Cartesian system	**Spherical system**	**Cylindrical system**
$h, h_2, h_3 \equiv 1,1,1$	$h, h_2, h_3 \equiv 1, r, r\sin\theta$	$h, h_2, h_3 \equiv 1, r, 1$
$\hat{e}_1, \hat{e}_2, \hat{e}_3 \equiv \hat{i}, \hat{j}, \hat{k}$	$\hat{e}_1, \hat{e}_2, \hat{e}_3 \equiv \hat{r}, \hat{\theta}, \hat{\phi}$	$\hat{e}_1, \hat{e}_2, \hat{e}_3 \equiv \hat{r}, \hat{\phi}, \hat{z}$
$u_1, u_2, u_3 \equiv x, y, z$	$u_1, u_2, u_3 \equiv r, \theta, \phi$	$u_1, u_2, u_3 \equiv r, \phi, z$

In General:

1) $$\nabla t = \frac{1}{h_1}\frac{\partial t}{\partial u_1}\hat{e}_1 + \frac{1}{h_2}\frac{\partial t}{\partial u_2}\hat{e}_2 + \frac{1}{h_3}\frac{\partial t}{\partial u_3}\hat{e}_3$$

2) $$\nabla \cdot \vec{A} = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u_1}(A_1h_2h_3) + \frac{\partial}{\partial u_2}(A_2h_3h_1) + \frac{\partial}{\partial u_3}(A_3h_1h_2)\right]$$

3) $$\nabla \times \vec{A} = \frac{1}{h_1h_2h_3}\begin{vmatrix} h_1\hat{e}_1 & h_2\hat{e}_2 & h_3\hat{e}_3 \\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_3} \\ A_1h_1 & A_2h_2 & A_3h_3 \end{vmatrix}$$

4) $$\nabla^2 t = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u_1}\left\{\frac{h_2h_3}{h_1}\left(\frac{\partial t}{\partial u_1}\right)\right\} + \frac{\partial}{\partial u_2}\left\{\frac{h_3h_1}{h_2}\left(\frac{\partial t}{\partial u_2}\right)\right\} + \frac{\partial}{\partial u_3}\left\{\frac{h_1h_2}{h_3}\left(\frac{\partial t}{\partial u_3}\right)\right\}\right]$$

Let: $\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$ ⟶ Cartesian system

$= A_1\hat{e}_r + A_2\hat{e}_\theta + A_3\hat{e}_\phi$ ⟶ Spherical system

$= A_1\hat{e}_r + A_2\hat{e}_\phi + A_3\hat{e}_z$ ⟶ Cylindrical

In Cartesian system:

1) $$\nabla t = \frac{\partial t}{\partial x}\hat{i} + \frac{\partial t}{\partial y}\hat{j} \quad \frac{\partial t}{\partial z}\hat{k}$$

2) $$\nabla \cdot \vec{A} = \frac{\partial A_1}{\partial x} + \frac{\partial A_2}{\partial y} + \frac{\partial A_3}{\partial z}$$

3) $$\nabla \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_1 & A_2 & A_3 \end{vmatrix}$$

4) $$\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

In Spherical system:

1) $$\nabla t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\phi}$$

2) $$\nabla \cdot \vec{A} = \frac{1}{r^2\sin\theta}\left[\frac{\partial}{\partial r}(A_1 r^2 \sin\theta) + \frac{\partial}{\partial \theta}(A_2 r \sin\theta) + \frac{\partial}{\partial \phi}(A_3 r)\right]$$

3) $$\nabla \times \vec{A} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A1 & rA_2 & r\sin\theta A_3 \end{vmatrix}$$

