Table 1.2

Power of 10	Prefix	Symbol
6	mega	M
3	kilo	k
– 2	centi	c
– 3	mili	m
– 6	micro	μ
– 9	nano	n
– 12	pico	p

Definitions of Some Important SI Units

(i) **Metre** 1 m = 1,650,763.73 wavelengths in vacuum, of radiation corresponding to orange red light of krypton-86.

(ii) **Second** 1s = 9,192,631,770 time periods of a particular radiation from cesium-133 atom.

(iii) **Kilogram** 1 kg = mass of 1 L volume of water at 4°C.

(iv) **Ampere** It is the current which when flows through two infinitely long straight conductors of negligible cross-section placed at a distance of 1 m in vacuum produces a force of 2×10^{-7} N/m between them.

(v) **Kelvin** 1 K = 1 / 273.16 part of the thermodynamic temperature of triple point of water.

(vi) **Mole** It is the amount of substance of a system which contains as many elementary particles (atoms, molecules, ions etc.) as there are atoms in 12 g of carbon-12.

(vii) **Candela** It is luminous intensity in a perpendicular direction of a surface of $\left(\dfrac{1}{600000}\right) m^2$ of a black body at the temperature of freezing platinum under a pressure of 1.013×10^5 N/m^2.

(viii) **Radian** It is the plane angle between two radii of a circle which cut-off on the circumference, an arc equal in length to the radius.

(ix) **Steradian** The steradian is the solid angle which having its vertex at the centre of the sphere, cut-off an area of the surface of sphere equal to that of a square with sides of length equal to the radius of the sphere.

Dimensions

Dimensions of a physical quantity are the powers to which the fundamental quantities must be raised to represent the given physical quantity.

For example, $\text{density} = \dfrac{\text{mass}}{\text{volume}} = \dfrac{\text{mass}}{(\text{length})^3}$

or $\text{density} = (\text{mass})\,(\text{length})^{-3}$ ...(i)

Thus, the dimensions of density are 1 in mass and –3 in length. The dimensions of all other fundamental quantities are zero.

For convenience, the fundamental quantities are represented by one letter symbols. Generally mass is denoted by *M*, length by *L*, time by *T* and electric current by *A*.

The thermodynamic temperature, the amount of substance and the luminous intensity are denoted by the symbols of their units K, mol and cd respectively. The physical quantity that is expressed in terms of the base quantities is enclosed in square brackets.

Thus, Eq. (i) can be written as

$$[\text{density}] = [ML^{-3}]$$

Such an expression for a physical quantity in terms of the fundamental quantities is called the dimensional formula.

Here, it is worthnoting that constants such as 5, π or trigonometrical functions such as sin θ, cos θ etc., have no units and dimensions.

$$[\sin\theta] = [\cos\theta] = [\tan\theta] = [\log x]$$
$$= [e^x] = [M^0L^0T^0]$$

Table 1.3 given below gives the dimensional formulae and SI units of some physical quantities frequently used in physics.

Table 1.3

S. No.	Physical Quantity	SI Units	Dimensional Formula
1.	Velocity = displacement/time	m/s	$[M^0LT^{-1}]$
2.	Acceleration = velocity/time	m/s^2	$[M^0LT^{-2}]$
3.	Force = mass × acceleration	kg-m/s^2 = newton or N	$[MLT^{-2}]$
4.	Work = force × displacement	kg-m^2/s^2 = N-m = joule or J	$[ML^2T^{-2}]$
5.	Energy	J	$[ML^2T^{-2}]$
6.	Torque = force × perpendicular distance	N-m	$[ML^2T^{-2}]$
7.	Power = work/time	J/s or watt	$[ML^2T^{-3}]$
8.	Momentum = mass × velocity	kg-m/s	$[MLT^{-1}]$
9.	Impulse = force × time	N-s	$[MLT^{-1}]$
10.	Angle = arc/radius	radian or rad	$[M^0L^0T^0]$
11.	Strain = $\dfrac{\Delta L}{L}$ or $\dfrac{\Delta V}{V}$	No units	$[M^0L^0T^0]$
12.	Stress = force/area	N/m^2	$[ML^{-1}T^{-2}]$
13.	Pressure = force/area	N/m^2	$[ML^{-1}T^{-2}]$

