Preface

The Sanskrit word Veda is derived from the root Vid, meaning to know without limit. The word Veda covers all Veda-sakhas known to humanity. The Veda is a repository of all knowledge, fathomless, ever revealing as it is delved deeper.

Swami Bharati Krishna Tirtha (1884-1960), former Jagadguru Sankaracharya of Puri culled a set of 16 Sutras (aphorisms) and 13 Sub - Sutras (corollaries) from the Atharva Veda. He developed methods and techniques for amplifying the principles contained in the aphorisms and their corollaries, and called it Vedic Mathematics.

According to him, there has been considerable literature on Mathematics in the Veda-sakhas. Unfortunately most of it has been lost to humanity as of now. This is evident from the fact that while, by the time of Patanjali, about 25 centuries ago, 1131 Veda-sakhas were known to the Vedic scholars, only about ten Veda-sakhas are presently in the knowledge of the Vedic scholars in the country.

The Sutras apply to and cover almost every branch of Mathematics. They apply even to complex problems involving a large number of mathematical operations. Application of the Sutras saves a lot of time and effort in solving the problems, compared to the formal methods presently in vogue. Though the solutions appear like magic, the application of the Sutras is perfectly logical and rational. The computation made on the computers follows, in a way, the principles underlying the Sutras. The Sutras provide not only methods of calculation, but also ways of thinking for their application.

This book on Vedic Mathematics seeks to present an integrated approach to learning Mathematics with keenness of observation and inquisitiveness, avoiding the monotony of accepting theories and working from them mechanically. The explanations offered make the processes clear to the learners. The logical proof of the Sutras is detailed in algebra, which eliminates the misconception that the Sutras are a jugglery.

Application of the Sutras improves the computational skills of the learners in a wide area of problems, ensuring both speed and accuracy, strictly based on rational and logical reasoning. The knowledge of such methods enables the teachers to be more resourceful to mould the students and improve their talent and creativity. Application of the Sutras to specific problems involves rational thinking, which, in the process, helps improve intuition that is the bottom - line of the mastery of the mathematical geniuses of the past and the present such as Aryabhatta, Bhaskaracharya, Srinivasa Ramanujan, etc.

Case (iii) :

When one number is less and another is more than the base, we can use (x-a)(x+b) = x(x−a+ b)−ab. and the procedure is evident from the examples given.

Find the following products by Nikhilam formula.

1) 7 X 4 **2) 93 X 85** **3) 875 X 994**

4) 1234 X 1002 **5) 1003 X 997** **6) 11112 X 9998**

7) 1234 X 1002 **8) 118 X 105**

Nikhilam in Division

Consider some two digit numbers (dividends) and same divisor 9. Observe the following example.

i) 13 ÷ 9 The quotient (Q) is 1, Remainder (R) is 4

since 9) 13 (1
9
____
4

ii) 34 ÷ 9, Q is 3, R is 7.

iii) 60 ÷ 9, Q is 6, R is 6.

iv) 80 ÷ 9, Q is 8, R is 8.

Now we have another type of representation for the above examples as given hereunder:

i) Split each dividend into a left hand part for the Quotient and right - hand part for the remainder by a slant line or slash.

Eg. 13 as 1 / 3, 34 as 3 / 4 , 80 as 8 / 0.

ii) Leave some space below such representation, draw a horizontal line.

In the same way

```
             897 )  11  /  422
             ---
             103      1  /  03
                         /   206
                    -----------
                      12 /  658
```

gives 11,422 ÷ 897, Q = 12, R=658.

In this way we have to multiply the quotient by 2 in the case of 8, by 3 in the case of 7, by 4 in the case of 6 and so on. i.e., multiply the Quotient digit by the divisors complement from 10. In case of more digited numbers we apply Nikhilam and proceed. Any how, this method is highly useful and effective for division when the numbers are near to bases of 10.

*** Guess the logic in the process of division by 9.**

*** Obtain the Quotient and Remainder for the following problems.**

1) 311 ÷ 9   **2) 120012 ÷ 9**   **3) 1135 ÷ 97**

4) 2342 ÷ 98   **5) 113401 ÷ 997**

6) 11199171 ÷ 99979

Observe that by nikhilam process of division, even lengthier divisions involve no division or no subtraction but only a few multiplications of single digits with small numbers and a simple addition. But we know fairly well that only a special type of cases are being dealt and hence many questions about various other types of problems arise. The answer lies in Vedic Methods.

side. Thus third digit = 3.

iv) (1X 3) + (2 X 1) = 3 + 2 = 5. the carried over 1 of above step is added i.e., 5 + 1 = 6 . It is retained. Thus fourth digit = 6

v) (1 X 1) = 1. As there is no carried over number from the previous step it is retained. Thus fifth digit = 1