Compare (1) & (2)

$$\vec{E} = -\nabla V$$

i) Taking 'curl' on both sides

$$\nabla \times E = \nabla \times (-\nabla V)$$

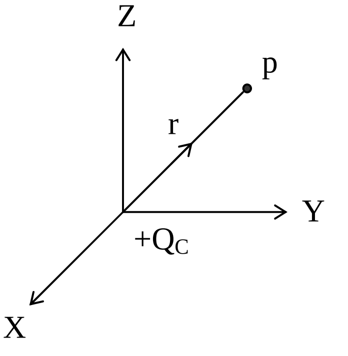
$$\therefore \quad \nabla \times \vec{E} = 0 \longrightarrow$$ **Maxwell's 3rd Equation**

ii) Taking 'divergence' on both sides

$$\nabla \cdot E = \nabla \cdot (-\nabla V)$$
$$= \nabla^2 V \neq 0$$

$$\therefore \quad \nabla \cdot \vec{E} \neq 0$$

∴ Therefore, an electrostatic field is irrotational (or) conservative but not solenoidal.



Electric potential due to a point charge (Absolute potential):

We know,

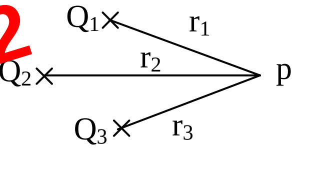
$$V(p) = -\int_{\theta}^{p} \vec{E} \cdot \vec{dl}$$

Due to finite charge, replace reference point θ with infinity (∞).

$$V(p) = -\int_{\infty}^{r} (Q / 4\pi\varepsilon_0 r^2) \cdot dr$$

$$\therefore \quad V(p) = Q / 4\pi\varepsilon_0 r$$

Electric potential due to a discrete charges:



$$V(p) = V(Q_1) + V(Q_2) + V(Q_3) + \ldots\ldots$$

$$= Q / 4\pi\varepsilon_0 r_1 + Q / 4\pi\varepsilon_0 r_2 + \ldots\ldots$$

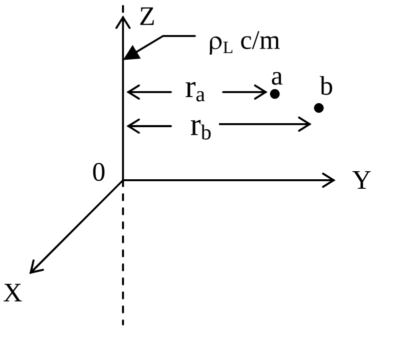
Electric Potential due to a continuous charge distribution:

$V(p) = \int (\rho_L\, dl) / 4\pi\varepsilon_0 r$ → for line charge distribution

$= \oiint_S (\rho_s\, da) / 4\pi\varepsilon_0 r$ → for surface charge distribution

$= \int_V (\rho_v\, dv) / 4\pi\varepsilon_0 r$ → for volume charge distribution

Electric potential due to an infinite line charge distribution:



Consider an infinite line charge placed along the Z – axis.

∴ Electric potential difference,

$$\therefore \quad V = (\rho_L / 2\pi\varepsilon_0)\, ln\, (r_b/r_a)$$

Solution of Laplace equation in spherical co – ordinates:

$$\nabla^2 V = 0$$

$$\Rightarrow 1/r^2 \text{Sin}\theta \; [\; \partial/\partial r(r^2 \sin\theta \; \partial V/\partial r) + \partial/\partial\theta(\text{Sin}\theta \; \partial V/\partial\theta) + \partial/\partial\phi[(1/\text{Sin}\theta) \; \partial V/\partial\phi)] = 0$$

Case1: 'V' is a function of 'r' only

$$\therefore \quad \boxed{V = -A / r + B}$$

Case2: 'V' is a function of 'θ' only

$$\therefore \quad \boxed{V = A \; ln \tan(\theta/2) + B}$$

Case3: 'V' is a function of 'ϕ' only

$$\therefore \quad \boxed{V = A\phi \; + B}$$

Solution of Laplace equation in Cylindrical Co – ordinates:

$$\nabla^2 V = 0$$

$$\Rightarrow 1/r[1/\partial r(r \; \partial V/\partial r) + \partial/\partial\phi(1/r \; . \; \partial V/\partial\phi) + \partial/\partial z(r \; \partial V/\partial z)] = 0$$

