A2 Maths Revision Notes

Polynomials

Long Division: $x+1 \overline{)x^2+5x+4}$ ← Dividend; Divisor: $x+1$; Quotient: $x+4$

$$x-1 \overline{)x^3-3x^2-x+3}$$

$$\begin{array}{r} x^2-2x-3 \\ x-1 \overline{)x^3-3x^2-x+3} \\ x^3-x^2 \\ \hline -2x^2-x \\ -2x^2+2x \\ \hline -3x+3 \\ -3x+3 \end{array}$$

① How many times does x go into x^3?

② Multiply through

③ Subtract + Repeat

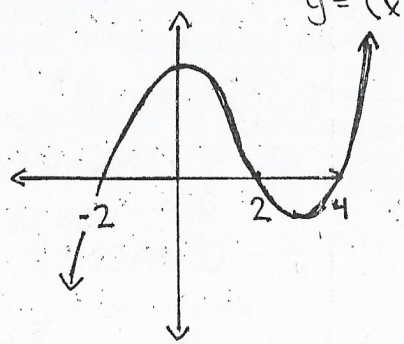
⚠ Sometimes there is a remainder!

Factor Theorem: If $(x-t)$ is a factor of $(f(x))$ then $f(t)=0$

Remainder Theorem: The remainder of $\frac{f(x)}{(x-p)}$ is $f(p)$

Sketching: sketch $x^3-4x^2-4x+16=y$

$y=(x-2)(x+2)(x-4)$



Should be able to mirror.

Differentiation

Refresher → $y = ax^n$ so $\frac{dy}{dx} = anx^{n-1}$

→ $(ax+b)^n$ so $\frac{dy}{dx} = n(a)(ax+b)^{n-1}$

Product Rule $\boxed{\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}}$

$y = 20x(x-1)^6$ → $u: 20x$ $v: (x-1)^6$

$\frac{du}{dx}: 20$ $\frac{dv}{dx}: 6(x-1)^5$

$\frac{dy}{dx} = 20x(6)(x-1)^5 + (x-1)^6(20)$

$120x(x-1)^5 + 20(x-1)^6$

$= 20(x-1)^5[6x + (x+1)]$

$\frac{dy}{dx} = 20(x-1)^5[7x+1]$

Quotient Rule $\boxed{\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}}$

$y = \frac{x+1}{x^2+2x+3}$ → $u: x+1$ $v: x^2+2x+3$

$\frac{du}{dx} = 1$ $\frac{dv}{dx} = 2x+2$

$\frac{dy}{dx} = \frac{x^2+2x+3 - (2x+2)(x+1)}{(x^2+2x+3)^2}$

$\frac{dy}{dx} = \frac{-x^2-2x+1}{(x^2+2x+3)^2}$

Natural Logs

$y = e^x \rightarrow \frac{dy}{dx} = e^x$

$y = \ln(f(x)) \rightarrow \frac{dy}{dx} = \frac{f'(x)}{f(x)}$

y	$\frac{dy}{dx}$
e^x	$\frac{dy}{dx}$
Ae^x	Ae^x
e^{bx}	be^{bx}
$\ln(Ax)$	$\frac{1}{x}$

Differentiation

Differentiating Trigonometry:

$y = \sin x$	$\frac{dy}{dx} = \cos x$	$y = \cos x$	$\frac{dy}{dx} = -\sin x$
$y = \tan x$	$\frac{dy}{dx} = \sec^2 x$	$y = \cot x$	$\frac{dy}{dx} = -\operatorname{cosec}^2 x$
$y = \operatorname{cosec} x$	$\frac{dy}{dx} = -\cot x \operatorname{cosec} x$	$y = \sec x$	$\frac{dy}{dx} = -\tan x \sec x$

Proof: $y = \sec x \Rightarrow \frac{dy}{dx} = -\tan x \sec x$

$\sec x = \frac{1}{\cos x}$ → $u = 1$ $\frac{du}{dx} = 0$ $v = \cos x$ $\frac{dv}{dx} = -\sin x$

$\frac{dy}{dx} = \frac{0 - \sin x}{\cos^2 x} = \frac{-\sin x}{\cos^2 x} = -\tan x \sec x$

Implicit Differentiation:

→ If you differentiate y, you get $\frac{dy}{dx}$

$\boxed{y \rightarrow \frac{dy}{dx}}$ $\boxed{\sin y \rightarrow \frac{dy}{dx}\cos y}$

Find gradient of $x^3 - x^2y - y^3 = 3$ $x = 2$ $y = 1$

$3x^2$ $-x^2 \frac{dy}{dx}$ $-2xy$ $-3\frac{dy}{dx}y^2$

→ $3x^2 - x^2\frac{dy}{dx} - 2xy - 3\frac{dy}{dx}y^2 = 0$

PLUG IN $x=2$ $y=1$

$3(4) - 4\frac{dy}{dx} - 4 - 3\frac{dy}{dx} = 0$

$8 = 7\frac{dy}{dx}$ → $\frac{dy}{dx} = \frac{8}{7}$ Answer