124 X 132 = 16368.

Let us work another problem by placing the carried over digits under the first row and proceed.

```
  234
 x 316
 ------
 61724
  1222
 ------
 73944
```

i) 4 X 6 = 24 : 2, the carried over digit is placed below the second digit.

ii) (3 X 6) + (4 x 1) = 18 + 4 = 22 ; 2, the carried over digit is placed below third digit.

iii) (2 X 6) + (3 X 1) + (4 X 3) = 12 + 3 + 12 = 27 ; 2, the carried over digit is placed below fourth digit.

iv) (2 X 1) + (3 X 3) = 2 + 9 = 11; 1, the carried over digit is placed below fifth digit.

v) (2 X 3) = 6.

vi) Respective digits are added.

Note :

1. We can carry out the multiplication in urdhva - tiryak process from left to right or right to left.

2. The same process can be applied even for numbers having more digits.

3. urdhva –tiryak process of multiplication can be effectively used in multiplication regarding algebraic expressions.

i.e.,, 12 122 5
- 2

Step 3 : Write the 1st digit below the horizontal line drawn under thedividend. Multiply the digit by –2, write the product below the 2nd digit and add.

i.e.,, 12 122 5
-2 -2

10

Since 1 x –2 = -2and 2 + (-2) = 0

Step 4 : We get second digits' sum as '0'. Multiply the second digits' sum thus obtained by –2 and writes the product under 3rd digit and add.

12 122 5
- 2 -20

102 5

Step 5 : Continue the process to the last digit

i.e., 12 122 5
-2 -20 -4

102 1

Step 6: The sum of the last digit is the Remainder and the result to its left is Quotient.
Thus Q = 102 andR = 1

Example 2 : Divide 1697 by 14.

14 1 6 9 7
- 4 -4–8–4

1 2 1 3

Q = 121, R = 3.

Example 3 : Divide 2598 by 123.

Note that the divisor has 3 digits. So we have to set up the last two

Find the Quotient and Remainder for the problems using paravartya – yojayet method.

1) 1234 ÷ 112   2) 11329 ÷ 1132

3) 12349÷ 133   4) 239479÷1203

Now let us consider the application of paravartya – yojayet in algebra.

Example 1 : Divide $6x^2 + 5x + 4$ by $x - 1$

$$\begin{array}{l|l} \underline{X-1} & 6x^2 + 5x + 4 \\ 1 & \underline{\quad 6 + 11 \quad} \\ & 6x + 11 + 15 \end{array}$$

Thus $Q = 6x+11$, $R=15$.

Example 2 : Divide $x^3 - 3x^2 + 10x - 4$ by $x - 5$

$$\begin{array}{l|l} \underline{X-5} & x^3 - 3x^2 + 10x - 4 \\ 5 & \underline{\quad 5 + 10 \quad 100} \\ & x^2 + 2x + 20 + 96 \end{array}$$

Thus $Q = x^2 + 2x + 20$, $R = 96$.

The procedure as a mental exercise comes as follows :

i) x^3 / xgives x^2 i.e.,, 1 the first coefficient in the Quotient.

ii) Multiply 1 by + 5,(obtained after reversing the sign of second term in the Quotient) and add to the next coefficient in the dividend. It gives 1 X(+5) = +5, adding to the next coefficient, i.e.,, –3 + 5 = 2. This is next coefficient in Quotient.

iii) Continue the process : multiply 2 by +5, i.e.,, 2 X +5 =10, add to the next coefficient 10 + 10 = 20. This is next coefficient in Quotient. Thus Quotient is $x^2 + 2x + 20$

iv) Nowmultiply 20 by + 5i.e.,, 20 x 5 = 100. Add to the next (last) term, 100 + (-4) = 96, which becomesR,i.e.,, R =9.

Example 2 :

$$\frac{5}{x+1} + \frac{6}{x-21} = 0$$

gives

$$x = \frac{-(5)(-21) - (6)(1)}{5+6} = \frac{105 - 6}{11} = \frac{99}{11} = 9$$

I . Solve the following problems using the sutra Paravartya – yojayet.