Case1: 'V' is a function of 'r' only

$$\therefore \quad \boxed{V = A \; ln \; r + B}$$

Case2: 'V' is a function of 'ϕ' only

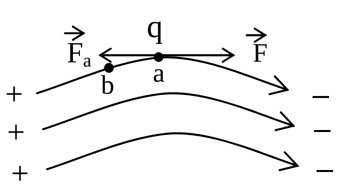
$$\therefore \quad \boxed{V = A\phi + B}$$

Case3: 'V' is a function of 'z' only

$$\therefore \quad \boxed{V = Az + B}$$

Note: Here A and B are arbitrary constants, whose values are determined by using appropriate boundary conditions.

Work Done:



A charge 'q' kept in the electric field experiences a force in the direction of electric field. F is the force experienced by the charge 'q'. F_a is the force applied in opposite direction. If the magnitude of F_a is equal to F, the charge remains in equilibrium. If F_a is slightly greater than F, the charge can be moved from point a to point b. The small work done to move the charge 'q' by a distance 'dl' is $F_a.dl$. Total work done in moving the charge from a to b can be obtained.

16. A line charge of $10^{-9}/2$ c/m lies on the Z – axis. Find r_{ab} if 'a' is at (2,0,0) and b is at (4,0,0)
a) 2V b) 4.24 V c) 6.24 V d) 8.24 V

17. A point charge of 0.4 nc is located at (2,3,3) in Cartesian system. Find r_{ab} if A is (2,2,3) and B is (-2,3,3).
a) 2.7 V b) 3.6 V c) 4.7 V d) 8.1 V

18. Determine the potential at (0,0,5) m caused by a total charge 10^{-8}c distributed uniformly along a disc of radius 5m lying in the Z = 0 plane and centered at the origin.
a) 12.2 b) 17 V c) 14.8 V d) 13.2 V

19. 3 point charges of 1C, 2C and 3C are located at the corner of an equilateral triangle of 1m side each. Find the energy stored in the system.
a) $9 / 4\pi\epsilon_0$ Joules b) $4\pi\epsilon_0 / 3$ Joules c) $11 / 4\pi\epsilon_0$ Joules d) 30×10^9 Joules

20. If the potential is given by $V = 5r^2$ where 'r' is distance from origin. How much charge is located with in a sphere of 1m radius centered at the origin.
a) $90\ \epsilon_0$ b) $-30\epsilon_0$ c) $30\ \epsilon_0$ d) $-30 / \epsilon_0$

Common data question

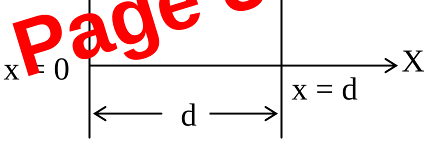
A spherical shell of radius 'a' contains a total charge of Q_0 uniformly distributed over its surface.

21. Find the potential inside the spherical shell
a) $Q_0^2 / 4\pi$ b) $Q_0 / 4\pi^2\varepsilon_0 a$ c) $Q_0 / 4\pi\varepsilon_0 a$ d) zero

22. Find the potential outside the spherical shell
a) $Q_0^2 / 4\pi$ b) $Q_0 / 4\pi\varepsilon_0 a$ c) zero d) $Q_0 / 4\pi\varepsilon_0 r$

Linked Question

Two parallel infinite conducting plates separated by a distance 'd' along the X – axis have a potential V_0 and zero respectively as shown.



