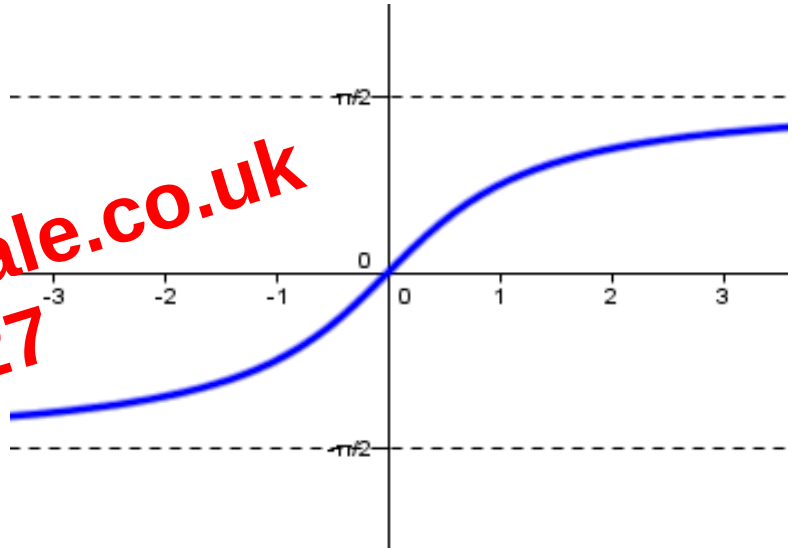
$$2.\ \lim_{x\to 0}\frac{x^2}{1-\cos x}\qquad \left(\frac{0}{0}\right)$$

$$\overset{L}{=}\lim_{x\to 0}\frac{2x}{\sin x}\qquad \left(\frac{0}{0}\right)$$

$$\overset{L}{=}\lim_{x\to 0}\frac{2}{\cos x}$$

$$=2$$

3. $$\lim_{x\to-\infty}\frac{\frac{\pi}{2}+\operatorname{Arctan}x}{\frac{1}{x}}\quad\left(\frac{0}{0}\right)$$



$$\overset{L}{=}\lim_{x\to-\infty}\frac{\frac{1}{1+x^2}}{-\frac{1}{x^2}}$$

$$=\lim_{x\to-\infty}\frac{-x^2}{1+x^2}\qquad\left(\frac{\infty}{\infty}\right)$$

$$\overset{L}{=}\lim_{x\to-\infty}\frac{-2x}{2x}$$

$$=\lim_{x\to-\infty}(-1)$$

$$=-1$$

Example 1.7.5

Evaluate $\lim_{x\to 0^+} x^{x^2}$.

$$\lim_{x\to 0^+} x^{x^2} \quad \left(0^0\right)$$

Let $y = x^{x^2}$

$$\ln y = \ln x^{x^2}$$

$$\ln y = x^2 \ln x$$

$$\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} x^2 \ln x \qquad \left(0\cdot\infty\right)$$

$$= \lim_{x\to 0^+} \frac{\ln x}{1/x^2} \qquad \left(\frac{\infty}{\infty}\right)$$

$$\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} x^2 \ln x \qquad (0\cdot\infty)$$

$$= \lim_{x\to 0^+} \frac{\ln x}{1/x^2} \qquad \left(\frac{\infty}{\infty}\right)$$

$$\overset{L}{=} \lim_{x\to 0^+} \frac{1/x}{-2/x^3}$$

$$= \lim_{x\to 0^+} \left(-\frac{x^2}{2}\right)$$

$$= 0$$