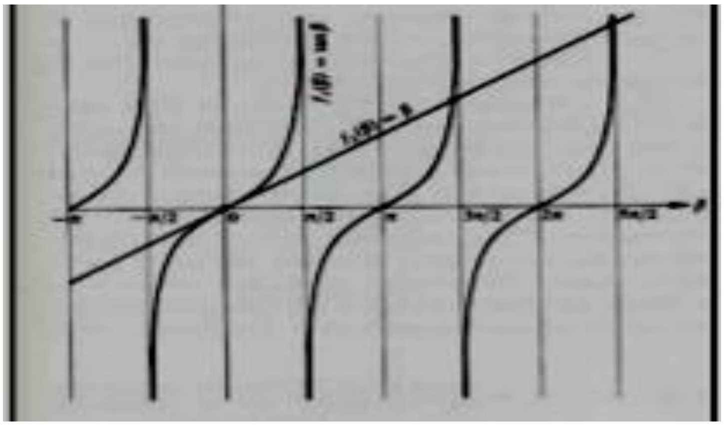
Wavelength of light in any given medium, decreases to1/μ times of wavelength in vacuum.

$$\beta \propto \lambda_m$$

$$\beta_m = \frac{\lambda_m D}{d}$$

$$\left[\beta_m = \frac{\lambda_0 D}{\mu d}\right]$$

So, it decreases 1/μ times.



From the expression for amplitude we have

$$R = \frac{A\sin\alpha}{\alpha} = \frac{A}{\alpha}\left[\alpha + \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} + \frac{\alpha^7}{7!} + \ldots\ldots\ldots\ldots\right]$$

$$= \frac{A}{\alpha} x\alpha\left[1 + \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} + \ldots\ldots\ldots\ldots\ldots\right] = A, \text{ since } \alpha \ll 1$$

Thus the intensity at the central principal maxima is I_0.

For $\alpha = \frac{3\pi}{2}$, $I_1 = I_0 \frac{Sin^2\alpha}{\alpha^2} = I0 \frac{Sin^2(\frac{3\pi}{2})}{\left(\frac{3\pi}{2}\right)^2} = \frac{I_0}{22}$

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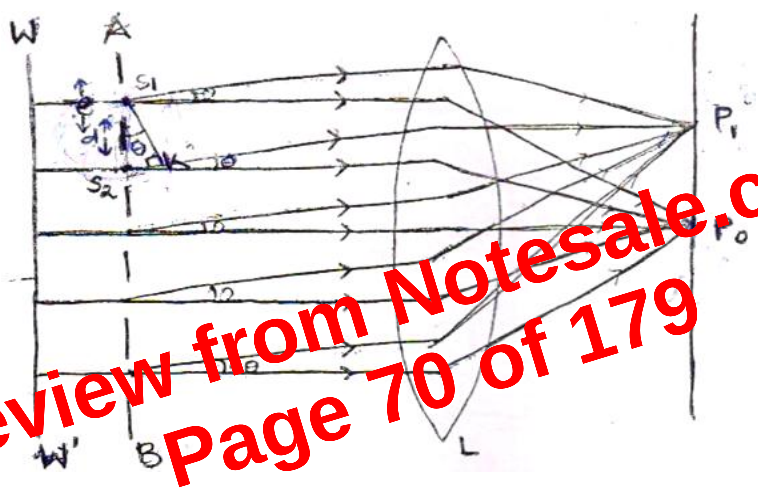
For $\alpha = \frac{5\pi}{2}$, $I_2 = I_0 \frac{Sin^2\alpha}{\alpha^2} = I0 \frac{Sin^2(\frac{5\pi}{2})}{\left(\frac{5\pi}{2}\right)^2} = \frac{I_0}{62}$ and so on ……

Intensity distribution curve:

The graph plotted between phase difference and intensity of the fringes is known as intensity distribution curve. The nature of the graph is as follows:

placed at its focal length. The rays of light which are allowed to incident normally on the lens are converged to a point "P_o" forming central principal maxima having high intensity and the rays of light which are diffracted through an angle are "θ" are converge to a point "P_1" forming a minima having less intensity as compared to central principal maxima. Again those rays of light which are diffracted through an angle "θ" are undergoes a path difference and hence a phase difference producing diffraction.