23. Find the expression for voltage distribution
a) $V = V_0(1 + d/x)$ b) $V = V_0(1 - x/d)$ c) $V = V_0(1 - d/x)$ d) 0

24. Find the electric field intensity
a) $(V_0 / x)\ \hat{i}$ b) $V_0\ \hat{i}$ c) $(V_0 / d)\ .\ \hat{i}$ d) $(x / V_0)\ .\ \hat{i}$

Key:

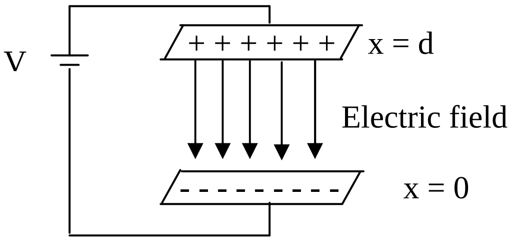
1.a	2.d	3.b	4.b	5.a	6.c	7.b	8.c	9.a	10.a	11.b	12.d	13.b
14.a	15.d	16.c	17.a	18.c	19.c	20.b	21.c	22.d	23.b	24.c		

TOPIC – 5: CAPACITANCE

Capacitor is formed using two conducting media with an insulator in between them.

Capacitance is the property of a dielectric to store electrical energy. An electric field is present between the plates since a voltage is applied between them. The dielectric is subjected to electric stress and strain. Therefore some energy can be stored in the dielectric. Capacitance is similar to inertia. The speed of a vehicle cannot change suddenly due to inertia. Similarly voltage across capacitor cannot change suddenly.

Capacitance of a parallel plate capacitor:



We know that, $c = Q/v$

$$= \frac{\left|\int \rho_s da\right|}{\left|-\int \vec{E}.\vec{dl}\right|}$$

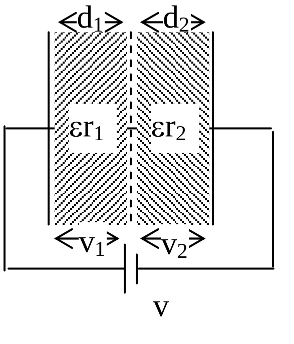
$$= \frac{\rho_s\,(\text{Area})}{\int \frac{\rho_s}{\varepsilon}\,\hat{i}\,.\,dx\,\hat{i}}$$

$$= \frac{\rho_s\,(A)}{\frac{\rho_s}{\varepsilon}\int_0^d dx}$$

$$= \frac{\rho_s\,(A)}{\frac{\rho_s}{\varepsilon}\,x\,d}$$

$$\therefore\ c = \frac{\varepsilon A}{d}$$ where A = cross section area of plate

Capacitance of parallel plate capacitor with two media



$V = V_1 + V_2$

$= E_1d_1 + E_2d_2$

$= (D/\varepsilon_1)\,d_1 + (D/\varepsilon_2)\,d_2$

$= (Q/A\varepsilon_1)\,d_1 + (Q/A\varepsilon_2)d_2$

$$= Q/A\left(\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2}\right)$$

$$V = \frac{CV}{A}\left(\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2}\right)$$

$$C = \frac{A}{\frac{d_1}{\varepsilon_0\varepsilon_{r1}} + \frac{d_2}{\varepsilon_0\varepsilon_{r2}}}$$

$$\boxed{W = 1/2\ cv^2}\ \text{Joules}$$

Energy density = $\frac{\text{Energy}}{\text{Volume}}$

$= \frac{1/2\ cv^2}{A \times d}$

$= 1/2 \times \frac{1}{A x d} \times \frac{\varepsilon A}{d} \times v^2$

$= (1/2)\ \varepsilon\ (v/d)^2$ [$\because$ v/d = E]

$= 1/2\ \varepsilon E^2$

$= 1/2\ (\varepsilon E)\ E$

$= 1/2$ DE

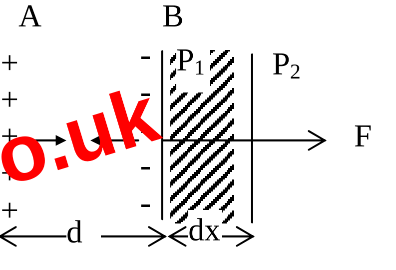
D and E can be written as D.E since D & E are in same direction

$\therefore$ Energy density = 1/2 D.E

Energy = 1/2 D.E x volume

$$\boxed{W = 1/2 \int_v \vec{D}.\vec{E}\ dv}$$

Force of Attraction between plates:



Between the oppositely charged plates there is a force of attraction. F is an externally applied force to move the plate B from p_1 to p_2. The work done is stored in the form of energy in the additional volume Adx.