1) $3x + 5 = 5x - 3$

2) $(2x/3) + 1 = x - 1$

3) $\dfrac{7x + 2}{3x - 5} = \dfrac{5}{8}$

4) $\dfrac{x + 1/3}{3x - 1} = 1$

5) $\dfrac{5}{x + 3} + \dfrac{2}{x - 4} = 0$

6) $(x + 1)(x + 2) = (x - 3)(x - 4)$

7) $(x - 7)(x - 9) = (x - 3)(x - 22)$

8) $(x + 7)(x + 9) = (x + 3)(x + 21)$

II)

1.Show that for the type of equations

$$\frac{m}{x+a} + \frac{n}{x+b} + \frac{p}{x+c} = 0, \quad \text{the solution is}$$

$$x = \frac{-mbc - nca - pab}{m(b+c) + n(c+a) + p(a+b)}, \text{ if } m + n + p = 0.$$

2. Apply the above formula to set the solution for the problem

Problem

$$\frac{3}{x+4} + \frac{2}{x+6} - \frac{5}{x+5} = 0$$

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some more simple solutions :

$$\frac{m}{x+a} + \frac{n}{x+b} = \frac{m+n}{x+c}$$

Now this can be written as,

$$\frac{m}{x+a} + \frac{n}{x+b} = \frac{m}{x+c} + \frac{n}{x+c}$$

$$\frac{m}{x+a} - \frac{m}{x+c} = \frac{n}{x+c} - \frac{n}{x+b}$$

$$\frac{m(x+c) - m(x+a)}{(x+a)(x+c)} = \frac{n(x+b) - n(x+c)}{(x+c)(x+b)}$$

$$\frac{mx + mc - mx - ma}{(x+a)(x+c)} = \frac{nx + nb - nx - nc}{(x+c)(x+b)}$$

$$\frac{m(c-a)}{x+a} = \frac{n(b-c)}{x+b}$$

$$m(c - a).x + m(c - a).b = n(b - c).x + n(b - c).a$$

$$x[m(c - a) - n(b - c)] = na(b - c) - mb(c - a)$$

$$\text{or } x[m(c - a) + n(c - b)] = na(b - c) + mb(a - c)$$

Hence $5x + 7 = 0$, $x - 3 = 0$

$5x = -7$, $x = 3$

i.e., $x = -7 / 5$, $x = 3$

Note that all these can be easily calculated by mere observation.

Example 8:

$$\frac{3x + 4}{6x + 7} = \frac{5x + 6}{2x + 3}$$

Observe that

$N_1 + N_2 = 3x + 4 + 5x + 6 = 8x + 10$

and $D_1 + D_2 = 6x + 7 + 2x + 3 = 8x + 10$

Further $N_1 \sim D_1 = (3x + 4) - (6x + 7)$

$= 3x + 4 - 6x - 7$

$= -3x - 3 = -3 (x + 1)$

$N_2 \sim D_2 = (5x + 6) - (2x + 3) = 3x + 3 = 3(x + 1)$

By 'Sunyam Samuccaye' we have

$8x + 10 = 0$ $\qquad$ $3(x + 1) = 0$

$8x = -10$ $\qquad$ $x + 1 = 0$

$x = -10 / 8$ $\qquad$ $x = -1$

$= - 5 / 4$

vi)'Samuccaya' with the same sense but with a different context and application .

Example 9:

$$\frac{1}{x - 4} + \frac{1}{x - 6} = \frac{1}{x - 2} + \frac{1}{x - 8}$$

Usually we proceed as follows.

$$\frac{x-6+x-4}{(x-4)\ (x-6)} = \frac{x-8+x-2}{(x-2)\ (x-8)}$$

$$\frac{2x-10}{x^2-10x+24} = \frac{2x-10}{x^2-10x+16}$$

$$(2x - 10) (x^2 - 10x + 16) = (2x - 10) (x^2 - 10x + 24)$$
$$2x^3-20x^2+32x-10x^2+100x-160 = 2x^3-20x^2+48x-10x^2+100x-240$$
$$2x^3 - 30x^2 + 132x - 160 = 2x^3 - 30x^2 + 148x - 240$$
$$132x - 160 = 148x - 240$$
$$132x - 148x = 160 - 240$$
$$- 16x = - 80$$
$$x = - 80 / - 16 = 5$$

Now 'Samuccaya' sutra, tell us that, if other elements being equal, the sum-total of the denominators on the L.H.S. and their total on the R.H.S. be the same, that total is zero.

Now $D_1 + D_2 = x - 4 + x - 6 = 2x - 10$, and
$D_3 + D_4 = x - 2 + x - 8 = 2x - 10$

By Samuccaya, $2x - 10$ gives $2x = 10$

$$x = \frac{10}{2} = 5$$

Example 10:

$$\frac{1}{x-8} + \frac{1}{x-9} = \frac{1}{x-5} + \frac{1}{x-12}$$

$D_1 + D_2 = x - 8 + x - 9 = 2x - 17$, and
$D_3 + D_4 = x - 5 + x - 12 = 2x - 17$
Now $2x - 17 = 0$ gives $2x = 17$

$$x = \frac{17}{2} = 8½$$

Example 11:

$$\frac{1}{x+7} - \frac{1}{x+10} = \frac{1}{x+6} - \frac{1}{x+9}$$

$$y^3 = y^2 + 4y - 4 \text{ for } y = x + 3$$
$$y = 1, 2, -2.$$

Hence $x = -2, -1, -5$

Thus purana is helpful in factorization.