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Let AB- be the transverse section of the plane transmission grating

ww'- be a plane wave front coming from infinite distance

e = width of the slit

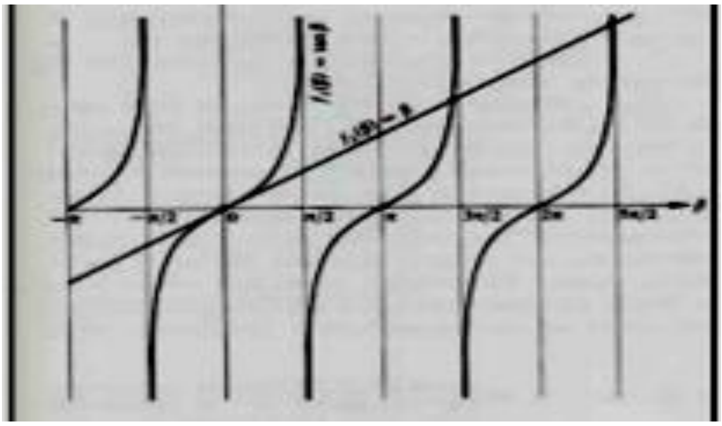
d = width of the opacity

(e+d) = grating element of the grating

N = be the no. of rulings present in the grating

Now the path difference between the deviated light rays is

$S_2K = S_1S_2 \mathrm{Sin}\theta = (e+d)Sin\theta$



Intensity distribution curve:

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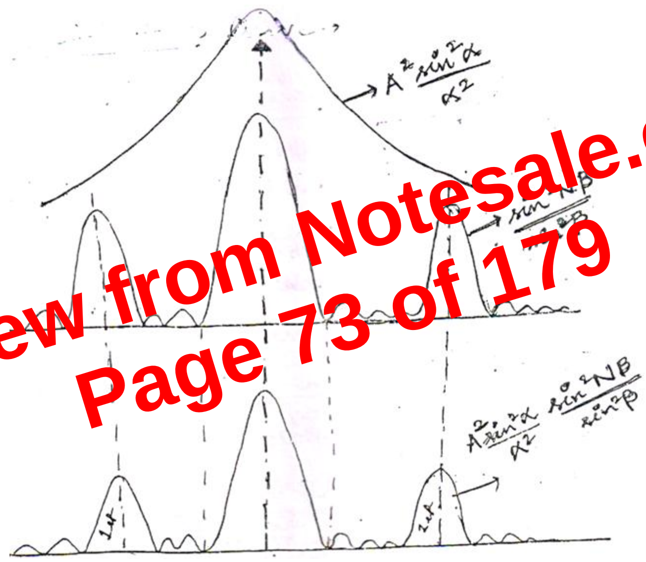

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Characteristics of the spectral lines or grating spectra:

1.The spectra of different order are situated on either side of central principal maximum
2.Spectral lines are straight and sharp
3.The spectra lines are more dispersed as we go to the higher orders.

i.e Third order spectra or multiple of 3 spectra will found to be missed or absent on the resulting diffraction pattern.

Dispersion:

The phenomenon of splitting of light wave into different order of spectra is known as dispersion.

Dispersive power:

The variation of angle of diffraction with the wave length of light is known as dispersive power. It is expressed as $\frac{d\theta}{d\lambda}$

Where $d\theta = \theta_1 - \theta_2$ = difference in angle of diffraction and $d\lambda = \lambda_1 - \lambda_2$=difference in wave length of light

Expression for dispersive power:

We have

$$(e+d)Sin\theta = \pm m\lambda$$

$$\frac{d}{d\lambda}\left[(e+d)Sin\theta = \pm m\lambda\right] = \frac{d}{d\lambda}(m\lambda)$$

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$$\Rightarrow (e+d)\frac{d}{d\lambda}(Sin\theta) = m\frac{d\lambda}{d\lambda}$$

$$\Rightarrow (e+d)\cos\theta\frac{d\theta}{d\lambda} = m$$

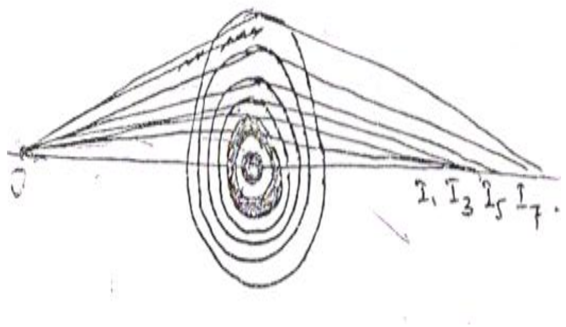
$$\Rightarrow \frac{d\theta}{d\lambda} = \frac{m}{(e+d)\cos\theta}$$

$$\Rightarrow \frac{d\theta}{d\lambda} \quad \alpha \quad m$$

$$\alpha \frac{1}{(e+d)}$$

$$\alpha \frac{1}{\cos\theta}$$

zone through the alternate transparent zones. So,the rays of light differ by a phase difference of π.