Work done = Additional energy

F dx = (Energy density) volume

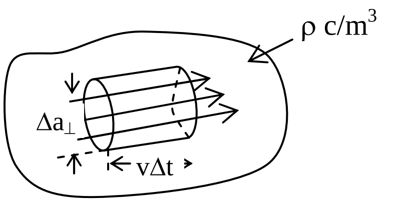
F dx = (1/2 εE^2) Adx

F = 1/2 εE^2 A Newtons

F/A = 1/2 εE^2 N/m^2

$\therefore$ $\boxed{\text{Force /unit area} = 1/2\ \varepsilon E^2}$

Volume Currents:



Flow of electric charges over a volume represents volume currents. Every volume current is associated with a mobile volume charge density ρ c/m^3. Considering an elementary cylinder within the volume current region, the amount of mobile charges contained at any instant is "$\rho\,\Delta a_{\perp}(V\,\Delta t)$".

All these elementary mobile charges coming out of the elementary cylinder in 'Δt' seconds is called current.

$$\Delta I = \frac{\rho \Delta a_{\perp} (V \cancel{\Delta t})}{\cancel{\Delta t}}$$

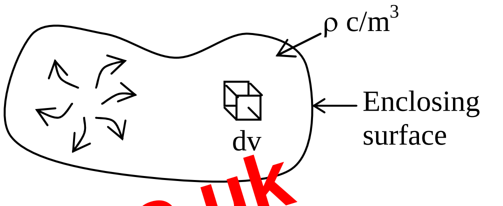
$$\Delta I = \rho V \Delta a_{\perp}$$

$$\frac{\Delta I}{\Delta a_{\perp}} = \rho V = J$$

$$\therefore \quad \boxed{\vec{J} = \rho \vec{V}} \quad A/m^2$$

Where $\vec{J}$ = Volume current density, A/m^2

Continuity Equation:



Let us consider a region carrying volume currents. For convenience let the charges flow outward. The net outward current through the enclosing surface can be obtained as.

$$\boxed{I = \oint_S \vec{J} \cdot \vec{da}} \quad \text{---} \quad (1) \quad [\text{from volume currents}]$$

And also, the rate of reduction of electric charges within the encloser.

$$= -\, d/dt \int_V \rho\, dv \quad \text{---} \quad (2)$$

According to the law of conservation of charges the above two equations are equal

$$\oint_S \vec{J} \cdot \vec{da} = -\, d/dt \int_V \rho\, dv$$

According to the fundamental theorem of divergence

$$\int_V (\nabla \cdot \vec{J})\, dv = -\, \partial/\partial t \int_V \rho\, dv \qquad [\text{ only one variable}]$$

Integration is done with respect to volume and differentiation is done with respect to time.

Multiply and divide with volume

$$P = VI \, . \, Al / Al$$

Rearrange the terms,

$$P = \frac{V}{l} \times \frac{l}{A} (Al)$$

Substitute $E = V/l$, $J = I/A$ and volume $= Al$

$$\therefore \; P = EJ \text{ volume}$$

EJ can be written as $\vec{E}.\vec{J}$ since E and J are in the same direction

$$P = (\vec{E} \, . \, \vec{J}) \text{ volume}$$

$$\therefore \quad \boxed{P = \int_V (\vec{E} \, . \, \vec{J}) \, dv}$$

According to Joule's law, energy dissipated per second is volume integral of dot product of the vectors E and J.

