

Example 3**Self Tutor**

Simplify, giving answers in simplest rational form:

a 7^0

b 3^{-2}

c $3^0 - 3^{-1}$

d $\left(\frac{5}{3}\right)^{-2}$

a $7^0 = 1$

b $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

c $3^0 - 3^{-1} = 1 - \frac{1}{3} = \frac{2}{3}$

d $\left(\frac{5}{3}\right)^{-2} = \left(\frac{3}{5}\right)^2 = \frac{9}{25}$

Notice that

$$\left(\frac{a}{b}\right)^{-2} = \left(\frac{b}{a}\right)^2$$



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Questions A

Saturday, 12 August 2017 3:27 PM

1 Which of the following numbers are *not* written in scientific notation?

a 3.7×10^4

b 4.2×10^{-7}

c 0.3×10^5

d 21×10^{11}

2 Copy and complete:

$1000 = 10^3$

$100 = 10^2$

$10 =$

$1 =$

$0.1 =$

$0.01 = 10^{-2}$

$0.001 =$

$0.0001 =$

3 Write as decimal numbers:

a 8.2×10^4

b 3.6×10^1

c 8.7×10^0

d 4.9×10^2

e 7.8×10^{-3}

f 5.5×10^{-2}

g 3.78×10^{-1}

h 2.02×10^{-3}

4 Write in scientific notation:

a 3900

b 17000

c 0.04

d 0.000 071

e 85

f 3.2

g 2 480 000

h 0.000 000 108

5 Write these calculator displays in scientific notation:

a $4.5E07$

b $3.8E-04$

c $2.1E05$

d $4.0E-03$

e $6.1E03$

f $1.6E-06$

g $3.9E04$

h $6.7E-02$



6 Write the numbers displayed in question 5 as decimal numbers.

For example, $3.9E06$ is 3.9×10^6
 $= 3.900\ 000 \times 1\ 000\ 000$
 $= 3\ 900\ 000$

In an exam it is **not** acceptable to write your answer as a calculator display.

7 Use your calculator to evaluate the following, giving your answer in scientific notation:

a $680\ 000 \times 73\ 000\ 000$

b 900^4

c $0.0006 \div 15\ 000$

d $(0.0007)^3$

e $5.3 \times 10^8 \times 6.4 \times 10^5$

f $2.6 \times 10^4 \times 3.7 \times 10^{-9}$

g $\frac{3.6 \times 10^4}{7.5 \times 10^{11}}$

h $\frac{4.9 \times 10^{-5}}{1.12 \times 10^6}$

Questions B

Saturday, 12 August 2017 3:24 PM

1 Solve for x :

a $3^x = 3$

e $3^x = \frac{1}{3}$

i $2^{x+2} = \frac{1}{4}$

m $4^{2x+1} = \frac{1}{2}$

b $3^x = 9$

f $5^x = \frac{1}{5}$

j $3^{x-1} = \frac{1}{27}$

n $9^{x-3} = 3$

c $2^x = 8$

g $2^x = \frac{1}{16}$

k $2^{x-1} = 32$

o $(\frac{1}{2})^{x-1} = 2$

d $5^x = 1$

h $5^{x+2} = 25$

l $3^{1-2x} = \frac{1}{27}$

p $(\frac{1}{3})^{2-x} = 9$

2 Solve for x :

a $5 \times 2^x = 40$

d $4 \times 5^x = 500$

g $2^{2-5x} = 4^x$

i $2^x \times 4^{2-x} = 8$

b $6 \times 2^{x+2} = 24$

e $8 \times (\frac{1}{2})^x = 1$

h $5^{x-1} = (\frac{1}{25})^x$

k $3^{x+1} \times 9^{-x} = (\frac{1}{3})^{x+1}$

c $3 \times (\frac{1}{2})^x = 12$

f $7 \times (\frac{1}{3})^x = 63$

i $9^{x-2} = (\frac{1}{3})^{3x-1}$

l $2^{x^2-2x} = 8$

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$2^{x^2-2x} = 8$
 $2^{x^2-2x} = 2^3$
 $x^2-2x = 3$
 $x^2-2x-3 = 0$
 $(x+1)(x-3) = 0$
 $x = -1$ $x = 3$