Hence, the resultant amplitude is sum of the individual amplitude due to light coming from alternate half period zones. Thus for any point object situated at infinite produces a bright image at a particular distance which is same as that of image produced by a convex lens. Thus a zone plate is equivalent to that of a convex lens.

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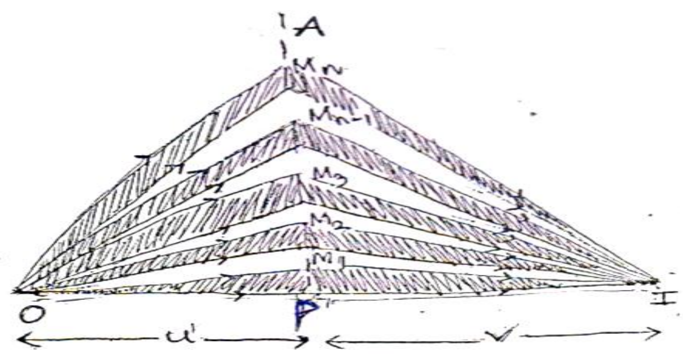
Theory of zone plate:

Let us consider a transverse section of a zone plate placed perpendicular to the plane of the paper. Let 'O' be a point object placed at a distance '$OP = u$' forms a real image 'I' at a distance '$PI = v$' from the zone plate.



$$\Rightarrow r^2{}_n\left(-\frac{1}{u}+\frac{1}{v}\right)= n\lambda$$

$$\Rightarrow r^2{}_n\left(\frac{1}{f}\right)= n\lambda \quad \Rightarrow f=\frac{r_n^2}{n\lambda} \qquad (5)$$

This is the required expression for primary focal length

Again, $f\alpha\frac{1}{\lambda} \quad \Rightarrow fx\lambda$ = constant

Area of zone plate:

The space enclosed between two consecutive zones is known as area of zone plate.

Let A_{n-1} and A_n be the area of (n-1)[th] and n[th] zone

Then $A = A_n - A_{n-1} = \pi r_n^2 - \pi r_{n-1}^2 = \frac{\pi uvn\lambda}{u+v} - \frac{\pi uv(n-1)\lambda}{u+v} = \frac{\pi uv\lambda}{u+v}$ = constat

Thus, the area of zone plate is independent of order of zone i.e the zones are equispaced.

Multiple foci of zone plate:

Now from the expression we have,

$$r^2{}_n\left(\frac{1}{u}+\frac{1}{v}\right)= n\lambda$$

If the object is situated at infinity (∞), then the first image at distance ,

$$v_1 = f_1 = \frac{r_n^2}{n\lambda}$$

If we divide the half period zones into half period elements having equal area, then the 1[st] half period zone will divided into three half period zones,2[nd] half period zone will divided into five half period elements and so on

The second brightest image will at

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path difference of λ/2, such zone plate is known as phase reverse zone plate.

Huygens's Principle:

About the propagation of the wave, Huygens suggested a theory which is based on a principle known as Huygens's principle.

It states that:-

1) Each point on a given wave front will act as centre of disturbances and emits small wavelets called secondary wave front in all the possible direction.
2) The forward tangent envelope to these wave lets gives the direction of new wave front.

Explanation/construction of secondary wave front:

To explain Huygens's principles let us consider a source of light emits waves in all directions. Let AB be the wave front at t=0. As the time advances each point on the given wave front AB will act as centre of disturbance and emit wave lets in all possible directions.

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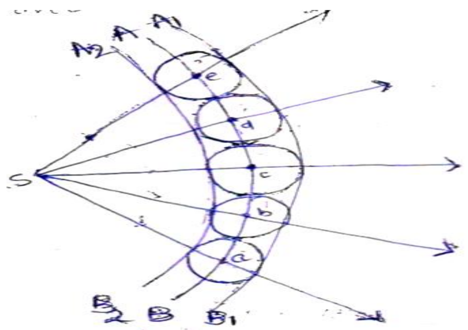

Taking a, b, c, d, e as centre and radii equal to 'ct' (c-velocity of light &'t' time), we can construct a large number of spheres which represents a centre of disturbance for the new wave. The length A_1B_1 represents the direction of new wave front.

1.It produces a path difference of λ /4 between O and E wave 2. The light emerging from a λ /4 plate maybe circularly elliptically or plane polarised. 3. In this case nicol may give a non zero minimum. 4. It is used for production of all type polarised light.	1.It produces a path difference of λ /2 between O and E ray. 2. The light emerging from a λ /2 plates is plane polarised for all orientation of the plate. 3. In this case nicol may give a zero minimum always. 4. It is used in polarism for half shade device.

