MODULE - IV
Functions

Notes

Solution :

(a) It is a function.

Domain= $\{1,3,4,8\}$, Range $=\{-2,7,-6,1\}$

(b) It is not a function. Because Ist two ordered pairs have same first elements.

(c) It is not a function.

Domain= $\{a,b,c,d\} \neq A$, Range $=\{b,c\}$

(d) It is a function.

Domain $=\{2,3,4,5,6\}$, Range $=\{4,9,16,25,36\}$

(e) It is not a function .

Domain= $\{1,2,3,4,5\} \neq A$, Range $=\{-1,-2,-3,-4,-5\}$

(f) It is a function .

$$\text{Domain}=\left\{\sin\frac{\pi}{6},\cos\frac{\pi}{6},\tan\frac{\pi}{6},\cot\frac{\pi}{6}\right\}, \qquad \text{Range}=\left\{\frac{1}{2},\frac{\sqrt{3}}{2},\frac{1}{\sqrt{3}},\sqrt{3}\right\}$$

(g) It is not a function.

First two ordered pairs have same first component and last two ordered pairs have also same first component.

Example 15.2 State whether each of the following relations represent a function or not.

(a)

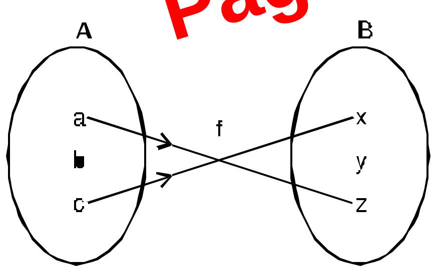


Fig.15.14

(b)

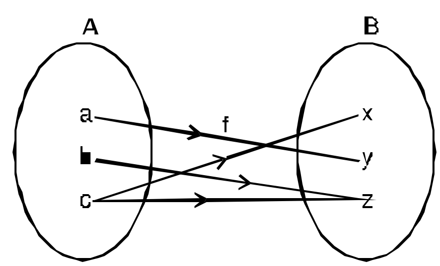


Fig.15.15

(c)

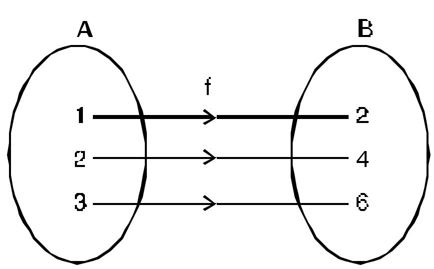


Fig. 15.16

(d)

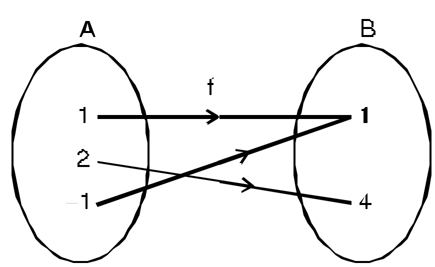
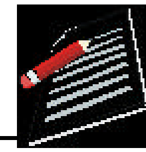


Fig.15.17



(c)

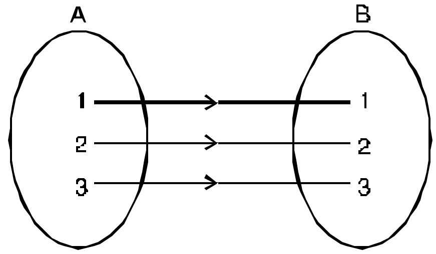


Fig. 15.24

(d)

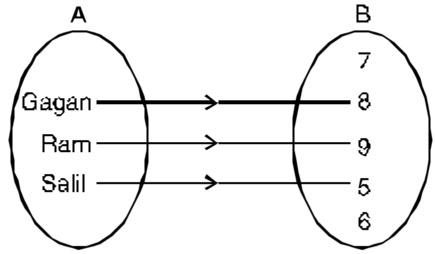


Fig. 15.25

(e)

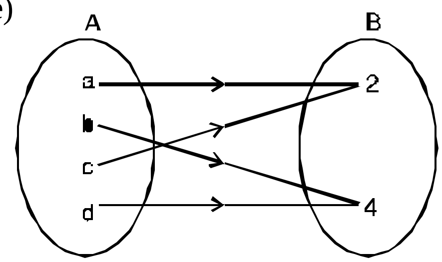


Fig. 15.26

15.14.1 Some More Examples on Domain and Range

Let us consider some functions which are only defined for a certain subset of the set of real numbers.