- 1 a $x = 1$ b $x = 2$ c $x = 3$ d $x = 0$
 e $x = -1$ f $x = -1$ g $x = -4$ h $x = 0$
 i $x = -4$ j $x = -2$ k $x = 6$ l $x = 2$
 m $x = -\frac{3}{4}$ n $x = \frac{7}{2}$ o $x = 0$ p $x = 4$
- 2 a $x = 3$ b $x = 0$ c $x = -2$ d $x = 3$
 e $x = 3$ f $x = -2$ g $x = \frac{2}{7}$ h $x = \frac{1}{3}$
 i $x = 1$ j $x = 1$ k no solution l $x = 3$ or -1

$$\textcircled{8} \quad (x+1)(x+2)(x+3)$$

$$(x+1)(x^2+5x+6)$$

$$x^3 + 5x^2 + 6x + x^2 + 5x + 6$$

$$x^3 + 6x^2 + 11x + 6$$

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Further Expansion

Sunday, 13 August 2017 1:14 PM

In this section we expand more complicated expressions by repeated use of the expansion laws.

Consider the expansion of $(a + b)(c + d + e)$.

$$\begin{aligned} \text{Now } (a + b)(c + d + e) \\ = (a + b)c + (a + b)d + (a + b)e \\ = ac + bc + ad + bd + ae + be \end{aligned}$$

$$\begin{aligned} \text{Compare: } \square(c + d + e) \\ = \square c + \square d + \square e \end{aligned}$$

Notice that there are 6 terms in this expansion and that each term within the first bracket is multiplied by each term in the second.

2 terms in first bracket \times 3 terms in second bracket \rightarrow 6 terms in expansion.

| | |
|--|-------------------|
| Example 8 | Self Tutor |
| Expand and simplify: $(x + 3)(x^2 + 2x + 4)$ | |
| $\begin{aligned} (x + 3)(x^2 + 2x + 4) \\ = x^3 + 2x^2 + 4x + 3x^2 + 6x + 12 \\ = x^3 + 5x^2 + 10x + 12 \end{aligned}$ <div style="display: flex; justify-content: space-between; margin-top: 5px;"> {all terms of 2nd bracket $\times x$} {all terms of 2nd bracket $\times 3$} </div> <div style="margin-top: 5px;"> {collecting like terms} </div> | |