Relaxation Time:

To study relaxation time we start with ohm's law and equation of continuity.

$$\vec{J} = \sigma \vec{E} \text{ and } \nabla \, . \, \vec{J} = - (\partial\rho / \partial t)$$

$$\nabla \, . \, \sigma E = - (\partial\rho / \partial t)$$

$$\nabla \, . \, \frac{\varepsilon}{\varepsilon} \sigma \vec{E} = - (\partial\rho / \partial t)$$

$$\frac{\sigma}{\varepsilon} \nabla \, . \, \vec{D} = - \frac{\partial\rho}{\partial t}$$

$$\frac{\sigma}{\varepsilon} \rho + \frac{\partial\rho}{\partial t} = 0$$

$$\therefore \quad \boxed{\frac{\partial\rho}{\partial t} + \frac{\sigma}{\varepsilon} \rho = 0}$$

$\rho = \rho_0 \, e^{-(\sigma/\varepsilon)t}$ where ρ_0 is charge density at t = 0.
The charge density decays exponentially as time passes with time constant equal to $\in/\sigma$ seconds. This time constant is called relaxation time.

Conductance – Capacitance Theorem:

G = Conductance, C = Capacitance

σ = Conductivity, ε = permittivity $\quad \rho$ = resistivity

According to conductance theorem, conductance of an insulated medium is equal to $\sigma/\in$ times the capacitance of the insulation provided between two conducting media.

$$G = (\sigma / \varepsilon) \, C$$

We know that $C = \in A / l$ and $R = \rho l / A \;\Rightarrow G = A / \rho l = \sigma A / l$

$$\therefore \frac{G}{C} = \frac{\sigma A}{\varepsilon A}$$

$$\boxed{G = (\sigma/\varepsilon) \, C}$$

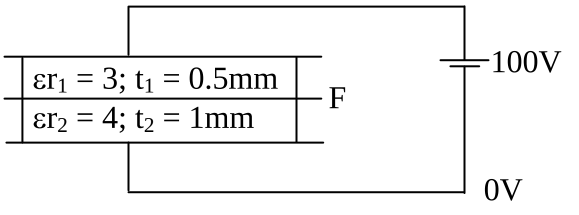
This theorem is very useful to obtain the expression for conductance of the configuration if capacitance of that configuration is already known. Conductance can be obtained by multiplying capacitance expression with $\sigma/\in$.

21. The intrinsal impedance of a lossy dielectric medium is given by
a) $\omega\mu j/\sigma$ b) $j\omega\varepsilon/\mu$ c) $\sqrt{j\omega\mu/(\sigma + j\omega\varepsilon)}$ d) $\sqrt{\mu/\varepsilon}$

22. An antenna, when radiating, has a highly directional radiation pattern. When the antenna is receiving, its radiation pattern
a) is more directive b) is less directive c) is same d) exhibits no directivity at all

Two Mark Questions

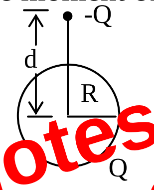
1. A composite parallel capacitor is made up of two different materials with different thickness (t_1 and t_2) as shown. The two different dielectric materials are separated by a conductivity foil F. The voltage of the conductivity foil is.
a) 52 V



b) 60V
c) 67 V
d) 33 V

2. A parallel plate capacitor has an electrode area of 100 mm^2, with a spacing of 0.1 mm between the electrodes. The dielectric between the plates is air with a permittivity of 8.85×10^{-12} F/m. The charge on the capacitor is 100V. The stored energy in the capacitor is
a) 8.85 PJ b) 440 PJ c) 22.1 nJ d) 44.3 nJ

3. A circular ring carrying a uniformly distributed charge Q and a point charges –Q on the axis of the ring are shown. The magnitude of the dipole moment of the charge system is
a) Qd
b) QR^2 / d
c) $Q\sqrt{R^2 + d^2}$
d) QR