Production and Analysis Polarised Light

1. Production of plane polarised light:

To produce plane polarised light a beam of ordinary light is sent through a Nicol prism in a direction almost parallel to the long edge of the prism. Inside the prism the beam is broken up to two components 'O' and 'E' ray. The 'O' component is totally reflected at the Canada balsam and is absorbed.

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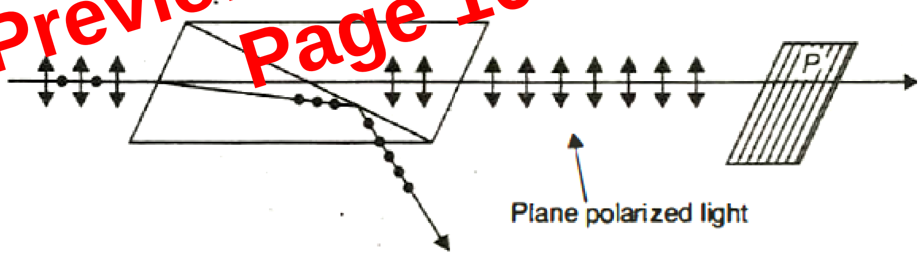


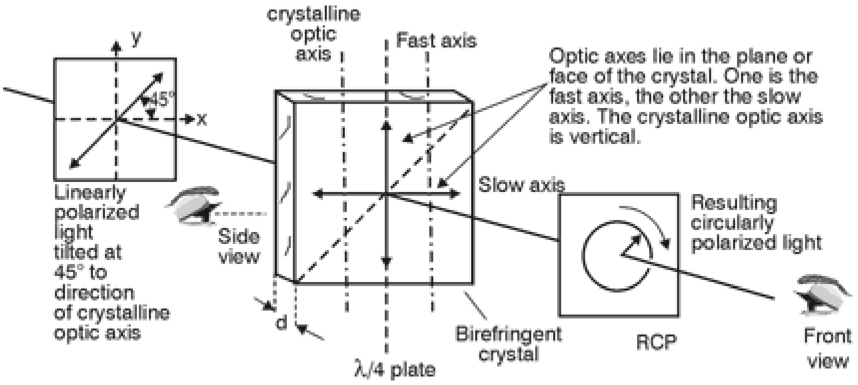
The 'E' component emerges out which is plane polarised with vibration parallel to the end faces of the Nicol.

2. Production of circularly polarised light:

The circularly polarised light can be produced by allowing plane-polarised light

obtained from the Nicol to fall normally on a quarter wave plate such that the

direction of vibration in the incident plane polarised light makes an angle of 45^0 with
the optic axis of the crystal.



Inside the plate the incident waves of amplitude A is divided into $E = A\cos 45^0$

$O = A\sin 45^0$ with a phase difference $\frac{\pi}{2}$ between them.

Let $A\cos 45^0 = A\sin 45^0$ Asin $45^0 = a$ of the axis of x

Let $x = a\sin(wt + \frac{\pi}{2}) = a\cos wt$ and $y = a\sin wt$

Eliminating t from both the equation, we have

$x^2 + y^2 = a^2$ which represents a circle.

Hence the light emerging from $\lambda/4$ plate is circularly polarised.

3. Production of elliptically polarised light:

The elliptically polarised light can be produced by allowing plane polarised light normally in a quarter wave plate such that the direction of vibration in the incident plane polarised light makes an angle other than 0^0,45^0 and 90^0 with the optical axis which is 30^0.

In this case the incident wave is divided inside the plate into E and O components of unequal amplitude $A\cos 30^0$ and $A\sin 30^0$ respectively which emerge from the plate with a phase difference of $\frac{\pi}{2}$.

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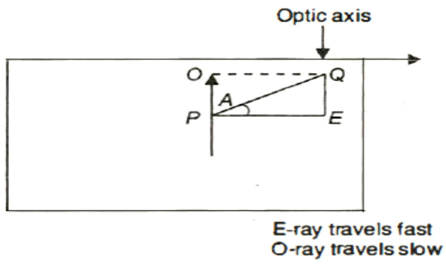

