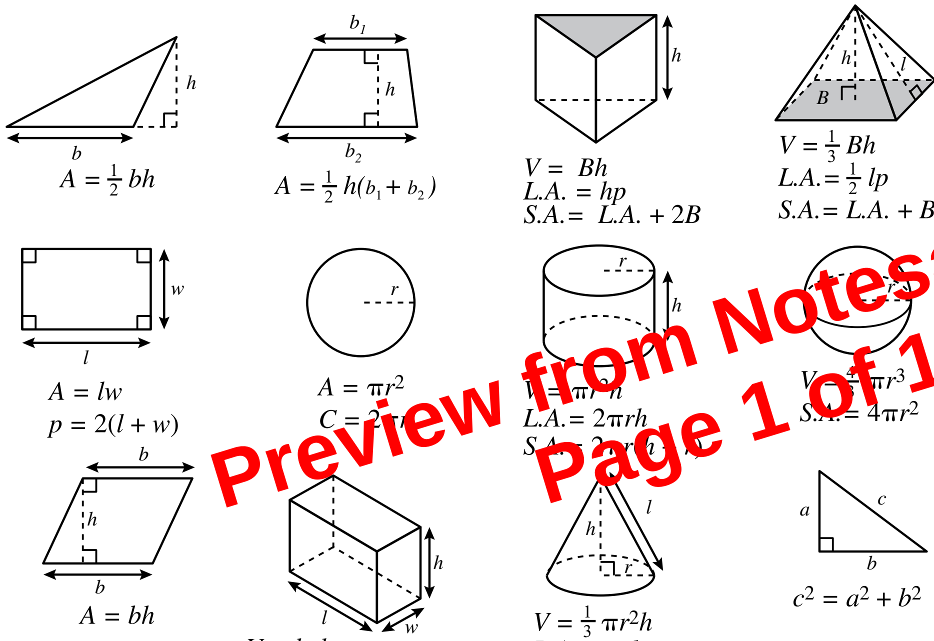
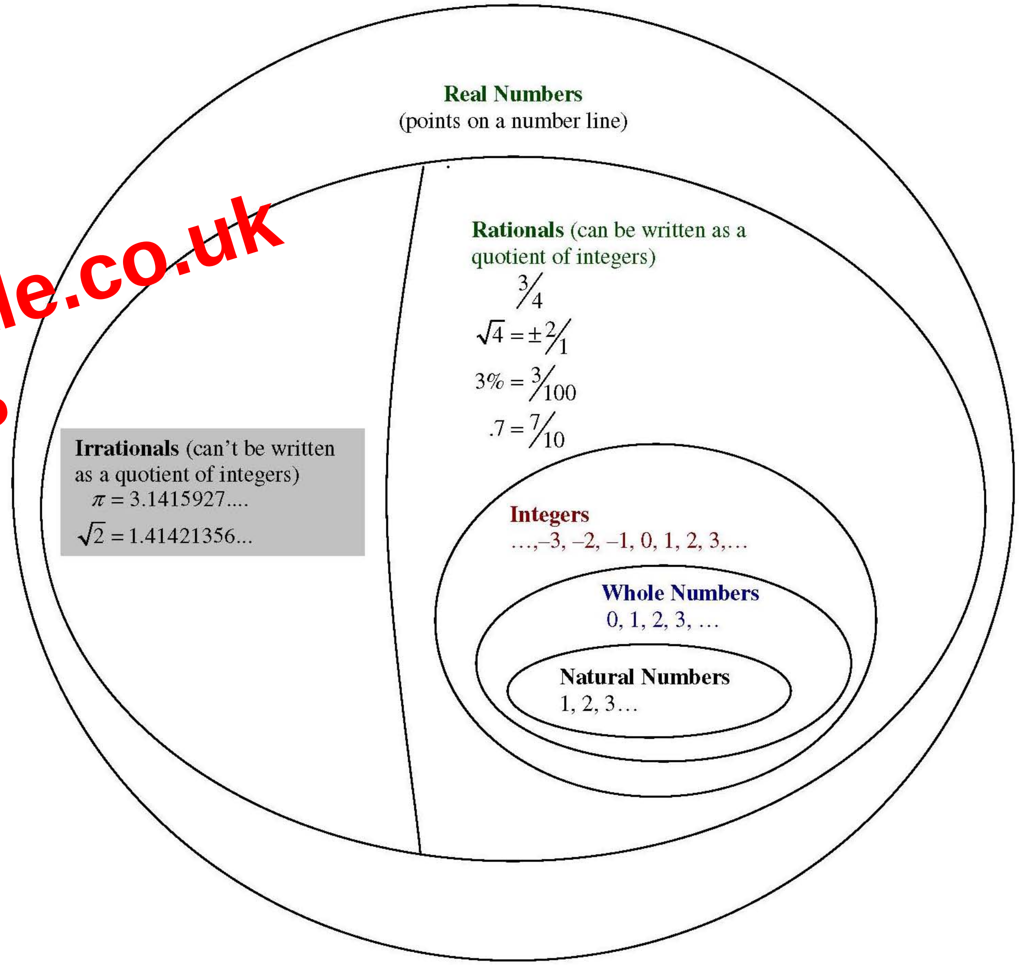
Geometric Formulas