S. No.	Physical Quantity	SI Units	Dimensional Formula
14.	Modulus of elasticity = stress/strain	N/m^2	$[ML^{-1}T^{-2}]$
15.	Frequency = 1/time period	per sec or hertz (Hz)	$[M^0L^0T^1]$
16.	Angular velocity = angle/time	rad/s	$[M^0L^0T^{-1}]$
17.	Moment of inertia = (mass) × (distance)^2	$kg\text{-}m^2$	$[ML^2T^0]$
18.	Surface tension = force/length	N/m	$[ML^0T^{-2}]$
19.	Gravitational constant = $\dfrac{\text{force} \times (\text{distance})^2}{(\text{mass})^2}$	$N\text{-}m^2/kg^2$	$[M^{-1}L^3T^{-2}]$
20.	Angular momentum	$kg\text{-}m^2/s$	$[ML^2T^{-1}]$
21.	Coefficient of viscosity	$N\text{-}s/m^2$	$[ML^{-1}T^{-1}]$
22.	Planck's constant	J-s	$[ML^2T^{-1}]$
23.	Specific heat (*s*)	J/kg-K	$[M^0L^2T^{-2}\theta^{-1}]$
24.	Coefficient of thermal conductivity (*K*)	watt/m-K	$[MLT^{-3}\theta^{-1}]$
25.	Gas constant (*R*)	J/mol-K	$[ML^2T^{-2}\theta^{-1}mol^{-1}]$
26.	Boltzmann constant (*k*)	J/K	$[ML^2T^{-2}\theta^{-1}]$
27.	Wien's constant (*b*)	m-K	$[L\theta]$
28.	Stefan's constant (σ)	$watt/m^2\text{-}K^4$	$[MLT^{-3}\theta^{-4}]$
29.	Electric charge	C	[AT]
30.	Electric intensity	N/C	$[MLT^{-3}A^{-1}]$
31.	Electric potential	volt (V)	$[ML^2T^{-3}A^{-1}]$
32.	Capacitance	farad (F)	$[M^{-1}L^{-2}T^4A^2]$
33.	Permittivity of free space	$C^2N^{-1}m^{-2}$	$[M^{-1}L^{-3}T^4A^2]$
34.	Electric dipole moment	C-m	[LTA]
35.	Resistance	Ohm	$[ML^2T^{-3}A^{-2}]$
36.	Magnetic field	tesla (T) or weber/m^2 (Wb/m^2)	$[MT^{-2}A^{-1}]$
37.	Coefficient of self-induction	henry (H)	$[ML^2T^{-2}A^{-2}]$

Key-Terms for Concepts

- Astronomical unit
  1 AU = mean distance of earth from sun
  $\approx 1.5 \times 10^{11}$ m
- Light year
  1 ly = distance travelled by light in vacuum in 1 year
  $= 9.46 \times 10^{15}$ m
- Parsec
  1 parsec $= 3.07 \times 10^{16}$ m = 3.26 light year
- X-ray unit
  $1\,U = 10^{-13}$ m
- 1 shake $= 10^{-8}$ s
- 1 bar $= 10^5\ N/m^2 = 10^5$ pascal
- 1 torr = 1 mm of Hg = 133.3 Pa
- 1 barn $= 10^{-28}\ m^2$
- 1 horse power = 746 W
- 1 pound = 453.6 g = 0.4536 kg

Example 1.1 *Find the dimensional formulae of*
(a) coefficient of viscosity η
(b) charge q
(c) potential V
(d) capacitance C, and
(e) resistance R.
Some of the equations containing these quantities are

$$F = -\eta A\left(\frac{\Delta v}{\Delta l}\right), \quad q = It, \quad U = VIt,$$

$$q = CV \quad \text{and} \quad V = IR$$

where A denotes the area, v the velocity, l is the length, I the electric current, t the time and U the energy.

Solution (a) $\eta = -\dfrac{F}{A}\dfrac{\Delta l}{\Delta v}$

$$\therefore \quad [\eta] = \frac{[F][l]}{[A][v]} = \frac{[MLT^{-2}][L]}{[L^2][LT^{-1}]}$$

$$= [ML^{-1}T^{-1}]$$

(b) $q = It$

$$\therefore \quad [q] = [I][t] = [AT]$$

Example 1.9 *Calculate percentage error in determination of time period of a pendulum*

$$T = 2\pi\sqrt{\frac{l}{g}}$$

where l and g are measured with ± 1% and ± 2% errors.