| | | | | | |
|---|---|---|--|--|---|
| Example 9 | Self Tutor | | | | |
| Expand and simplify: | | | | | |
| a $x(x + 1)(x + 3)$ | b $(x + 1)(x - 3)(x + 2)$ | | | | |
| <table style="width: 100%; border: none;"> <tr> <td style="width: 50%; padding: 5px;"> $\begin{aligned} \text{a } x(x + 1)(x + 3) \\ = (x^2 + x)(x + 3) \\ = x^3 + 3x^2 + x^2 + 3x \\ = x^3 + 4x^2 + 3x \end{aligned}$ </td> <td style="width: 50%; padding: 5px; vertical-align: top;"> <div style="margin-top: 5px;">{all terms in first bracket $\times x$}</div> <div style="margin-top: 5px;">{expand remaining factors}</div> <div style="margin-top: 5px;">{collect like terms}</div> </td> </tr> <tr> <td style="padding: 5px;"> $\begin{aligned} \text{b } (x + 1)(x - 3)(x + 2) \\ = (x^2 - 3x + x - 3)(x + 2) \\ = (x^2 - 2x - 3)(x + 2) \\ = x^3 - 2x^2 - 3x + 2x^2 - 4x - 6 \\ = x^3 - 7x - 6 \end{aligned}$ </td> <td style="padding: 5px; vertical-align: top;"> <div style="margin-top: 5px;">{expand first two factors}</div> <div style="margin-top: 5px;">{collect like terms}</div> <div style="margin-top: 5px;">{expand remaining factors}</div> <div style="margin-top: 5px;">{collect like terms}</div> </td> </tr> </table> | | $\begin{aligned} \text{a } x(x + 1)(x + 3) \\ = (x^2 + x)(x + 3) \\ = x^3 + 3x^2 + x^2 + 3x \\ = x^3 + 4x^2 + 3x \end{aligned}$ | <div style="margin-top: 5px;">{all terms in first bracket $\times x$}</div> <div style="margin-top: 5px;">{expand remaining factors}</div> <div style="margin-top: 5px;">{collect like terms}</div> | $\begin{aligned} \text{b } (x + 1)(x - 3)(x + 2) \\ = (x^2 - 3x + x - 3)(x + 2) \\ = (x^2 - 2x - 3)(x + 2) \\ = x^3 - 2x^2 - 3x + 2x^2 - 4x - 6 \\ = x^3 - 7x - 6 \end{aligned}$ | <div style="margin-top: 5px;">{expand first two factors}</div> <div style="margin-top: 5px;">{collect like terms}</div> <div style="margin-top: 5px;">{expand remaining factors}</div> <div style="margin-top: 5px;">{collect like terms}</div> |
| $\begin{aligned} \text{a } x(x + 1)(x + 3) \\ = (x^2 + x)(x + 3) \\ = x^3 + 3x^2 + x^2 + 3x \\ = x^3 + 4x^2 + 3x \end{aligned}$ | <div style="margin-top: 5px;">{all terms in first bracket $\times x$}</div> <div style="margin-top: 5px;">{expand remaining factors}</div> <div style="margin-top: 5px;">{collect like terms}</div> | | | | |
| $\begin{aligned} \text{b } (x + 1)(x - 3)(x + 2) \\ = (x^2 - 3x + x - 3)(x + 2) \\ = (x^2 - 2x - 3)(x + 2) \\ = x^3 - 2x^2 - 3x + 2x^2 - 4x - 6 \\ = x^3 - 7x - 6 \end{aligned}$ | <div style="margin-top: 5px;">{expand first two factors}</div> <div style="margin-top: 5px;">{collect like terms}</div> <div style="margin-top: 5px;">{expand remaining factors}</div> <div style="margin-top: 5px;">{collect like terms}</div> | | | | |

$$\textcircled{6} \quad \left(x - \frac{1}{2}\right)^4$$

$$\textcircled{7} \quad \left(\frac{x}{2} - \frac{1}{3}\right)^5$$

$$\textcircled{8} \quad (a+b)^n$$

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Extension

Friday, 18 August 2017 6:30 AM

- 5.** Find the coefficient of x^{-3} in the expansion of $(x-1)^3\left(\frac{1}{x}+x\right)^6$.
- 6.** Find the constant term in the expansion of $\left(x-\frac{1}{2x}\right)^{10}$.
- 7.** Find the constant term in the expansion of $\left(3x-\frac{1}{6x}\right)^{12}$.
- 8.** Find the term independent of x in the expansion of $(2-x)^3\left(\frac{1}{3x}-x\right)^6$.
- 9.** Find the term independent of x in the expansion of $\left(2x-\frac{1}{3}\right)^6\left(\frac{1}{2x}+x\right)^6$.

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5. 19 **6.** $-\frac{63}{8}$ **7.** $\frac{231}{16}$ **8.** $-\frac{130}{27}$ **9.** -20

$$\textcircled{4} \quad x^2 - 7x + 12$$
$$(x - 3)(x - 4)$$

$$\textcircled{5} \quad 3x^2 + 14x - 5$$

$$(3x - 1)(x + 5)$$

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$$\textcircled{6} \quad 2x^2 - 5x - 12$$
$$(2x + 3)(x - 4)$$

check

$$\begin{array}{r} 2x^2 - 8x + 3x - 12 \\ 2x^2 - 5x - 12 \end{array} \quad \checkmark$$

