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$$2 \lim_{x \rightarrow 0} \frac{x^2}{1 - \cos x} \quad \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\stackrel{L}{=} \lim_{x \rightarrow 0} \frac{2x}{\sin x} \quad \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\stackrel{L}{=} \lim_{x \rightarrow 0} \frac{2}{\cos x}$$

$$= 2$$

$$3. \lim_{x \rightarrow -\infty} \frac{\frac{\pi}{2} + \operatorname{Arctan} x}{\frac{1}{x}} \quad \left(\frac{0}{0} \right)$$

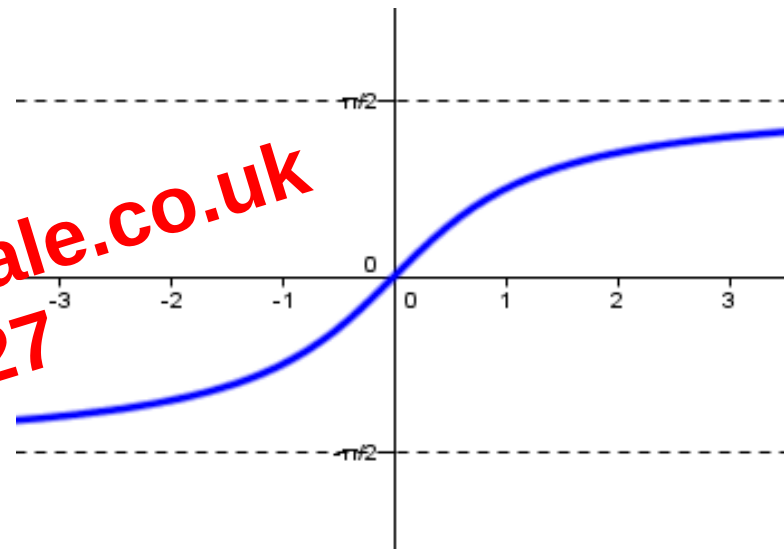
$$\stackrel{L}{=} \lim_{x \rightarrow -\infty} \frac{\frac{1}{1+x^2}}{-\frac{1}{x^2}}$$

$$= \lim_{x \rightarrow -\infty} \frac{-x^2}{1+x^2} \quad \left(\frac{\infty}{\infty} \right)$$

$$\stackrel{L}{=} \lim_{x \rightarrow -\infty} \frac{-2x}{2x}$$

$$= \lim_{x \rightarrow -\infty} (-1)$$

$$= -1$$



Example 1.7.5

Evaluate $\lim_{x \rightarrow 0^+} x^{x^2}$.

$$\lim_{x \rightarrow 0^+} x^{x^2} \quad (0^0)$$

Let $y = x^{x^2}$

$$\ln y = \ln x^{x^2}$$

$$\ln y = x^2 \ln x$$

$$\lim_{x \rightarrow 0^+} \ln y = \lim_{x \rightarrow 0^+} x^2 \ln x \quad (0 \cdot \infty)$$

$$= \lim_{x \rightarrow 0^+} \frac{\ln x}{\cancel{1} / x^2} \quad \left(\frac{\infty}{\infty} \right)$$

$$\lim_{x \rightarrow 0^+} \ln y = \lim_{x \rightarrow 0^+} x^2 \ln x \quad (0 \cdot \infty)$$

$$= \lim_{x \rightarrow 0^+} \frac{\ln x}{1/x^2} \quad \left(\frac{\infty}{\infty} \right)$$

$$\stackrel{L}{=} \lim_{x \rightarrow 0^+} \frac{1/x}{-2/x^3}$$

$$= \lim_{x \rightarrow 0^+} \left(-\frac{x^2}{2} \right)$$

$$= 0$$