Solution $\frac{\Delta T}{T} \times 100 = \pm\left(\frac{1}{2} \times \frac{\Delta l}{l} \times 100 + \frac{1}{2} \times \frac{\Delta g}{g} \times 100\right)$

$$= \pm\left(\frac{1}{2} \times 1 + \frac{1}{2} \times 2\right) = \pm 1.5\%$$

Least Count

The minimum measurement that can be measured accurately by an instrument is called the least count. The least count of a metre scale graduated in millimetre mark is 1 mm. The least count of a watch having seconds hand is 1 s.

Key-Terms for Concepts

- Least count of vernier callipers

$$= \begin{Bmatrix}\text{Value of 1 part of} \\ \text{main scale } (s)\end{Bmatrix} - \begin{Bmatrix}\text{Value of one part on} \\ \text{vernier scale } (v)\end{Bmatrix}$$

Least count of vernier callipers = 1MSD − 1VSD

where, MSD = Main Scale Division

VSD = Vernier Scale Division

- Least count $= \frac{\text{Value of 1 part on main scale } (s)}{\text{Number of parts on vernier scale } (n)}$
- Least count of screw guage

$$= \frac{\text{Pitch } (p)}{\text{Number of parts on circular scale } (n)}$$

Solved Examples

Example 1. *Check the correctness of the relation* $s = ut + \frac{1}{2}at^2$ *where u is initial velocity, a the acceleration, t the time and s the displacement.*

Solution Writing the dimensions of either side of the given equation.

LHS = s = displacement = $[M^0LT^0]$

RHS = ut = velocity × time = $[M^0LT^{-1}][T] = [M^0LT^0]$

and $\frac{1}{2}at^2$ = (acceleration) × (time)^2

$$= [M^0LT^{-2}][T]^2 = [M^0LT^0]$$

As LHS = RHS, formula is dimensionally correct.

Example 2. *Write the dimensions of a and b in the relation,*

$$P = \frac{b - x^2}{at}$$

where P is power, x the distance and t the time.

Solution The given equation can be written as,

$$Pat = b - x^2$$

Now, $[Pat] = [b] = [x^2]$

or $[b] = [x^2] = [M^0L^2T^0]$

and $[a] = \frac{[x^2]}{[Pt]} = \frac{[L^2]}{[ML^2T^{-3}][T]}$

$$= [M^{-1}L^0T^2]$$

Example 3. *The centripetal force F acting on a particle moving uniformly in a circle may depend upon mass (m), velocity (v) and radius (r) of the circle. Derive the formula for F using the method of dimensions.*

Solution Let $F = k(m)^x(v)^y(r)^z$ ...(i)

Here, k is a dimensionless constant of proportionality. Writing the dimensions of RHS and LHS in Eq. (i), we have

$$[MLT^{-2}] = [M]^x[LT^{-1}]^y[L]^z$$
$$= [M^xL^{y+z}T^{-y}]$$

Equating the powers of M, L and T of both sides, we have

$$x = 1,\ y = 2 \text{ and } y + z = 1$$

or $z = 1 - y = -1$

Putting the values in Eq. (i), we get

$$F = kmv^2r^{-1} = k\frac{mv^2}{r}$$

$$F = \frac{mv^2}{r} \quad \text{(where } k = 1)$$

Example 4. *Write down the number of significant figures in the following :*

(a) 6428 *(b)* 62.00 *m*

(c) 0.00628 *cm* *(d)* 1200 *N*

Solution (a) 6428 has four significant figures.

(b) 62.00 m has four significant figures.

(c) 0.00628 cm has three significant figures.

(d) 1200 N has four significant figures.

Example 5. Round off to four significant figures :

(a) 45.689 *(b)* 2.0082

Solution (a) 45.69

(b) 2.008

Example 6. *Add* 6.75×10^3 *cm to* 4.52×10^2 *cm with regard to significant figures.*

Solutions

Objective Problems (Level 1)