Further purana can be applied in solving Biquadratic equations also.

Solve the following using purana – apuranabhyam.

1. $\mathbf{x^3 - 6x^2 + 11x - 6 = 0}$
2. $\mathbf{x^3 + 9x^2 + 23x + 15 = 0}$
3. $\mathbf{x^2 + 2x - 3 = 0}$
4. $\mathbf{x^4 + 4x^3 + 6x^2 + 4x - 15 = 0}$

9. Calana - Kalanabhyam

In the book on **Vedic Mathematics** Sri Bharati Krishna Tirthaji mentioned the Sutra 'Calana - Kalanabhyam' at only two places. The Sutra means 'Sequential motion'.

i) In the first instance it is used to find the roots of a quadratic equation $7x^2 - 11x - 7 = 0$. Swamiji called the sutra as calculus formula. Its application at that point is as follows.Now by calculus formula we say: $14x - 11 = \pm\sqrt{317}$

A Note follows saying every Quadratic can thus be broken down into two binomial factors. An explanation in terms of first differential, discriminant with sufficient number of examples are given under the chapter 'Quadratic Equations'.

ii) At the Second instance under the chapter 'Factorization and Differential Calculus' for factorizing expressions of 3rd, 4th and 5th degree, the procedure is mentioned as'Vedic Sutras relating to Calana – Kalana – Differential Calculus'.

Further other Sutras 10 to 16 mentioned below are also used to get the required results. Hence the sutra and its various applications will be taken up at a later stage for discussion.

But sutra – 14 is discussed immediately after this item.

What happens if we take 4000 i.e. 4 X 1000 as working base?

$$\begin{array}{ll} 3998 & \overline{0002} \\ 4998 & 0998 \\ \hline \end{array}$$ Since 1000 is an operation

4996 / 1996

As 1000 is in operation, $\overline{1996}$ has to be written as $_1\overline{996}$ and 4000 as base, the L.H.S portion 5000 has to be multiplied by 4. i. e. the answer is

$$\begin{array}{r|l} 4996 & _1\overline{996} \\ \times \quad 4 & \\ \hline 19982 & 004 \end{array} \quad = 19982004$$

A simpler example for better understanding.

Example 6: 58 x 48

Working base 50 = 5 x 10 gives

$$\begin{array}{ll} 58 & +8 \\ 48 & \overline{2} \\ \hline 56 & \overline{16} \\ \times\ 5 & \\ \hline 280 & _1\overline{6} \end{array} \quad = 280 / 4 = 2784$$

Since 10 is in operation.

Use anurupyena by selecting appropriate working base and method.

Find the following product.

1. 46 x 46 **2. 57 x 57** **3. 54 x 45**

4. 18 x 18 **5. 62 x 48** **6. 229 x 230**

7. 47 x 96 **8. 87965 x 99996** **9. 49x499**

10. 389 x 512

Cubing of Numbers:

Example : Find the cube of the number 106.

We proceed as follows:

i) For 106, Base is 100. The surplus is 6.

Here we add double of the surplus i.e. 106+12 = 118.

(Recall in squaring, we directly add the surplus)

This makes the left-hand -most part of the answer.

i.e. answer proceeds like 118 / - - - - -

ii) Put down the new surplus i.e. 118-100=18 multiplied by the initial surplus

i.e. 6=108.

Since base is 100, we write 108 in carried over form 108 i.e. .

As this is middle portion of the answer, the answer proceeds like 118 / $_1$08 /....

iii) Write down the cube of initial surplus i.e. 6^3 = 216 as the last portion

i.e. right hand side last portion of the answer.

Since base is 100, write 216 as $_2$16 as 2 is to be carried over.

Answer is 118 / $_1$08 / $_2$16

Now proceeding from right to left and adjusting the carried over, we get the answer

119 / 10 / 16 = 1191016.

Eg.(1): 102^3 = (102 + 4) / 6 X 2 / 2^3

= 106 = 12 = 08

= 1061208.

Observe initial surplus = 2, next surplus =6 and base = 100.

Step (ii): Eliminate z and x, retain y and independent term
i.e., z = 0, x = 0 in the expression.
Then E = $6y^2 + 22y + 20 = (2y + 4)(3y + 5)$

Step (iii): Eliminate x and y, retain z and independent term
i.e., x = 0, y = 0 in the expression.
Then E = $2z^2 + 13z + 20 = (z + 4)(2z + 5)$

Step (iv): The expression has the factors (think of independent terms)
$= (3x + 2y + z + 4)(x + 3y + 2z + 5)$.

In this way either homogeneous equations of second degree or general equations of second degree in three variables can be very easily solved by applying 'adyamadyena' and 'lopanasthapanabhyam' sutras.

Solve the following expressions into factors by using appropriate sutras:

1. $x^2 + 2y^2 + 3xy + 2xz + 3yz + z^2$.
2. $3x^2 + y^2 - 4xy - yz - 2z^2 - zx$.
3. $2p^2 + 2q^2 + 5pq + 2p - 5q - 12$.
4. $u^2 + v^2 - 4u + 6v - 12$.
5. $x^2 - 2y^2 + 3xy + 4x - y + 2$.
6. $3x^2 + 4y^2 + 7xy - 2xz - 3yz - z^2 + 17x + 21y - z + 20$.

Highest common factor:

To find the Highest Common Factor i.e. H.C.F. of algebraic expressions, the factorization method and process of continuous division are in practice in the conventional system. We now apply' Lopana - Sthapana' Sutra, the 'Sankalana vyavakalanakam' process and the 'Adyamadya' rule to find out the H.C.F in a more easy and elegant way.

Example 1: Find the H.C.F. of $x^2 + 5x + 4$ and $x^2 + 7x + 6$.

1. Factorization method:

$x^2 + 5x + 4 = (x + 4)(x + 1)$
$x^2 + 7x + 6 = (x + 6)(x + 1)$

18. Gunita Samuccayah : Samuccaya Gunitah

In connection with factorization of quadratic expressions a sub-Sutra, viz. 'Gunita samuccayah-Samuccaya Gunitah' is useful. It is intended for the purpose of verifying the correctness of obtained answers in multiplications, divisions and factorizations. It means in this context:

'The product of the sum of the coefficients **sc** in the factors is equal to the sum of the coefficients **sc** in the product'

Symbolically we represent as **sc** of the product = product of the **sc** (in the factors)

Example 1: $(x + 3)(x + 2) = x2 + 5x + 6$

Now $(x + 3) (x + 2) = 4 \times 3 = 12$: Thus verified.

Example 2: $(x - 4)(2x + 5) = 2x^2 - 3x - 20$

Sc of the product $2 - 3 - 20 = -21$

Product of the **Sc** $= (1 - 4)(2 + 5) = (-3)(7) = -21$. Hence verified.

In case of cubics, biquadratics also the same rule applies.

We have $(x + 2)(x + 3)(x + 4) = x^3 + 9x^2 + 26x + 24$

Sc of the product $= 1 + 9 + 26 + 24 = 60$

Product of the **Sc** $= (1 + 2)(1 + 3)(1 + 4)$

$= 3 \times 4 \times 5 = 60$. Verified.

Example 3: $(x + 5)(x + 7)(x - 2) = x^3 + 10x^2 + 11x - 70$

$(1 + 5)(1 + 7)(1 - 2) = 1 + 10 + 11 - 70$

i.e., $6 \times 8 \times -1 = 22 - 70$
i.e., $-48 = -48$ Verified.

We apply and interpret **So** and **Sc** as sum of the coefficients of the odd powers and sum of the coefficients of the even powers and derive that **So** = **Sc** gives (x + 1) is a factor for thee concerned expression in the variable x. **Sc** = 0 gives (x - 1) is a factor.

Beejank of 18 → 1 + 8 → 9

Product of the Beejanks = 3 X 9 → 27 → 2 + 7 → 9

Beejank of 4266 → 4 + 2 + 6 + 6 → 18 → 1 + 8 → 9

Hence verified.

vii) 24^2 = 576

Beejank of 24 → 2 + 4 → 6

square of it 6^2 → 36 → 9

Beejank of result = 576 → 5 + 7 + 6 → 18 → 1 + 8 → 9

Hence verified.

viii) 356^2 = 126736

Beejank of 356 → 3 + 5 + 6 → 5

Square of it = 5^2 = 25 → 2+ 5 → 7

Beejank of result 126736 → 1 + 2 + 6 + 7 + 3 + 6 → 1 + 6 → 7

(∵ 7 + 2 = 9; 6 + 3 = 9) hence verified.

ix) Beejank in Division:

Let P, D, Q and R be respectively the dividend, the divisor, the quotient and the remainder.

Further the relationship between them is P = (Q X D) + R

eg 1: 187 ÷ 5

we know that 187 = (37 X 5) + 2 now the Beejank check.

We know that 187 = (37 X 5) +2 now the Beejank check.

187 → 1 + 8 + 7 → 7(∵ 1 + 8 = 9)

(37 X 5) + 2 → Beejank [(3 + 7) X 5] + 2

→ 5 + 2 → 7

Hence verified.

eg 2: 7986 ÷ 143

7896 = (143 X 55) + 121

Beejank of 7986 → 7 + 9 + 8 + 6 → 21

(∵ 9 is omitted) → 2 + 1 → 3

Beejank of 143 X 55 → (1 + 4 + 3) (5 + 5)

→ 8 X 10 → 80 → (8 + 0) → 8

Beejank of (143 X 55) + 121 → 8 + (1 + 2 + 1)

→ 8 + 4 → 12 → 1 + 2 → 3

hence verified.

Check the following results by Beejank method

1. 67 + 34 + 82 = 183 **2. 4381 - 3216 = 1165**

3. 63^2 = 3969 **4. $(1234)^2$ = 1522756**

5. (86X17) + 34 = 1496 **6. 2556 ÷ 127 gives Q =20, R = 16**

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i) Vinculum : The numbers which by presentation contains both positive and negative digits are called vinculum numbers.

ii) Conversion of general numbers into vinculum numbers.

We obtain them by converting the digits which are 5 and above 5 or less than 5 without changing the value of that number.

Consider a number say 8. (Note it is greater than 5). Use it complement (purak - rekhank) from 10. It is 2 in this case and add 1 to the left (i.e. tens place) of 8.

Thus 8 = 08 = $1\bar{2}$.

The number 1 contains both positive and negative digits

$11276 = 113\overline{24}$

i.e. $113\overline{24} = 11300 - 24 = 11276$.

The conversion can also be done by the sutra sankalana vyavakalanabhyam as follows.

eg 4: 315.

sankalanam (addition) = 315+315 = 630.

Vyvakalanam (subtraction) = $630 - 315 = 32\overline{5}$

Working steps :

$0 - 5 = \overline{5}$

$3 - 1 = 2$

$6 - 3 = 3$

Let's apply this sutra in the already taken example 47768

Samkalanam = 47768 + 47768 = 95536

Vyavakalanam = 95536 - 47768.

Steps :

$$\left.\begin{array}{l} 6 - 8 = \overline{2} \\ 3 - 6 = \overline{3} \\ 5 - 7 = \overline{2} \\ 5 - 7 = \overline{2} \\ 9 - 4 = 5 \end{array}\right\} = 5\overline{2232}$$

Consider the convertion by sankalanavyavakalanabhyam and check it by Ekadhika and Nikhilam.

eg 5: 12637

1. Sankalana .... gives, 12637 + 12637 = 25274

$25274 - 12637 = (2 - 1) / (5 - 2) / (2 - 6) / (7 - 3) / (4 - 7) = 13\overline{4}4\overline{3}$

$2\bar{7}$

iv) the obtained answer is to be normalized as per rules already explained. rules already explained.

i.e., $2\bar{7}$ = (2 - 1) (10- 7) = 13 Thus we get 7 + 6 = 13.

eg 2 : Add 973 and 866.

973 = $1\ 0\ \bar{3}\ 3$ $\qquad\qquad$ $1\ 0\ \bar{3}\ 3$

866 = $1\ \bar{1}\ \overline{3\ 4}$ $\qquad\qquad$ $1\ \bar{1}\ \bar{3}\ \bar{4}$

$\qquad\qquad\qquad\qquad$ $2\ \bar{1}\ \bar{6}\ \bar{1}$

But $2\overline{161}$ = 2000 - 161 = 1839.

Thus 973+866 by vinculum method gives 1839 which is correct.

Observe that in this representation the need to carry over from the previous digit to the next higher level is almost not required.

eg 3 : Subtract 1828 from 4247.

i.e.,, 4247
-1828

Step (i) : write –1828 in Bar form i.e.,, $\overline{1828}$

(ii) : Now we can add 4247 and $\overline{1828}$ i.e.,,

4247
$+\overline{1828}$

$3\bar{6}2\bar{1}$

since $7 + \bar{8} = \bar{1}$, $4 + \bar{2} = 2$, $2 + \bar{8} = \bar{6}$, $4 + \bar{1} = 3$

(iii) Changing the answer $3\bar{6}2\bar{1}$ into normal form using Nikhilam, we get

2. Addition and Subtraction

ADDITION:

In the convention process we perform the process as follows.

234 + 403 + 564 + 721

write as 234
403
564
721

Step (i): 4 + 3 + 4 + 1 = 12 2 retained and 1 is carried over to left.

Step (ii): 3 + 0 + 6 + 2 = 11 the carried '1' is added

i.e., Now 2 retained as digit in the second place (from right to left) of the answer and 1 is carried over to left.

step (iii): 2 + 4 + 5 + 7 = 18 carried over '1' is added

i.e., 18 + 1 = 19. As the addition process ends, the same 19 is retained in the left had most part of the answer.

thus 234
403
564
+721

1922 is the answer

we follow sudhikaran process Recall 'sudha' i.e., dot (.) is taken as an upa-sutra (No: 15)

consider the same example

234
403
564
+ 721

i) Carry out the addition column by column in the usual fashion, moving from bottom to top.

(a) 1 + 4 = 5, 5 + 3 = 8, 8 + 4 = 12 The final result is more than 9. The tenth place '1' is dropped once number in the unit place i.e., 2 retained. We say at this stage sudha and a dot is above the top 4. Thus column (1) of addition (right to left)

.
4
3
4
1
___
2

b) Before coming to column (2) addition, the number of dots are to be counted, This shall be added to the bottom number of column (2) and we proceed as above.

Thus second column becomes

.
3 dot=1, 1 + 2 = 3
0 3 + 6 = 9
6 9 + 0 = 9
2 9 + 3 = 12
___
2

2 retained and '.' is placed on top number 3

c) proceed as above for column (3)

2 i)dot = 1 ii) 1 + 7 = 8
4 iii)8 + 5 = 13 iv) Sudha is said.

.
5 A dot is placed on 5 and proceed
7 with retained unit place 3.
___
9 v) 3+4=7,7+2=9 Retain 9 in 3rd digit i.e.,in 100th place.

d) Now the number of dots is counted. Here it is 1 only and the number is carried out left side ie. 1000th place

Thus
. .
234
403
.
564
+721
______
1922 is the answer.

Though it appears to follow the conventional procedure, a careful observation and practice gives its special use.

eg (1):

```
 .
437
.  .
624
 .
586
+162
______
1809
```

Steps 1:

i) 2 + 6 = 8, 8 + 4 = 12 so a dot on 4 and 2 + 7 = 9 the answer retained under column (i)

ii) One dot from column (i) treated as 1, is carried over to column (ii),

thus 1 + 6 = 7, 7 + 8 = 15 A' dot'; is placed on 8 for the 1 in 15 and the 5 in 15 is added to 2 above.

5 + 2 = 7, 7 + 3 = 10 i.e. 0 is written under column (ii) and a dot for the carried over 1 of 10 is placed on the top of 3.

(iii) The number of dots counted in column (iii) are 2

Hence the number 2 is carried over to column (ii) Now in column (iii)

2 + 1 = 3, 3 + 5 = 8, 8 + 6 = 14 A dot for 1 on the number 6 and 4 is retained to be added 4 above to give 8. Thus 8 is placed under column (iii).

iv) Finally the number of dots in column (iii) are counted. It is '1' only. So it carried over to 1000th place. As there is no fourth column 1 is the answer for 4th column. Thus the answer is 1809.

Example 3:

```
  3647
  4086
  5718
+ 1692
______
 15143
```

Check the result verify these steps with the procedure mentioned above.

The process of addition can also be done in the down-ward direction i.e., addition of numbers column wise from top to bottom

Thus answer is 12772.

Add the following numbers use 'Sudhikaran' whereever applicable.

1.

$$\begin{array}{r} 486 \\ 395 \\ 721 \\ +609 \\ \hline \\ \hline \end{array}$$

2.

$$\begin{array}{r} 5432 \\ 3691 \\ 4808 \\ +6787 \\ \hline \\ \hline \end{array}$$

3.

$$\begin{array}{r} 968763 \\ 476509 \\ +584376 \\ \hline \\ \hline \end{array}$$

Check up whether 'Sudhkaran' is done correctly. If not write the correct process. In either case find the sums.

1.

$$\begin{array}{rrr} & 4 & \dot{3} \\ & \dot{7} & \dot{8} \\ + & 6 & 9 \\ \hline \\ \hline \end{array}$$

2.

$$\begin{array}{rrr} 3 & \dot{7} & \dot{9} \\ 8 & \dot{5} & 4 \\ \dot{7} & \dot{6} & \dot{7} \\ 4 & 2 & 6 \\ \hline \\ \hline \end{array}$$

3.

$$\begin{array}{rrrrrr} & 7 & 8 & \dot{9} & 2 & 1 \\ & 2 & \dot{7} & 2 & 7 & 2 \\ + & 7 & 2 & 6 & \dot{8} & 4 \\ \hline \\ \hline \end{array}$$

SUBTRACTION:

The 'Sudha' Sutra is applicable where the larger digit is to be subtracted from the smaller digit. Let us go to the process through the examples.

