Quadratic Equation Examples:

Completing the Square:

$px^2 + qx + r = 0$

$x^2 + \frac{qx}{p} = -\frac{r}{p}$

$\left(\frac{q}{2p}\right)^2 = \frac{q^2}{4p^2}$

$x^2 + \frac{qx}{p} + \frac{q^2}{4p^2} = -\frac{r}{p} + \frac{q^2}{4p^2}$

$\left(x + \frac{q}{2p}\right)^2 = \frac{q^2 - 4pr}{4p^2}$

$x + \frac{q}{2p} = \sqrt{\frac{q^2 - 4pr}{4p^2}}$

$x = -\frac{q}{2p} \pm \frac{\sqrt{q^2 - 4pr}}{2p}$

$\therefore x = \frac{-q \pm \sqrt{q^2 - 4pr}}{2p}$

$5x^2 - 20x + 20 = 0$

$\frac{5x^2 - 20x + 20}{5} = 0$

$x^2 - 4x + 4 = 0$

$\left(\frac{-4}{2}\right)^2 = 4$

$x^2 - 4x + 4 = -4 + 4$

$(x-2)(x-2) = 0$

$\therefore x = 2$

Squaring Both Sides:

$\sqrt{10 + 2x} = x + 1$

$(\sqrt{10 + 2x})^2 = (x+1)^2$

$10 + 2x = x^2 + 2x + 1$

$x^2 - 9 = 0$

$(x+3)(x-3) = 0$

$\therefore x = 3 \text{ or } x = -3$

$\{-3 + 1 = -2 \,;\, x \neq -3\}$

$*$ *A square number cannot be negative*

'K' Method:

$x = 3$ *is a root of the equation*

$4x^2 + tx - 15 = 0$

solve for t & find the other root:

$4(3)^2 + t(3) - 15 = 0$

$3t = 15 - 36$

$t = -21 \div 3$

$t = -7$

$4x^2 - 7x - 15 = 0$

$(x-3)(4x+5) = 0$

$\therefore x = 3 \text{ or } x = -\frac{5}{4}$

Equations With Fractions:

$x + \frac{2}{3x} = 2$

$LCD: 3x$

$x(3x) + 2 = 2(3x)$

$3x^2 - 6x + 2 = 0$

$\therefore x = \frac{3 + \sqrt{3}}{3} \text{ or } x = \frac{3 - \sqrt{3}}{3}$

$\{R: x \neq 0\}$

Simultaneous Equations:

$5x^2 - 3xy + 3x - 2y^2 + 4y = 5$ ***and*** $y + 2x = 1$

$y = 1 - 2x$

$5x^2 - 3x(1-2x) + 3x - 2(1-2x)^2 + 4(1-2x) - 5 = 0$

$5x^2 - 3x + 6x^2 + 3x - 2 + 8x - 8x^2 + 4 - 8x - 5 = 0$

$3x^2 = 3$

$x^2 = 1$

$(x+1)(x-1) = 0$

$\therefore x = 1 \text{ or } x = -1$

The Quadratic Formula:

$x^2 - 2px + 3p = 0$

$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(3p)}}{2(1)}$

$x = \frac{4 \pm \sqrt{4 - 12p}}{2}$

$\therefore x = 2 \pm \frac{\sqrt{4 - 12p}}{2}$

Quadratic Inequalities:

State for which values of x the expression

$\sqrt{x^2 - 16}$ *is real:*

$\sqrt{x^2 - 16} \geq 0$

$x^2 - 16 \geq 0^2$

$x^2 \geq 16$

$x \geq \pm 4$

$x \in (-\infty; -4] \cup [4; \infty)$

$x \leq -4 \text{ or } x \geq 4$