- Triangle: $A = \frac{1}{2}bh$
- Trapezoid: $A = \frac{1}{2}h(b_1 + b_2)$
- Prism: $V = Bh$, $L.A. = hp$, $S.A. = L.A. + 2B$
- Pyramid: $V = \frac{1}{3}Bh$, $L.A. = \frac{1}{2}lp$, $S.A. = L.A. + B$
- Rectangle: $A = lw$, $p = 2(l + w)$
- Circle: $A = \pi r^2$, $C = 2\pi r$
- Cylinder: $V = \pi r^2 h$, $L.A. = 2\pi rh$, $S.A. = 2\pi rh + 2\pi r^2$
- Sphere: $V = \frac{4}{3}\pi r^3$, $S.A. = 4\pi r^2$
- Parallelogram: $A = bh$
- Rectangular prism: $V = lwh$, $S.A. = 2lw + 2lh + 2wh$
- Cone: $V = \frac{1}{3}\pi r^2 h$, $L.A. = \pi rl$, $S.A. = \pi r(l + r)$
- Right triangle: $c^2 = a^2 + b^2$



1.6. Properties of Real Numbers				
	For Addition	**For Subtraction**	**For Multiplication**	**For Division**
Commutative	$a + b = b + a$	$a - b \neq b - a$	$ab = ba$	$a/b \neq b/a$
Associative	$(a+b)+c = a+(b+c)$	$(a-b)-c \neq a-(b-c)$	$(ab)c = a(bc)$	$(a \div b) \div c \neq a \div (b \div c)$
Identity	$0+a = a$ & $a+0 = a$	$a - 0 = a$	$a \bullet 1 = a$ & $1 \bullet a = a$	$a \div 1 = a$
Inverse	$a + (-a) = 0$ & $(-a) + a = 0$	$a - a = 0$	$1/a \bullet a = 1$ & $a \bullet 1/a = 1$ if $a \neq 0$	$a \div a = 1$ if $a \neq 0$
Distributive Property	$a(b + c) = ab + ac$	$a(b - c) = ab - ac$	$-a(b + c) = -ab - ac$	$-a(b - c) = -ab + ac$

1.7. Properties of Equality	
Addition Property of Equality	If $a = b$ then $a + c = b + c$
Multiplication Property of Equality	If $a = b$ then $ac = bc$
Multiplication Property of 0	$0 \bullet a = 0$ and $a \bullet 0 = 0$

The laws of Nature are but the mathematical thoughts of God. (Euclid)

"The essence of mathematics is not to make simple things complicated, but to make complicated things simple." -S. Gudder

"Do not worry about your problems with mathematics, I assure you mine are far greater." -Albert Einstein

"Math is like love -- a simple idea but it can get complicated."

Work : If a force of $F(x)$ moves an object in $a \le x \le b$, the work done is $W = \int_a^b F(x)\,dx$

Average Function Value : The average value of $f(x)$ on $a \le x \le b$ is $f_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx$

Arc Length Surface Area : Note that this is often a Calc II topic. The three basic formulas are,

$L = \int_a^b ds$ $\quad SA = \int_a^b 2\pi y\,ds$ (rotate about x-axis) $\quad SA = \int_a^b 2\pi x\,ds$ (rotate about y-axis)

where ds is dependent upon the form of the function being worked with as follows.

$ds = \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$ if $y = f(x)$, $a \le x \le b$ $\quad ds = \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$ if $x = f(t), y = g(t)$, $a \le t \le b$

$ds = \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$ if $x = f(y)$, $a \le y \le b$ $\quad ds = \sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\,d\theta$ if $r = f(\theta)$, $a \le \theta \le b$

With surface area you *may* have to substitute in for the x or y depending on your choice of ds to match the differential in the ds. With parametric and polar you will always need to substitute.