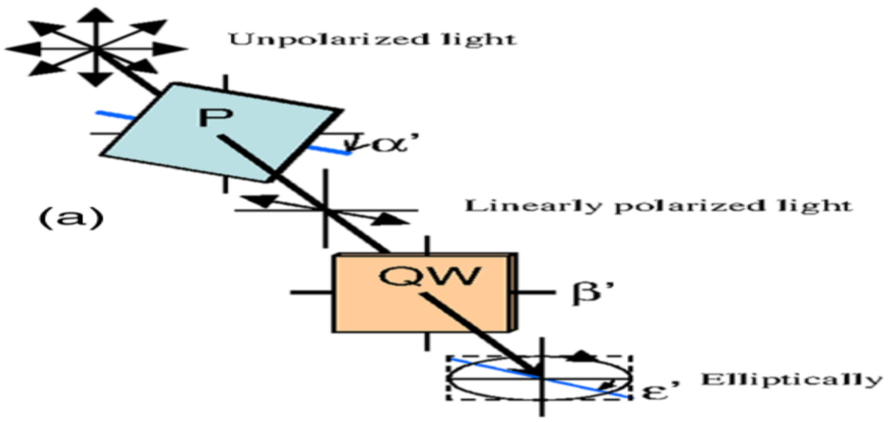

If we take $A\cos 30^0$ = a and $A\sin 30^0$ = b,then the emerging component can be written as,

$x = a\sin(wt + \frac{\pi}{2}) = a\cos wt$ and $y = b\sin wt$

Now eliminating 't' from both the equation we have

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ which the equation of an ellipse. Hence the emerging light coming from

$\lambda/4$ plate is elliptically polarised

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Analysis of different polarised light:

The whole analysis of different type of polarised light can be represented in algorithm form with figure as follows:

In a dielectric medium Gauss' law is given by

$$\phi_E = \int_S \vec{E}.\vec{dS} = \frac{q_{net}}{\varepsilon}$$

ε - Permittivity of the medium.

In terms of displacement vector Gauss' law is given by

$$\phi_E = \int_S \vec{D}.\vec{dS} = q_{net}$$

Notes:

- The charges enclosed by the surface may be point charges or continuous charge distribution.
- The net electric flux may be outward or inward depending upon the sign of charges.
- Electric flux is independent of shape & size of Gaussian surface.
- The Gaussian surface can be chosen to have a suitable geometrical shape for evaluation of flux.
- Limitation of Gauss' Law

  (a)Since flux is a scalar quantity Gauss' law enables us to find the magnitude of electric field only.

  (b) The applicability of the law is limited to situations with simple geometrical symmetry

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Gauss' Law in Differential form

Gauss' law is given by

$$\int_S \vec{E}.\vec{dS} = \frac{q_{net}}{\varepsilon}$$

For a charge distribution

$$q_{net} = \int_V \rho \, dV \qquad where\ \rho \rightarrow volume\ charge\ density$$

Using Gauss divergence theorem

$$\oint_S \vec{E}.\vec{dS} = \int_V \vec{\nabla}.\vec{E}\, dV$$

So $$\frac{1}{\varepsilon_0}\int_V \rho \, dV = \int_V \vec{\nabla}.\vec{E}\, dV$$

Or $$\int_V (\vec{\nabla}.\vec{E} - \frac{\rho}{\varepsilon_0})\, dV = 0$$

$$\Rightarrow \quad \vec{\nabla}.\vec{E} - \frac{\rho}{\varepsilon_0} = 0$$

$$\Rightarrow \quad \vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$$

This is the differential form of Gauss' law.

Magnetic Intensity (H) and Magnetic Induction $(\vec{B})$

The magnetic intensity $(\vec{H})$ is related to the magnetic field induction $(\vec{B})$ by

$$(\vec{H}) = \frac{(\vec{B})}{\mu_0}$$

Unit: in SI system $(\vec{H})$ is in amp/m and $(\vec{B})$ in tesla.

Magnetic Flux (ϕ_m)

The magnetic flux over a given surface area S is given by

$$(\phi_m) = \int_S \vec{B}.\vec{dS} = \int_S B\, dS \cos\alpha$$

where $\alpha \rightarrow$ *angle between magnetic field* $\vec{B}$ *and normal to the surface*

Unit of flux: 1 weber in SI

1 maxwell in cgs(emu)

So 1T= 1 weber/m^2

1 gauss= 1maxwell/cm^2

Gauss' Law in magnetism

Since isolated magnetic pole does not exist, by analogy with Gauss' law of electrostatics, Gauss' law of magnetism is given by

$$\oint_S \vec{B}.\vec{dS} = 0$$

Using Gauss divergence theorem

$$\oint_S \vec{B}.\vec{dS} = \int_V \vec{\nabla}.\vec{B}\, dV = 0$$

$$\Rightarrow \quad \vec{\nabla}.\vec{B} = 0$$

This is the differential form of Gauss' law of magnetism.