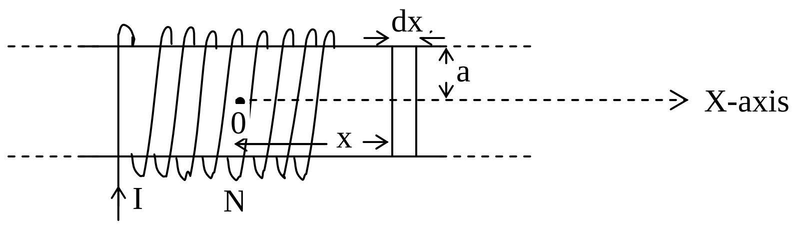
4. Find the polarization in a dielectric material with $\varepsilon_r = 2.8$ if $D = 3 \times 10^{-7}$ c/m^2.
a) 1.93×10^{-7} c/m^2 b) 10^{-19} c/m^2 c) 6.602×10^{-2} c/m^2 d) 0

5. Determine the value of electric field in a dielectric material for which χ is 3.5 and P is 2.3×10^{-7} c/m^2.
a) 7.9×10^{-2} b) 62.1×10^{-3} c) 74.3×10^{2} d) 83×10^{3}

6. Calculate the emerging angle by which the vector E changes its direction as it passes from a medium with $\varepsilon_r = 100$ into air making an angle of 45° with the interface as it enters
a) 90° b) 0.57° c) 0.89° d) 45°

7. Electric flux lines are incident in the porcelain insulator of $\in_r = 6$ at an angle of 45°. The electric field in the insulator is 1000V/m. Determine the electric field in the air and the angle at which flux lines are emerging out
a) 0.46°, 400 V/cm b) 2.25°, 4000 V/cm c) 7.2°, 4925 V/cm d) 9.46°, 4302 V/cm

Magnetic field due to an infinite circular solenoidal along its axis:



Let us consider an infinite circular solenoidal of radius ‘a’ with ‘n’ no. of turns per unit length (n=N/l) and carrying a current I. Let the axis of a solenoid coincides with x-axis and origin coincides with the point of observation. Consider an elemental thickness ‘dx’ at a distance ‘x’ from the origin.

Therefore, the elemental magnetic flux density due to this elemental section at point of observation ‘o’ is given by

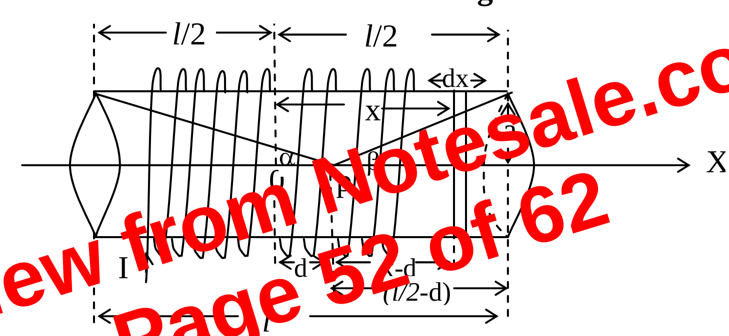
$$dB = \frac{\mu_0 (ndx) Ia^2}{2(a^2 + x^2)^{3/2}}$$

∴ Net magnetic field $\boxed{B = \mu_0 nI}$

The magnetic field due to an infinite circular solenoid is totally confined within the solenoid, uniform and axially directed and is equal to $B = \mu_0 nI$.

The direction of the magnetic field depends on the sense of current carrying by the solenoid and the right hand screw rule.

Magnetic field due to a finite circular solenoid along its axis:



Let us consider a finite circular solenoid of radius ‘a’ and length ‘l’. let ‘n’ be the no. of turns per unit length and ‘I’ be the carrying current. Assume that the solenoid axis coincides with the x-axis and the origin coincides with centre. Let ‘p’ be the point of observation at a distance ‘d’ from the centre.

$$\therefore \quad \boxed{\vec{B} = \frac{\mu_0 nI}{2}(\cos\beta + \cos\alpha)}$$

Corollary-1:

Magnetic field due to an infinite circular solenoid

i.e $\alpha = 0,\ \beta = 0$

$$\therefore \quad \boxed{B = \mu_0 nI}$$