Improper Integral

An improper integral is an integral with one or more infinite limits and/or discontinuous integrands. Integral is called convergent if the limit exists and has a finite value and divergent if the limit doesn't exist or has infinite value. This is typically a Calc II topic.

Infinite Limit

1. $\int_a^{\infty} f(x)\,dx = \lim_{t\to\infty}\int_a^t f(x)\,dx$
2. $\int_{-\infty}^{b} f(x)\,dx = \lim_{t\to-\infty}\int_t^b f(x)\,dx$
3. $\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{c} f(x)\,dx + \int_c^{\infty} f(x)\,dx$ provided BOTH integrals are convergent.

Discontinuous Integrand

1. Discont. at a: $\int_a^b f(x)\,dx = \lim_{t\to a^+}\int_t^b f(x)\,dx$
2. Discont. at b : $\int_a^b f(x)\,dx = \lim_{t\to b^-}\int_a^t f(x)\,dx$
3. Discontinuity at $a < c < b$: $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ provided both are convergent.

Comparison Test for Improper Integrals : If $f(x) \ge g(x) \ge 0$ on $[a,\infty)$ then,

1. If $\int_a^{\infty} f(x)\,dx$ conv. then $\int_a^{\infty} g(x)\,dx$ conv.
2. If $\int_a^{\infty} g(x)\,dx$ divg. then $\int_a^{\infty} f(x)\,dx$ divg.

Useful fact : If $a > 0$ then $\int_a^{\infty} \frac{1}{x^p}\,dx$ converges if $p > 1$ and diverges for $p \le 1$.

Approximating Definite Integrals

For given integral $\int_a^b f(x)\,dx$ and a n (must be even for Simpson's Rule) define $\Delta x = \frac{b-a}{n}$ and divide $[a,b]$ into n subintervals $[x_0,x_1]$, $[x_1,x_2]$, … , $[x_{n-1},x_n]$ with $x_0 = a$ and $x_n = b$ then,

Midpoint Rule : $\int_a^b f(x)\,dx \approx \Delta x\left[f(x_1^*)+f(x_2^*)+\cdots+f(x_n^*)\right]$, x_i^* is midpoint $[x_{i-1},x_i]$

Trapezoid Rule : $\int_a^b f(x)\,dx \approx \frac{\Delta x}{2}\left[f(x_0)+2f(x_1)++2f(x_2)+\cdots+2f(x_{n-1})+f(x_n)\right]$

Simpson's Rule : $\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}\left[f(x_0)+4f(x_1)+2f(x_2)+\cdots+2f(x_{n-2})+4f(x_{n-1})+f(x_n)\right]$

Conic Section Formulas

Parabola:

$y = a(x-h)^2 + k$
If a > 0, opens up
If a < 0, opens down
Vertex: (h, k)
Focus: (h, k+p)
Directrix: y = k-p
Axis of Symmetry: x = h

$x = a(y-k)^2 + h$
If a > 0, opens right
If a < 0, opens left
Vertex: (h, k)
Focus: (h+p, k)
Directrix: x = h-p
Axis of Symmetry: y = k

$a = \frac{1}{4p}$ $\quad p = \frac{1}{4a}$

Ellipse:

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Center: (0,) 0
Foci: (c, 0), (-c, 0)
Vertices: (a, 0), (-a, 0)
y Intercepts: (0, b), (0, -b)
Major xis: xA xis a
Minor Axis: y axis
Length of Major Axis: 2a
Length of Minor Axis: 2b

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

Center: (0,) 0
Foci: (0, c), (0, -c)
Vertices: (0, a), (0, -a)
x Intercepts: (b, 0), (-b, 0)
Major xis: yA xis a
Minor Axis: x axis
Length of Major Axis: 2a
Length of Minor Axis: 2b

$c^2 = a^2 - b^2$, $a > b > 0$

Hyperbola:

Transverse Axis: Horizontal

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Center: (0, 0)
Foci: (c, 0), (-c, 0)
Vertices: (a, 0), (-a, 0)

Asymptotes: $y = \pm \frac{b}{a}x$

Transverse Axis: Vertical

$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Center: (0, 0)
Foci: (0, c), (0, -c)
Vertices: (0, a), (0, -a)

Asymptotes: $y = \pm \frac{a}{b}x$

$c^2 = a^2 + b^2$

Circle:

$(x-h)^2 + (y-k)^2 = r^2$

Center: (h, k)
Radius: r