1. Leap year, year and shake are the units of time.

3. 1 light year $= (3 \times 10^5)\,(365)\,(24)\,(3600)$

$$= 9.416 \times 10^{12} \text{ km}$$

8. Impulse = change in linear momentum.

13. Solid angle, strain and dielectric constant are dimensionless constant.

14. Since $(mvr) = n \cdot \dfrac{h}{2\pi}$

and $E = h\nu$

So, unit of h = joule second = angular momentum

17. Wb/m^2 and tesla are the units of magnetic field.

21. Impulse = Force × time

24. Young's modulus and pressure have the same dimensions.

26. Action is a force.

28. Relative density $= \dfrac{\text{Density of substance}}{\text{Density of water at } 4°\text{C temperature}}$

= Dimensionless

36. $m \propto v^a \rho^b g^c$. Writing the dimensions on both sides

$$[\text{M}] = [\text{LT}^{-1}]^a\,[\text{ML}^{-2}]^b\,[\text{LT}^{-2}]^c$$

$$[\text{M}] = [\text{M}^b\text{L}^{a-3b+c}\text{T}^{-a-2c}]$$

$\therefore$ $b = 1$

$a - 3b + c = 0$

$-a - 2c = 0$

Solving these we get

$a = 6$

Hence, $m \propto v^6$

37. Since $p^x Q^y c^z$ is dimensionless. Therefore,

$$[\text{ML}^{-1}\text{T}^{-2}]^x\,[\text{MT}^{-3}]^y\,[\text{LT}^{-1}]^z = [\text{M}^0\text{L}^0\text{T}^0]$$

Only option (b) satisfies this expression

So $x = 1,\ y = -1,\ z = 1$

38. Since units of length, velocity and force and doubled

Hence, $[\text{m}] = \dfrac{[\text{force}]\,[\text{time}]}{[\text{velocity}]}$, $[\text{time}] = \dfrac{[\text{length}]}{[\text{velocity}]}$

Hence unit of mass, and time remains same.

Momentum is doubled.

40. Since, $R = \dfrac{\rho l}{A}$, where ρ is specific resistance.

$\therefore$ $[\rho] = \left[\dfrac{RA}{l}\right]$, $R = \dfrac{V}{i}$, $V = \dfrac{W}{Q}$

$$[\rho] = [\text{ML}^3\text{T}^{-1}\text{Q}^{-2}]$$

41. $i = i_0\,\{1 - e^{-t/(L/R)}\}$

Where $\dfrac{L}{R}$ is time constant and its dimension is same as for time.

42. CR is time constant.

44. ωk is dimensionless.

45. $[a] = \left[\dfrac{F}{t}\right]$ and $[b] = \left[\dfrac{F}{t^2}\right]$

47. $\dfrac{1}{2}\varepsilon_0 E^2$ is energy density or energy per unit volume.

48. $p = \dfrac{a - t^2}{bx}$, where p-pressure, t-time

$$[pbx] = [a] = [t^2]$$

Hence, $[b] = \dfrac{[t^2]}{[px]}$

Dimensions of $\dfrac{a}{b} = [px] = [\text{MT}^{-2}]$

49. Velocity gradient is change in velocity per unit length.

50. Unit of emf e is volt.

51. $[a] = \left[\dfrac{v}{t}\right] : [b] = [vt] : [c] = [t]$

54. $F = \dfrac{1}{4\pi\varepsilon_0} \times \dfrac{q_1 q_2}{r^2}$

$\Rightarrow$ $\varepsilon_0 = \dfrac{1}{4\pi} \times \dfrac{q_1 q_2}{Fr^2}$

$\Rightarrow$ $\varepsilon_0 = \dfrac{(\text{coulomb})^2}{\text{newton-metre}^2}$

55. From definition of time constant $t = RC$, where R is resistance and C is capacitance.

$\therefore$ $R = \dfrac{t}{C}$

$$= \frac{[\text{T}]}{[\text{M}^{-1}\text{L}^{-2}\text{T}^4\text{A}^2]}$$

$$R = [\text{ML}^2\text{T}^{-3}\text{A}^{-2}]$$

56. $M = NIA$

57. Since, $R = \dfrac{\rho l}{A}$, where ρ is specific resistance

$[\rho] = \left[\dfrac{RA}{l}\right]$, $R = \dfrac{V}{i}$, $V = \dfrac{W}{Q}$

$$[\rho] = [\text{ML}^3\text{T}^{-1}\text{Q}^{-2}]$$

68. $R = 0.16$ mm

Hence, $A = \pi R^2$

$= \dfrac{22}{7} \times (0.16)^2$

$= 0.080384$

Since radius has two significant figure so answer also will have two significant figures.

$\therefore$ $A = 0.080$

73. Minimum number of significant figure should be 1.

75. Radius of ball = 5.2 cm

$$V = \frac{4}{3}\pi R^3$$