13. Expand and fully simplify using the Binomial Expansion:

$$\left(\frac{x^2}{2}\right)^5 + 5 \cdot \left(\frac{x^2}{2}\right)^4 \cdot \left(-\frac{1}{2}\right) + 10 \cdot \left(\frac{x^2}{2}\right)^3 \cdot \left(-\frac{1}{2}\right)^2 + 10 \cdot \left(\frac{x^2}{2}\right)^2 \cdot \left(-\frac{1}{2}\right)^3 + 5 \cdot \left(\frac{x^2}{2}\right) \cdot \left(-\frac{1}{2}\right)^4 + \left(-\frac{1}{2}\right)^5$$

$$= \frac{x^{10}}{32} - \frac{5x^8}{16x} + \frac{10x^6}{8x^2} - \frac{10x^4}{4x^3} + \frac{5x^2}{2x^4} - \frac{1}{x^5}$$

$$= \frac{x^{10}}{32} - \frac{5x^7}{16} + \frac{5}{4}x^4 - \frac{5}{2}x + \frac{5}{2x^2} - \frac{1}{x^5}$$

14. Simplify $\frac{n!}{(n-2)!}$

$n(n-1)$

15. Simplify the following:

$$\frac{x^2 - 5x + 6}{2x^2 - 5x + 3} = \frac{(x-3)(x-2)}{(2x+1)(x-3)}$$

$$= \frac{x-2}{2x+1}$$

error corrected during test (extra time given)

16. Find the constant term in the expansion of the following. Simplify your answer as far as possible.

* $(3x^2 + \frac{1}{x})^8$

$$\sum_r \binom{8}{r} (3x^2)^{8-r} \cdot \left(\frac{1}{x}\right)^r$$

16-2r-2r = 16-4r

r=4

$$\binom{8}{4} \cdot 3^4 = 70 \cdot 81 = 5670$$

17. Fully factorise the following:

$$25^x + 2 \times 5^x - 3$$

$$(5^x - 1)(5^x + 3)$$

19. Solve the following for x:

$$(x^2 + x - 57)^{3x+3} = (x^2 + x - 57)^{10x}$$

$$x^2 + x - 57 = 1$$

$$x^2 + x - 58 = 0$$

$$x = \frac{-1 \pm \sqrt{1 - 4 \cdot 1 \cdot -58}}{2}$$

$$= \frac{-1 \pm \sqrt{233}}{2}$$

$$3x^2 - 10x + 3 = 0$$

$$(3x-1)(x-3) = 0$$

$$x = \frac{1}{3} \quad x = 3$$

also try

$$x^2 + x - 57 = -1$$

$$x^2 + x - 56 = 0$$

$$x = \frac{-1 \pm \sqrt{1 - 4 \cdot 1 \cdot -56}}{2} = \frac{-1 \pm \sqrt{225}}{2}$$

20. Find the coefficient of x^6 in the expansion of $(2-x)(3x+1)^9$. Simplify your answer.

$$\binom{9}{r} (3x)^{9-r} \cdot 1^r$$

9-r=5
r=4

$$\binom{9}{4} \cdot 3^5 \cdot x^{-1}$$

also

9-r=6
r=3

$$\binom{9}{3} \cdot 3^6 \cdot x^2$$

$$= 91854$$

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$$x^2 \times x^3 = x^5$$

Example 11**Self Tutor**

Expand and simplify:

a $(\sqrt{3} + 2)^2$

b $(\sqrt{3} - \sqrt{7})^2$

a
$$\begin{aligned} & (\sqrt{3} + 2)^2 \\ &= (\sqrt{3})^2 + 2 \times \sqrt{3} \times 2 + 2^2 \\ &= 3 + 4\sqrt{3} + 4 \\ &= 7 + 4\sqrt{3} \end{aligned}$$

b
$$\begin{aligned} & (\sqrt{3} - \sqrt{7})^2 \\ &= (\sqrt{3})^2 + 2 \times \sqrt{3} \times (-\sqrt{7}) + (-\sqrt{7})^2 \\ &= 3 - 2\sqrt{21} + 7 \\ &= 10 - 2\sqrt{21} \end{aligned}$$

Example 12**Self Tutor**

Expand and simplify:

a $(3 + \sqrt{2})(3 - \sqrt{2})$

b $(2\sqrt{3} - 5)(2\sqrt{3} + 5)$

a
$$\begin{aligned} & (3 + \sqrt{2})(3 - \sqrt{2}) \\ &= 3^2 - (\sqrt{2})^2 \\ &= 9 - 2 \\ &= 7 \end{aligned}$$

b
$$\begin{aligned} & (2\sqrt{3} - 5)(2\sqrt{3} + 5) \\ &= (2\sqrt{3})^2 - 5^2 \\ &= (4 \times 3) - 25 \\ &= 12 - 25 \\ &= -13 \end{aligned}$$

Did you notice that these answers are **integers**?

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Exercise 4F - Equality of Surds

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Solve for x and y given that they are rational:

$x + y\sqrt{2} = 3 + 2\sqrt{2}$

$-x + y\sqrt{2} = 11 - 3\sqrt{2}$

$x + y\sqrt{2} = -3\sqrt{2}$

$15 - 4\sqrt{2} = x + y\sqrt{2}$

$x + y\sqrt{2} = 6$

$x + y\sqrt{2} = 0$

Solve for x and y given that they are rational:

$(x + y\sqrt{2})(2 - \sqrt{2}) = 1 + \sqrt{2}$

$(2 - 3\sqrt{2})(x + y\sqrt{2}) = \sqrt{2}$

Find rationals a and b such that:

$(a + \sqrt{2})(2 - \sqrt{2}) = 4 - b\sqrt{2}$

$(a + b\sqrt{2})^2 = 33 + 20\sqrt{2}$

$(x + y\sqrt{2})(3 + \sqrt{2}) = 1$

$(x + y\sqrt{2})(3 - \sqrt{2}) = -4\sqrt{2}$

$(a + 3\sqrt{2})(3 - \sqrt{2}) = 6 + b\sqrt{2}$

$(a + b\sqrt{2})^2 = 41 - 24\sqrt{2}$

Find $\sqrt{11 - 6\sqrt{2}}$. **Hint:** $\sqrt{2}$ is never negative.

a Write $\sqrt{11 + 4\sqrt{6}}$ in the form $a\sqrt{2} + b\sqrt{3}$ where $a, b \in \mathbb{Q}$.

b Can $\sqrt{11 + 4\sqrt{6}}$ be written in the form $a + b\sqrt{6}$ where $a, b \in \mathbb{Q}$? Explain your answer.

(2c)

$$(2 - 3\sqrt{2})(x + y\sqrt{2}) = \sqrt{2}$$

$$2x + 2y\sqrt{2} - 3\sqrt{2}x - 6y = \sqrt{2}$$

$$2x - 6y = 0$$

$$2y - 3x = 1$$

$$x - 3y = 0$$

$$2y - 3(3y) = 1$$

$$x = 3y$$

$$2y - 9y = 1$$

$$-7y = 1$$

$$y = -\frac{1}{7}$$

$$x = -\frac{3}{7}$$

- | | |
|------------------------------------|-------------------------------------|
| 1 a $x=3, y=2$ | b $x=15, y=-4$ |
| c $x=-11, y=-3$ | d $x=6, y=0$ |
| e $x=0, y=-3$ | f $x=y=0$ |
| 2 a $x=2, y=\frac{3}{2}$ | b $x=\frac{3}{7}, y=-\frac{1}{7}$ |
| c $x=-\frac{3}{7}, y=-\frac{1}{7}$ | d $x=-\frac{8}{7}, y=-\frac{12}{7}$ |
| 3 a $a=3, b=1$ | b $a=4, b=5$ |
| c $a=5, b=2$ or $a=-5, b=-2$ | |
| d $a=3, b=-4$ or $a=-3, b=4$ | |

4 Let $\sqrt{11 - 6\sqrt{2}} = a + b\sqrt{2}$; $a, b \in \mathbb{Q}$
 Show $a + b\sqrt{2} = 3 - \sqrt{2}$ or $-3 + \sqrt{2}$
 But $11 - 6\sqrt{2} > 0$
 $\therefore \sqrt{11 - 6\sqrt{2}} = 3 - \sqrt{2}$

5 a $\sqrt{11 + 4\sqrt{6}} = 2\sqrt{2} + \sqrt{3}$
 b No. (Suppose it can be written in the form $a + b\sqrt{6}$.)

