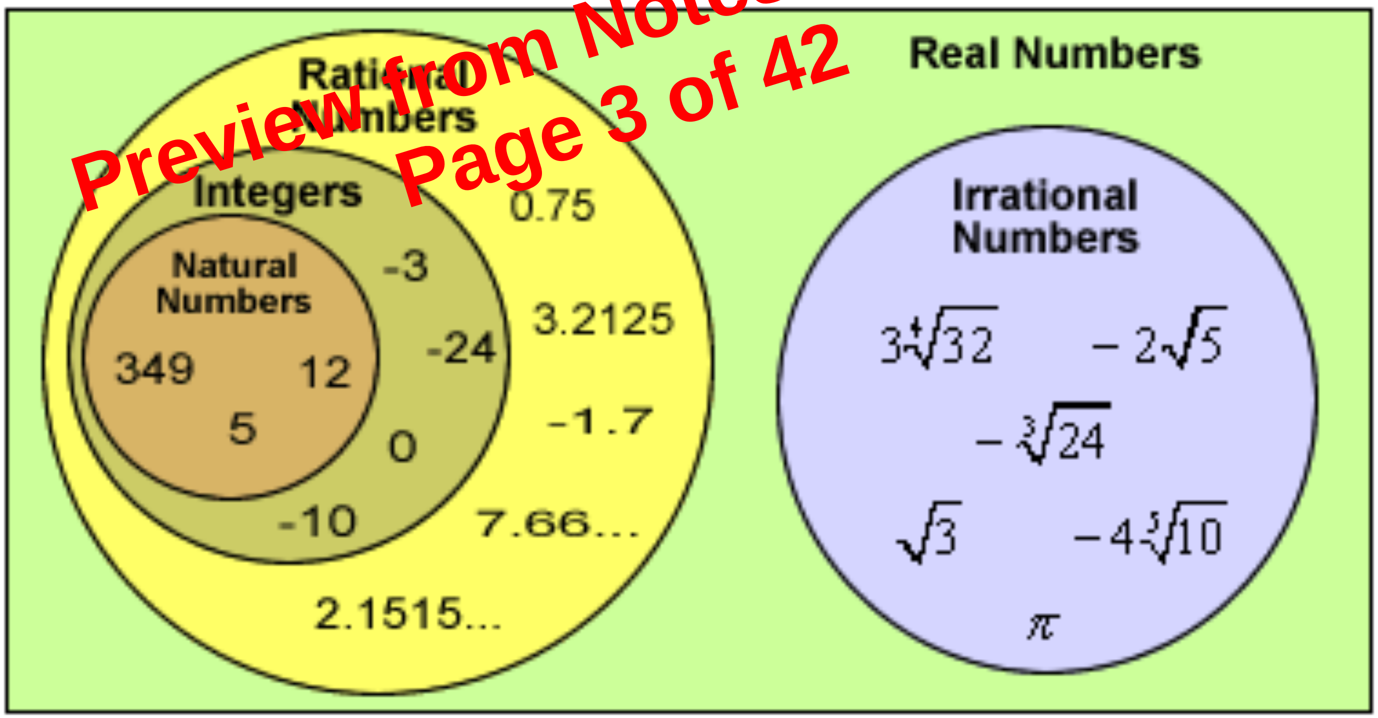
Surds

- A **surd** is an irrational number that is represented by a root sign or a radical sign, for example $\sqrt{\ }$, $\sqrt[3]{\ }$, $\sqrt[4]{\ }$.
- **Surds:** roots of numbers that do not have an exact answer, so they are irrational numbers. Surds themselves are exact numbers, for example, $\sqrt{6}$ or $\sqrt[3]{5}$.
- *Note:* An irrational number written in surd form gives an exact value of the number; whereas the same number written in decimal form (for example, to 4 decimal places) gives an approximate value.

What Are Rational and Irrational Numbers?

- Isn't Maths Already Rational?



In short, **rational numbers** are those that can be expressed as **fractions**, and **irrational numbers** are those that can not. If you have forgotten what counts as a fraction, it can be expressed as below:

$\frac{p}{q}$ *where* p and q are integers and q not being 0

Example

- **Simplify the following surds. Assume that x and y are positive real numbers.**

a. $\sqrt{384}$

b. $3\sqrt{405}$

c. $-\frac{1}{8}\sqrt{175}$

d. $5\sqrt{180x^3y^5}$

Multiplication and division of surds

- Multiplying surds
- To multiply surds, multiply together the expressions under the radical signs. For example, $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$, where a and b are <u>*positive*</u> real numbers.
- When multiplying surds it is best to first simplify them (if possible). Once this has been done and a mixed surd has been obtained, the coefficients are multiplied with each other and then the surds are multiplied together. For example,

$$m\sqrt{a} \times n\sqrt{b} = mn\sqrt{ab}.$$

Dividing surds

- To divide surds, divide the expressions under the radical signs; $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$, where a and b are whole numb

- When dividing surds it is best to simplify them (if possible) first. Once this has been done, the coefficients are divided next and then the surds are divided.

Expanding Binomial Surd Expressions

- **Example : Expand and simplify** (√3−√10)(√3+√12)

$$
\begin{aligned}
&(\sqrt{3}-\sqrt{10})(\sqrt{3}+\sqrt{12}) \\
&= \sqrt{3}\times\sqrt{3}+\sqrt{3}\times\sqrt{12}-\sqrt{10}\times\sqrt{3}-\sqrt{10}\times\sqrt{12} \\
&= (\sqrt{3})^2+\sqrt{3\times 12}-\sqrt{10\times 3}-\sqrt{10\times 12} \\
&= 3+\sqrt{36}-\sqrt{30}-\sqrt{120} \\
&= 3+6-\sqrt{30}-\sqrt{4\times 30} \\
&= 9-\sqrt{30}-2\sqrt{30} \\
&= 9-3\sqrt{30}
\end{aligned}
$$

- Complete **Lesson 10 – Surds and the binomial expansion**

Rationalising denominators using conjugate surds

Conjugate surds: surds that, when multiplied together, result in a rational number. For example, $(\sqrt{a} + \sqrt{b})$ and $(\sqrt{a} - \sqrt{b})$ are conjugate surds, because
$(\sqrt{a} + \sqrt{b}) \times (\sqrt{a} - \sqrt{b}) = a - b.$

- This fact is used to rationalise denominators containing a sum or a difference of surds.
- To rationalise the denominator that contains a sum or a difference of surds, multiply both numerator and denominator by the conjugate of the denominator.
- Two examples are given below: