Find the derivatives of inverse functions.

The inverse of an ordered pair (x, y) is (y, x).

$f(x) = y \Leftrightarrow x = f^{-1}(y)$.

Theorem: Let $f(x)$ represent a differentiable on an interval. If $f^{-1}(x)$ denotes the inverse of $f(x)$, then $f^{-1}(x)$ is differentiable at any x and

$$\left[f^{-1}(x)\right]' = \frac{1}{f'[f^{-1}(x)]}$$

where $\left[f^{-1}(x)\right]'$ denotes the derivative of inverse of $f(x)$, $f'(x)$ is the derivative of $f(x)$ and $f'\left[f^{-1}(x)\right] \neq 0$.

Proof:
If $f^{-1}(x)$ is the inverse of $f^{-1}(x)$, then $f[f^{-1}(x)] = f^{-1}[f(x)] = x$.

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Differentiating $f[f^{-1}(x)] = x$ applying chain rule:

$$\frac{df[f^{-1}(x)]}{dx} = \frac{dx}{dx}$$

$\dfrac{df[f^{-1}(x)]}{df^{-1}(x)} \cdot \dfrac{df^{-1}(x)}{dx} = 1$, but $\dfrac{df[f^{-1}(x)]}{df^{-1}(x)} = f'[f^{-1}(x)]$ and $\dfrac{df^{-1}(x)}{dx} = [f^{-1}(x)]'$

Thus $f'[f^{-1}(x)] \cdot [f^{-1}(x)]' = 1$ equation (1)

Dividing equation (1) by $f'[f^{-1}(x)]$,

$[f^{-1}(x)]' = \dfrac{1}{f'[f^{-1}(x)]}$, where $[f^{-1}(x)]'$ is the derivative of the inverse of $f(x)$ and

$f'[f^{-1}(x)] \neq 0$.

The formula $[f^{-1}(x)]' = \dfrac{1}{f'[f^{-1}(x)]}$ can be used to find the derivative of an inverse