Procedure:

i) If the digit to be subtracted is larger, a dot (sudha) is given to its left.

ii) The purak of this lower digit is added to the upper digit or purak-rekhank of this lower digit is subtracted.

$$
\begin{aligned}
195^2 &= (195 - 5) \,/\, 5^2 \\
&= 190 \,/\, 25 \\
&\quad \times 2 \qquad \text{operating on } 100 \\
&= 38025
\end{aligned}
$$

v) By ‘antyayor dasakepi’ and ‘Ekadhikena’ sutras

Since in 195 x 195, 5 + 5 = 10 gives

195^2 = 19 x 20 / 5 x 5 = 380 / 25 = 38025.

vi) Now "urdhva-tiryagbhyam" gives

```
                                   195
                                X  195
------------------------------------------------------------------------------------------
(1 x 1) / (1 x 9) + (1 x 9) / (1 x 5) + (9 x 9) + (5 x 1) / (9 x 5) + (5 x 9) / (5 x 5)
   1    /     9 + 9         /        5 + 81 + 5           /        45 + 45        /  25
=  1    /       8           /            1                /           0           /   5
              1                          9                            9               2
```

By the carryovers the answer is 38025

Example 2 : 98 X 92

i) ‘Nikhilam’ sutra

```
     98   -2
  x  92   -8
 ______________
   90 / 16 = 9016
```

ii) ‘Antyayordasakepi’ and ‘Ekadhikena’ sutras.

98 X 92 Last digit sum= 8+2 =10 remaining digit (s) = 9 samesutras work.

∴98 X 92 = 9 X (9 + 1) / 8X2 = 90/16 = 9016.

iii) urdhava-tiryak sutra

```
  98
x 92
```

Step(4):

8988) 12 / 1134

1012   1 012   [2+ 1 = 3 and 3x1012 = 3036]

3036

Now **final Step**

8988) 12 / 1134

1012   1   012

3036(Column wise addition)

13 / 4290

Thus 121134 ¸ 8988 gives Q = 13 and R = 4290.

iii) Paravartya method: Recall that this method is suitable when the divisor is nearer but more than the base.

Example 3: 32894 ÷ 1028.

The divisor has 4 digits. So the last 3 digits of the dividend are set apart for the remainder and the procedure follows.

1 0 2 8	32	8 9 4	i)	3 x (0, -2, -8)
0 -2 -8	0	-6 -24		= 0, -6, -24
		0 -4 -16	ii)	2 – 0 = 2 and
				2 x (0, -2, -8)
	32	2 -19 -12		= 0, -4, -16
			iii)	8 – 6 + 0 = 2
				9 – 24 – 4 = - 19
				4 – 16 = - 12

Now the remainder contains -19, -12 i.e. negative quantities. Observe that 32 is quotient. Take 1 over from the quotient column i.e. 1x1028 = 1028 over to the right side and proceed thus: 32 - 1 = 31 becomes the Q and R = 1028+200 - 190 - 12 =1028-2 =1026.

Thus 3289 ÷ 1028 gives Q = 31 and R = 1026.

The same problem can be presented or thought of in any one of the following forms.

Let a and b be two digits.

Consider the row $a^3 \quad a^2b \quad ab^2 \quad b^3$

the first is a^3 and the numbers are in the ratio a:b

since $a^3:a^2b=a^2b:b^3=a:b$

Now twice of a^2b, ab^2 are $2a^2b$, $2ab^2$

$$\begin{array}{l} a^3 + \ a^2b + ab^2 + b^3 \\ \qquad 2a^2b + 2ab^2 \\ \hline a^3 +3a^2b + 3ab^2 + b^3 = (a + b)^3. \end{array}$$

Thus cubes of two digit numbers can be obtained very easily by using the vedic sutra 'anurupyena'. Now cubing can be done by using the vedic sutra 'Yavadunam'.

Example 3: Consider 106^3.

i) The base is 100 and excess is 6. In this context we double the excess and then add.

i.e. 106 + 12 = 118. ($\because$ 2 X 6 =12)

This becomes the left - hand - most portion of the cube.

i.e. 106^3 = 118 / - - - -

ii) Multiply the new excess by the initial excess

i.e. 18 x 6 = 108 (excess of 118 is 18)

Now this forms the middle portion of the product of course 1 is carried over, 08 in the middle.

i.e. 106^3 = 118 / 08 / - - - - -
$\qquad\qquad\qquad$ 1

iii) The last portion of the product is cube of the initial excess.

i.e. 6^3 = 216.

16 in the last portion and 2 carried over.

i.e. 106^3 = 118 / 081 /16 = 1191016
$\qquad\qquad\qquad$ 1 $\quad$ 2

Example 4: Find 1002^3.

i) Base = 1000. Excess = 2. Left-hand-most portion of the cube becomes 1002+(2x2)=1006.