Ampere's Circuital law

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Statement:-The line integral of magnetic field along a closed loop is equal to $\mu_0$timesthenetelectriccurrentenclosedbyloop.

$$\oint_C \vec{B}.\vec{dl} = \mu_0 I$$

Where $I \rightarrow$ *net current enclosed by the loop*

$C \rightarrow$ *closed path enclosing the current (called ampere loop).*

In terms of magnetic intensity

$$\oint_C \vec{H}.\vec{dl} = I$$

Ampere's Law in Differential form

Ampere's law is

$$\oint_C \vec{B}.\vec{dl} = \mu_0 I \text{ ----------------------------------(i)}$$

Using Stoke's theorem, we have

$$\oint_C \vec{B}.\vec{dl} = \int_S (\vec{\nabla}\times\vec{B}).\vec{ds} \text{ ---------------------------(ii)}$$

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In terms of current density J

$$\mu_0 I = \mu_0 \int_S \vec{J}.\vec{ds} \text{ -------------------------------------(iii)}$$

Using (ii) and (iii) in equation (i) we have

$$\int_S (\vec{\nabla}\times\vec{B}).\vec{ds} = \mu_o \int_S \vec{J}.\vec{ds} = \int_S (\mu_o \vec{J}).\vec{ds}$$

$$\Rightarrow \vec{\nabla}\times\vec{B} = \mu_o \vec{J}$$

This is Ampere's circuital law in differential form.

Faraday's Law of electromagnetic induction

Or $$\int_S (\vec{\nabla}\times\vec{E}+\frac{\partial\vec{B}}{\partial t}).\vec{ds}=0$$

$$\Rightarrow \quad \vec{\nabla}\times\vec{E}+\frac{\partial\vec{B}}{\partial t}=0$$

This is differential form of Faraday's law electromagnetic induction.

Equation of Continuity

The electric current through a closed surface S is

$$I=\oint_S \vec{J}.\vec{ds}$$

Using Gauss divergence theorem

$$I=\oint_S \vec{J}.\vec{ds}=\int_V \vec{\nabla}.\vec{J}\,dV \text{ ---------------- (i)}$$

Where S is boundary of volume V

Now $I=-\frac{\partial q}{\partial t} \rightarrow$ *rate of decrease of charge from the volume through surface S*

$$\Rightarrow \quad I=-\frac{\partial}{\partial t}\int_V \rho\, dV=-\int_V \frac{\partial\rho}{\partial t}dV \text{ ---------------------(ii)}$$

From (i) and (ii)

$$\int_V \vec{\nabla}.\vec{J}\,dV=-\int_V \frac{\partial\rho}{\partial t}dV$$

$$\Rightarrow \quad \int_V (\vec{\nabla}.\vec{J}+\frac{\partial\rho}{\partial t})dV=0$$

$$\Rightarrow \quad \vec{\nabla}.\vec{J}+\frac{\partial\rho}{\partial t}=0$$

This is equation of continuity.

Displacement Current

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Maxwell associated a current (known as displacement current) with the time varying electric field.

A parallel plate capacitor connected to a cell is considered. During charging field $\vec{E}$ between varies.

Let $q \rightarrow$ instantaneous charge on capacitor plates.

$A \rightarrow$ area of each plate

We know that the electric field between the capacitor plates is

$$E = \frac{q}{\varepsilon_0 A}$$

$$\Rightarrow \quad \frac{dE}{dt} = \frac{1}{\varepsilon_0 A}\frac{dq}{dt}$$

$$\Rightarrow \quad \varepsilon_0 A \frac{dE}{dt} = \frac{dq}{dt}$$

$\Rightarrow \quad I_d = \varepsilon_0 A \dfrac{dE}{dt} \quad where\ I_d \rightarrow$ displacement current between the plates

I_d exists till $\vec{E}$ varies with time.

In general, whenever there is a time-varying electric field, a displacement current exists,

$$I_d = \varepsilon_0 \frac{\partial}{\partial t}\int_S \vec{E}.\vec{ds} = \varepsilon_0 \frac{\partial \phi_E}{\partial t}$$

Where ϕ_E is electric flux.

Modification of Ampere's circuital law

Taking displacement current into account Ampere's Circuital law is modified as

$$\oint_C \vec{B}.\vec{dl} = \mu_o \left(I + I_d\right)$$

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Relative magnitudes of $\vec{E}$ and $\vec{B}$

Now taking magnitudes

$$\left|(\vec{k}\times\hat{e})\right| = \left|\frac{B_0\omega}{E_0}\hat{b}\right|$$

$$\Rightarrow k = \frac{\omega B_0}{E_0}$$

$$\Rightarrow \frac{E_0}{B_0} = \frac{\omega}{k} = c, \quad \text{where } c \rightarrow \text{velocity of light}$$

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

Now using $B_0 = \mu_0 H_0$

$$\frac{E_0}{H_0} = \mu_0 c = \mu_0\frac{1}{\sqrt{\mu_0\varepsilon_0}} = \frac{\mu_0}{\varepsilon_0} = Z_0$$

The quantity Z_0 has the dimension of electrical resistance and it is called the impedance of vacuum.