Example 15.23 Find the domain of each of the following functions :

(a) $y = \frac{1}{x}$ (b) $y = \frac{1}{x-2}$ (c) $y = \frac{1}{(x+2)(x-3)}$

Solution : The function $y = \frac{1}{x}$ can be described by the following set of ordered pairs.

$$\left\{ \text{.......,} \left(-2, -\frac{1}{2}\right), (-1,-1), (1,1) \left(2, \frac{1}{2}\right), \text{....} \right\}$$

Here we can see that x can take all real values except 0 because the corresponding image, i.e., $\frac{1}{0}$ is not defined.

$\therefore$ Domain $= R - \{0\}$ [Set of all real numbers except 0]

Note : Here range = R - { 0 }

(b) x can take all real values except 2 because the corresponding image, i.e., $\frac{1}{(2-2)}$ does not exist.

$\therefore$ Domain $= R - \{2\}$

(c) Value of y does not exist for $x = -2$ and $x = 3$

$\therefore$ Domain $= R - \{-2, 3\}$

(b) (i) $f(x) = x - 2,\ 0 \le x \le 4$ (ii) $f(x) = 3x + 4,\ -1 \le x \le 2$

(c) (i) $f(x) = x^2,\ -5 \le x \le 5$ (ii) $f(x) = 2x,\ -3 \le x \le 3$

(iii) $f(x) = x^2 + 1,\ -2 \le x \le 2$ (iv) $f(x) = \sqrt{x},\ 0 \le x \le 25$

(d) (i) $f(x) = x + 5,\ x \in R$ (ii) $f(x) = 2x - 3,\ x \in R$

(iii) $f(x) = x^3,\ x \in R$ (iv) $f(x) = \dfrac{1}{x},\ \{x : x < 0\}$

(v) $f(x) = \dfrac{1}{x-2},\ \{x : x \le 1\}$ (vi) $f(x) = \dfrac{1}{3x-2},\ \{x : x \le 0\}$

(vii) $f(x) = \dfrac{2}{x},\ \{x : x > 0\}$ (viii) $f(x) = \dfrac{x}{x+5},\ \{x : x \ne -5\}$

15.15 CLASSIFICATION OF FUNCTIONS

Let f be a function from A to B. If every element of the set B is the image of at least one element of the set A i.e. if there is no unpaired element in the set B then we say that the function f maps the set A onto the set B. Otherwise we say that the function maps the set A into the set B.

Functions for which each element of the set A is mapped to a different element of the set B are said to be ***one-to-one***.

One-to-one function

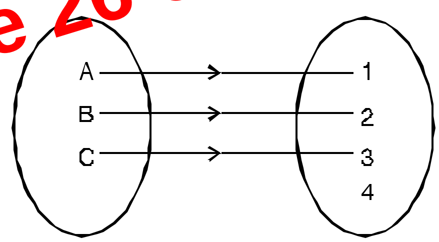


Fig.15.27

The domain is $\{A, B, C\}$

The co-domain is $\{1, 2, 3, 4\}$

The range is $\{1, 2, 3\}$

A function can map more than one element of the set A to the same element of the set B. Such a type of function is said to be ***many-to-one.***

Many-to-one function

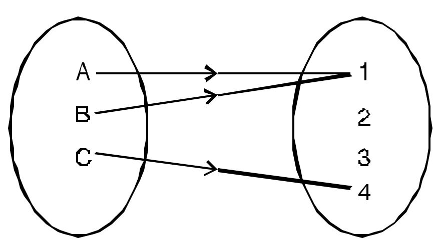


Fig. 15.28

($\because$ It is only increasing function on this interval)

But $\quad \forall\ x_1, x_2 \in (-\infty, 0)$

$$x_1 < x_2 \quad \Rightarrow \quad F(x_1) > F(x_2)$$

$\Rightarrow \quad$ F is a ***Monotonic Function*** on $[-\infty, 0]$

($\because$ It is only a decreasing function on this interval)

Therefore if we talk of the whole domain given function is not monotonic on R but it is monotonic on $(-\infty, 0)$ and $(0, \infty)$.

Again consider the function $F : R \rightarrow R$ defined by $f(x) = x^3$.