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$$(4a) \frac{1+\sqrt{2}}{1-\sqrt{2}} + \frac{1-\sqrt{2}}{1+\sqrt{2}}$$

make into a single fraction

$$\frac{(1+\sqrt{2})(1+\sqrt{2})}{(1-\sqrt{2})(1+\sqrt{2})} + \frac{(1-\sqrt{2})(1-\sqrt{2})}{(1+\sqrt{2})(1-\sqrt{2})}$$

$$= \frac{(1+\sqrt{2})^2 + (1-\sqrt{2})^2}{(1-\sqrt{2})(1+\sqrt{2})}$$

$$= \frac{1+2\sqrt{2}+2+1-2\sqrt{2}+2}{-1}$$

$$= -6$$

now try another one

$$(6) \sqrt{\frac{3+2\sqrt{2}}{3-2\sqrt{2}}} = \sqrt{\frac{3+2\sqrt{2}}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}}}$$

$$= \sqrt{\frac{(3+2\sqrt{2})^2}{9-8}}$$

$$= \sqrt{\frac{(3+2\sqrt{2})^2}{1}}$$

$$= \underline{3+2\sqrt{2}}$$

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Solving Examples

① Solve Linear ↙ 1 answer

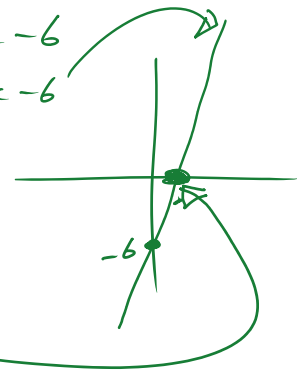
$$2x - 6 = 0$$

$$2x = 6$$

$$\underline{x = 3}$$

$$f(x) = 2x - 6$$

$$y = 2x - 6$$



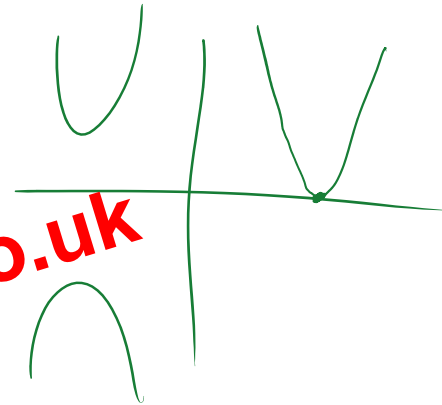
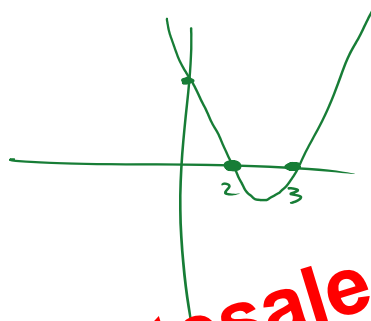
② Solve Quadratic ↙

$$x^2 - 5x + 6 = 0$$

$$(x - 3)(x - 2) = 0$$

$$x - 3 = 0 \text{ or } x - 2 = 0$$

$$\underline{x = 3} \quad \underline{x = 2}$$



③ Solve Quadratic ↙

$$2x^2 - 5x + 2 = 0$$

$$(2x - 1)(x - 2) = 0$$

$$2x - 1 = 0 \quad \underline{x = 2}$$

$$2x = 1$$

$$\underline{x = \frac{1}{2}}$$

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9b) $3x^4 - 24x^3 + 48x^2 = 0$
 $3x^2(x^2 - 8x + 16) = 0$
 $\downarrow \quad \downarrow$
 $3x^2 = 0 \quad (x-4)^2 = 0$
 $x^2 = 0 \quad x = 4$
 $x = 0$

9c) $2x^6 + 98x^2 = 30x^4$
 $2x^6 - 30x^4 + 98x^2 = 0$
 $2x^2(x^4 - 15x^2 + 49) = 0$
 $\downarrow \quad \downarrow$
 $2x^2 = 0 \quad (x^2 - 11)(x^2 - 4) = 0$
 $x^2 = 0 \quad x^2 = 11 \quad x^2 = 4$
 $x = 0$ $x = \pm\sqrt{11}$ $x = \pm 2$

10a) $\sqrt{x+2} = x$
 $x+2 = x^2$
 $0 = x^2 - x - 2$
 $0 = (x+1)(x-2)$

$x = -1$ $x = 2$
 check answers for validity

$\sqrt{-1+2} = -1$?