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Phase relation between $\vec{E}$ and $\vec{B}$

In an electromagnetic wave electric and magnetic field are in phase.

Either electric field or magnetic field can be used to describe the electromagnetic wave.

Electromagnetic Energy Density

The electric energy per unit volume is

$$u_E = \frac{1}{2}\vec{E}.\vec{D} = \frac{1}{2}\varepsilon E^2 \quad --------(1)$$

The magnetic energy per unit volume is

$$u_B = \frac{1}{2}\vec{B}.\vec{H} = \frac{1}{2}\mu H^2 \quad --------(2)$$

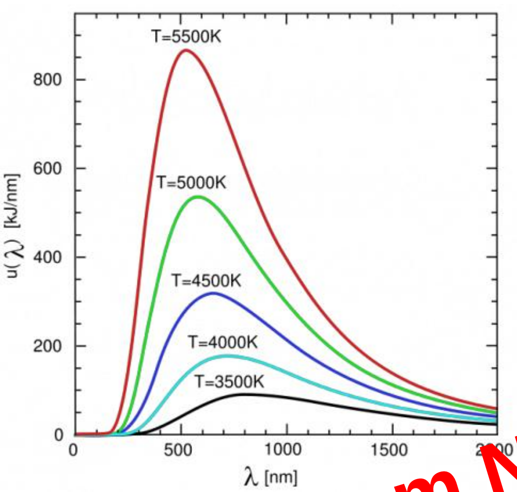
Therefore, $$u(\nu)d\nu = \frac{8\pi\nu^3}{c^3}\frac{1}{1+\frac{h\nu}{kT}-1}$$

$$u(\nu)d\nu=\frac{8\pi\nu^2}{c^3}kTd\nu$$

which is called Rayleigh-Jeans law.

In high frequency lim($\nu \rightarrow \infty$)

i.e$\nu \rightarrow \infty, \frac{h\nu}{kT} \rightarrow \infty, e^{h\nu/kT} - 1 \cong e^{h\nu/kT}$



Therefore, $u(\nu)d\nu = \frac{8\pi h\nu^3}{c^3}e^{-h\nu/kT}d\nu$, which is called Wein's radiation formula.

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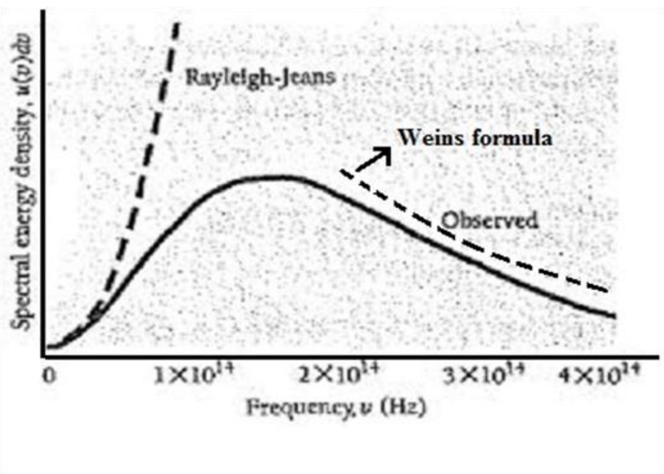