Clearly $\forall\ x_1\ x_2 \in$ domain

$$x_1 < x_2 \quad \Rightarrow F(x_1) < F(x_2)$$

$\therefore$ Given function is ***monotonic*** on R i.e. on the whole domain.

15.17.2 Even Function

A function is said to be an even function if for each x of domain

$$F(-x) = F(x)$$

For example, each of the following is an ***even function***.

(i) If $F(x) = x^2$ then $F(-x) = (-x)^2 = x^2 = F(x)$

(ii) If $F(x) = \cos x$ then $F(-x) = \cos(-x) = \cos x = F(x)$

(iii) If $F(x) = |x|$ then $F(-x) = |-x| = |x| = F(x)$

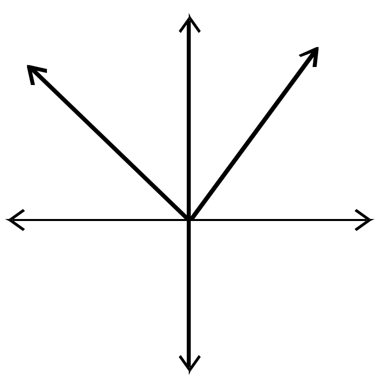


Fig. 15.45

The graph of this even function (modulus function) is shown in the figure above.

Observation

Graph is symmetrical about y-axis.

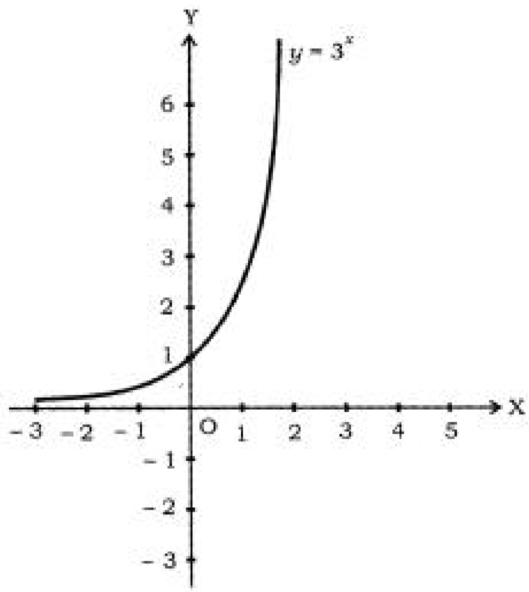


Fig. 15.50

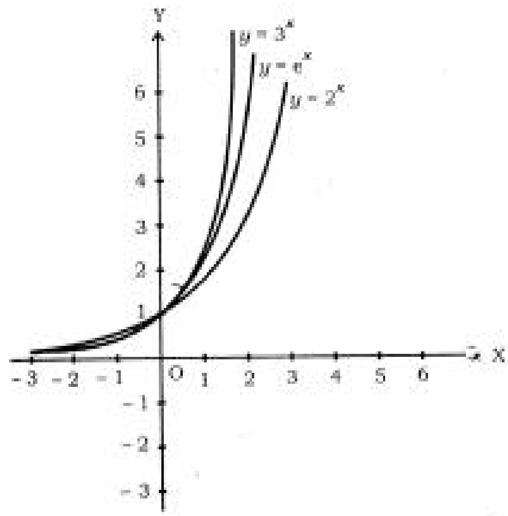


Fig. 15.51

15.17.9 Logarithmic Functions

Consider now the function

$$y = e^x \qquad .....(3)$$

We write it equivalently as

$$x = \log_e y$$

Thus, $y = \log_e x$ .....(4)

is the inverse function of $y = e^x$

The base of the logarithm is not written if it is e and so $\log_e x$ is usually written as log x.

As $y = e^x$ and $y = \log x$ are inverse functions, their graphs are also symmetric w.r.t. the line $y = x$

The graph of the function $y = \log x$ can be obtained from that of $y = e^x$ by reflecting it in the line y = x.

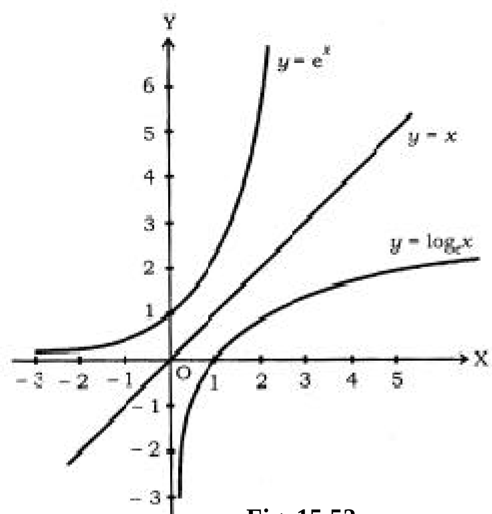


Fig. 15.52

Note

(i) The learner may recall the laws of indices which you have already studied in the Secondary Mathematics :

If a > 0, and m and n are any rational numbers, then

$$a^m \cdot a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$a^0 = 1$$

(ii) The corresponding laws of logarithms are

$$\log_a(mn) = \log_a m + \log_a n$$

$$\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$$

$$\log_a(m^n) = n \log_a m$$

$$\log_b m = \frac{\log_a m}{\log_a b}$$

or $$\log_b m = \log_a m \log_b a$$

Here $a, b > 0,\ a \neq 1,\ b \neq 1$.

CHECK YOUR PROGRESS 15.8

1. Tick mark the correct statement.

   (i) Function $f(x) = 2x^4 + 7x^2 + 9x$ is an even function.

   (ii) Odd function is symmetrical about y-axis.

   (iii) $f(x) = x^{1/2} - x^3 + x^5$ is a polynomial function.

   (iv) $f(x) = \dfrac{x-3}{3+x}$ is a rational function for all $x \in R$.

   (v) $f(x) = \dfrac{\sqrt{5}}{3}$ is a constant function.

   (vi) $f(x) = \dfrac{1}{x}$

   Domain of the function is the set of real numbers except 0.

   (vii) Greatest integer function is neither even nor odd.

2. Which of the following functions are even or odd functions ?

   (a) $f(x) = \dfrac{x^2-1}{x+1}$ (b) $f(x) = \dfrac{x^2}{5+x^2}$ (c) $f(x) = \dfrac{1}{x^2+5}$

   (d) $f(x) = \dfrac{2}{x^3}$ (e) $f(x) = \dfrac{x}{x^2+1}$ (f) $f(x) = \dfrac{5}{x-5}$

MODULE - IV
Functions

Notes

15.19 INVERSE OF A FUNCTION

(A) Consider the relation

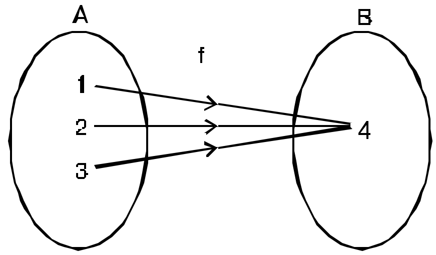


Fig. 15.54

This is a many-to-one function. Now let us find the inverse of this relation. Pictorially, it can be represented as

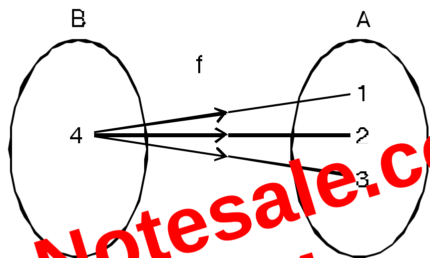


Fig. 15.55

Clearly this relation does not represent a function. (Why ?)

(B) Now take another relation

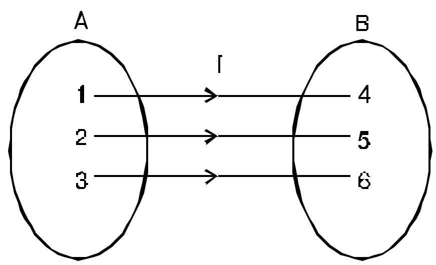


Fig.15.56

It represents one-to-one onto function. Now let us find the inverse of this relation, which is represented pictorially as

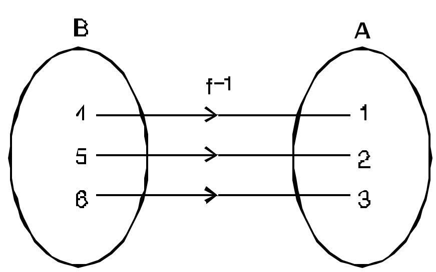


Fig. 15.57