$\sqrt{2+2} = 2$?

$\sqrt{1} = 1$
 \uparrow

$\sqrt{4} = 2$
 yes

this means

$+\sqrt{1} = -1$?

no, so invalid

$x = 2$ only

10b) $\sqrt{x+13} - \sqrt{7-x} = 2$
 $\sqrt{x+13} = 2 + \sqrt{7-x}$
 $x+13 = (2 + \sqrt{7-x})^2$
 $x+13 = 4 + 4\sqrt{7-x} + 7-x$
 $0 = 4\sqrt{7-x} - 2 - 2x$

$2+2x = 4\sqrt{7-x}$

$4 + 8x + 4x^2 = 16(7-x)$

$4 + 8x + 4x^2 = 112 - 16x$

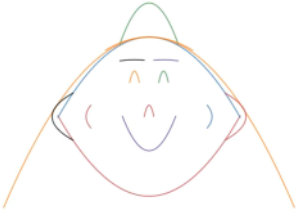
$4x^2 + 24x - 108 = 0$

$x^2 + 6x - 27 = 0$

$(x-3)(x+9) = 0$

$x = 3$ $x = -9$

\uparrow
 does not work
 when 'plugged'
 back in
 invalid answer



Mahnur's Ms Shalica



Yusuf's no name



Lydia's Herman the Tourist Sloth



Ayisha's sad face



Felix's ya boi

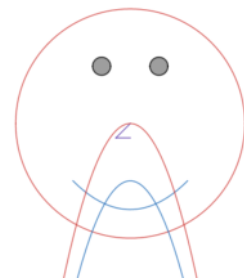
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Gabb's boys



Catharina's pointy head



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ugh x axis

ted by factor a

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3Adv Lesson 12: Advanced Quadratics

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B: Sum and Product of Roots

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If $ax^2 + bx + c = 0$ has roots α and β , then $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$.

For example: If α and β are the roots of $2x^2 - 2x - 1 = 0$
then $\alpha + \beta = 1$ and $\alpha\beta = -\frac{1}{2}$.

Proof:

If α and β are the roots of $ax^2 + bx + c = 0$,
then $ax^2 + bx + c = a(x - \alpha)(x - \beta)$
 $= a(x^2 - [\alpha + \beta]x + \alpha\beta)$
 $\therefore x^2 + \frac{b}{a}x + \frac{c}{a} = x^2 - [\alpha + \beta]x + \alpha\beta$

Equating coefficients,

$$\alpha + \beta = -\frac{b}{a} \quad \text{and} \quad \alpha\beta = \frac{c}{a}$$

Example 11

Self Tutor

Find the sum and product of the roots of $25x^2 - 20x + 1 = 0$.
Check your answer by solving the quadratic.

If α and β are the roots then $\alpha + \beta = -\frac{b}{a} = \frac{20}{25} = \frac{4}{5}$

$$\text{and } \alpha\beta = \frac{c}{a} = \frac{1}{25}$$

Check: $25x^2 - 20x + 1 = 0$ has roots

$$\frac{20 \pm \sqrt{400 - 4(25)(1)}}{50} = \frac{20 \pm \sqrt{300}}{50} = \frac{20 \pm 10\sqrt{3}}{50} = \frac{2 \pm \sqrt{3}}{5}$$

$$\text{These have sum} = \frac{2 + \sqrt{3}}{5} + \frac{2 - \sqrt{3}}{5} = \frac{4}{5} \quad \checkmark$$

$$\text{and product} = \left(\frac{2 + \sqrt{3}}{5}\right) \left(\frac{2 - \sqrt{3}}{5}\right) = \frac{4 - 3}{25} = \frac{1}{25} \quad \checkmark$$