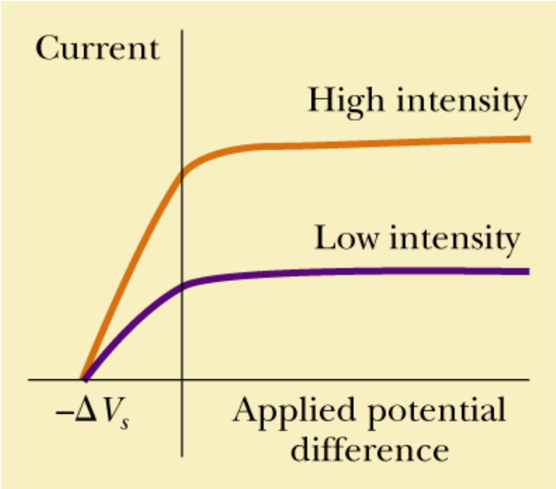

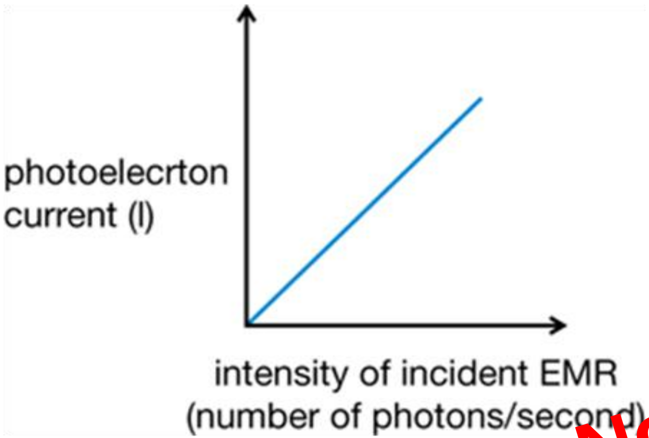

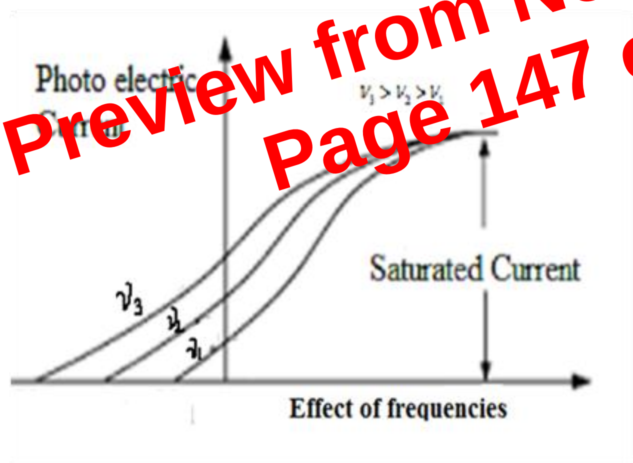

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<u>Laws of Photoelectric effect</u>

- It is an instantaneous process.
- It is directly proportional to intensity of incident light.
- Photocurrent is independent of frequency of incident light.

$$E = \sqrt{P^2c^2 + m_0^2c^4}$$

$$\Rightarrow P = \frac{\sqrt{E^2 - m_0^2c^4}}{c}$$

$$\therefore \boxed{\lambda = \frac{hc}{\sqrt{E^2 - m_0^2c^4}}}$$

* Experimental confirmation of matter wave was demonstrated by Davision-Germer experiment.
* The wave nature of electron was demonstrated by division and Germer.

Heisenberg's Uncertainty Principle:

It states that it is impossible to measure simultaneously the position and the corresponding component of its linear momentum with unlimited accuracy.

If Δx= uncertainty in x-component of the position of a particle

Δp_x= uncertainty in x-component of its linear momentum

then, $\Delta x \cdot \Delta p_x \geq \frac{\hbar}{2}$

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Similarly for y and z-component

$$\Delta y \cdot \Delta p_y \geq \frac{\hbar}{2}, \Delta z \cdot \Delta p_z \geq \frac{\hbar}{2}$$

Again uncertainty in energy and time is given by

$$\Delta t \cdot \Delta E \geq \frac{\hbar}{2}$$

Application of the uncertainty principle;

i. Ground state energy of harmonic oscillator

The energy of the 1-D harmonic oscillator is given as

$$E = \frac{P^2}{2m} + \frac{1}{2}m\omega^2x^2 \qquad (1)$$

Let us assume that in the ground state, the linear momentum P and position x of the oscillator are of the order of their uncertainties.

i.e $\Delta P \approx P$ and $\Delta x \approx x$

According to principle

$$\Delta x.\Delta p_x \geq \frac{\hbar}{2}$$
$$\Rightarrow \Delta x.\Delta p_x \approx \hbar$$
$$\Rightarrow p.x \approx \hbar$$

$$\Rightarrow p \approx \frac{\hbar}{2} \qquad (2)$$

Using eq^n(2) in eq^n(1),we get

$$E = \frac{\hbar^2}{2mx^2} + \frac{1}{2}m\omega^2 x^2 \qquad (3)$$

Since the energy E of the oscillator is minimum in the ground state, so

$$\left(\frac{\partial E}{\partial x}\right)_{x=x_0} = 0$$

$$\Rightarrow 0 = \left(\frac{\partial E}{\partial x}\right)_{x=x_0} = \frac{-\hbar^2}{mx_0^3} + m\omega^2 x_0$$

$$\Rightarrow x_0^2 = \frac{\hbar}{m\omega} \qquad (4)$$

Where x_0 corresponds to the ground state.

Using eq^n (4) in eq^n (3), we get

$$E_0 = \frac{\hbar^2}{2m\left(\frac{\hbar}{m\omega}\right)} + \frac{1}{2}m\omega^2\frac{\hbar}{m\omega} = \hbar\omega \qquad (5)$$

Thus the minimum energy of 1-D harmonic oscillator cannot be zero.

ii. Non-existence of electron in the nucleus

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