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Extension Questions

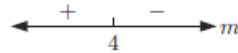
Friday, 15 September 2017 8:31 AM

- 2** The equation $kx^2 - (1 + k)x + (3k + 2) = 0$ is such that the sum of its roots is twice their product. Find k and the two roots.
- 3** The quadratic equation $ax^2 - 6x + a - 2 = 0$, $a \neq 0$, has one root which is double the other.
- Let the roots be α and 2α . Hence find two equations involving α .
 - Find a and the two roots of the quadratic equation.
- 4** The quadratic equation $kx^2 + (k - 8)x + (1 - k) = 0$, $k \neq 0$, has one root which is two more than the other. Find k and the two roots.

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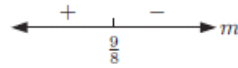
- 2** $k = -\frac{3}{5}$, roots are -1 and $\frac{1}{3}$
- 3 a** $3\alpha = \frac{6}{a}$, $2\alpha^2 = \frac{a-2}{a}$
- b** $a = 4$, roots are $\frac{1}{2}$ and 1 or $a = -2$, roots are -1 and -2
- 4** $k = 4$, roots are $-\frac{1}{2}$ and $\frac{3}{2}$ or $k = 16$, roots are $-\frac{5}{4}$ and $\frac{3}{4}$

3 a $\Delta = 16 - 4m$



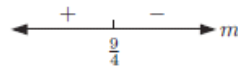
- i $m = 4$ ii $m < 4$ iii $m > 4$

b $\Delta = 9 - 8m$



- i $m = \frac{9}{8}$ ii $m < \frac{9}{8}$ iii $m > \frac{9}{8}$

c $\Delta = 9 - 4m$



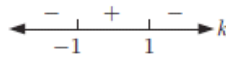
- i $m = \frac{9}{4}$ ii $m < \frac{9}{4}$ iii $m > \frac{9}{4}$

4 a $\Delta = k^2 + 8k$



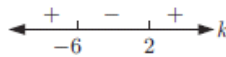
- i $k < -8$ or $k > 0$ ii $k \leq -8$ or $k \geq 0$
 iii $k = -8$ or 0 iv $-8 < k < 0$

b $\Delta = 4 - 4k^2$



- i $-1 < k < 1$ ii $-1 \leq k \leq 1$
 iii $k = \pm 1$ iv $k < -1$ or $k > 1$

c $\Delta = k^2 + 4k - 12$



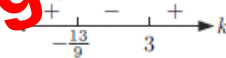
- i $k < -6$ or $k > 2$ ii $k \leq -6$ or $k \geq 2$
 iii $k = -6$ or 2 iv $-6 < k < 2$

d $\Delta = k^2 - 4k - 12$



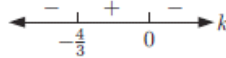
- i $k < -2$ or $k > 6$ ii $k \leq -2$ or $k \geq 6$
 iii $k = -2$ or 6 iv $-2 < k < 6$

e $\Delta = 9k^2 - 14k - 39$



- i $k < -\frac{13}{9}$ or $k > 3$ ii $k \leq -\frac{13}{9}$ or $k \geq 3$
 iii $k = -\frac{13}{9}$ or 3 iv $-\frac{13}{9} < k < 3$

f $\Delta = -3k^2 - 4k$



- i $-\frac{4}{3} < k < 0$ ii $-\frac{4}{3} \leq k \leq 0$
 iii $k = -\frac{4}{3}$ or 0 iv $k < -\frac{4}{3}$ or $k > 0$

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3Adv Lesson 14 Division of Polynomials

Friday, 22 September 2017 6:58 AM

① Addition ✓

$$(x^3 + x^2 - 3x + 5) + (2x^2 - x - 3) = x^3 + 3x^2 - 4x + 2$$

② Subtraction ✓

$$x^2 + 7x - 2 - (x^5 - 2x + 1) = x^2 + 7x - 2 - x^5 + 2x - 1 \\ = -x^5 + x^2 + 9x - 3$$

③ Multiplication ✓

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④ Division ?