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Page 1 of 105

## Unit 1

# Derivatives of and Integrals Yielding Transcendental Functions

# Theorem

Preview from Notesale.co.uk  
Page 13 of 105

$$\int \frac{1}{\sqrt{a^2 - u^2}} du = \text{Arcsin} \frac{u}{a} + C$$

$$\int \frac{1}{a^2 + u^2} du = \frac{1}{a} \text{Arctan} \frac{u}{a} + C$$

$$\int \frac{1}{u\sqrt{u^2 - a^2}} du = \frac{1}{a} \text{Arcsec} \frac{u}{a} + C$$

$$\text{If } y = \log_a x$$

$$\log_a x = \frac{\log_m x}{\log_m a}$$

$$y = \frac{\ln x}{\ln a}$$

$$\frac{dy}{dx} = \frac{1}{\ln a} \cdot \frac{1}{x} = \frac{1}{x \ln a}$$

# Theorem

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Page 31 of 105

If  $u$  is a differentiable function of  $x$ , then

$$D_x (\log_a u) = \frac{1}{u \ln a} \cdot D_x u$$

# Theorem

Preview from Notesale.co.uk  
Page 39 of 105

$$\int \tan u \, du = \ln|\sec u| + C$$

$$\int \cot u \, du = \ln|\sin u| + C$$

$$\int \sec u \, du = \ln|\sec u + \tan u| + C$$

$$\int \csc u \, du = \ln|\csc u - \cot u| + C$$

# Required Exercises

Preview from Notesale.co.uk  
Page 41 of 105

Answer Exercises in TC7.

Exercises 5.2 (5-36) on page 449-450

Exercises 5.3 (15-44) on page 457

Check your answers on A-155.

# Example 1.3.1

Differentiate the function given by  $y = \frac{(2x-1)^5 (x^2+1)^4}{\sqrt{x^3+1}}$ .

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Page 47 of 105

$$y = \frac{(2x-1)^5 (x^2+1)^4}{(x^3+1)^{1/2}}$$

$$|y| = \left| \frac{(2x-1)^5 (x^2+1)^4}{\sqrt{x^3+1}} \right|$$

$$|y| = \frac{|2x-1|^5 |x^2+1|^4}{|x^3+1|^{1/2}}$$

## Example 1.4.4

Evaluate  $\int 2^x \csc(2^x) dx$ .

$$\int 2^x \csc(2^x) dx$$

$$= \frac{1}{\ln 2} \int \csc u \, du$$

$$= \frac{1}{\ln 2} \cdot \ln |\csc u - \cot u| + C$$

$$= \frac{\ln |\csc(2^x) - \cot(2^x)|}{\ln 2} + C$$

Let

$$u = 2^x$$

$$du = 2^x \ln 2 \, dx$$

$$\frac{du}{\ln 2} = 2^x \, dx$$

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*Page 75 of 105*

$$P(x) = (\cosh x, \sinh x)$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$\frac{\cosh^2 x}{\cosh^2 x} - \frac{\sinh^2 x}{\cosh^2 x} = \frac{1}{\cosh^2 x}$$

$$1 - \tanh^2 x = \operatorname{sech}^2 x$$

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Page 77 of 105

$$y = \sinh x$$

$$y = \frac{e^x - e^{-x}}{2}$$

$$\frac{dy}{dx} = \frac{1}{2} \cdot (e^x - e^{-x}(-1))$$

$$= \frac{e^x + e^{-x}}{2}$$

$$= \cosh x$$

Preview from Notesale.co.uk  
Page 78 of 105

$$y = \cosh x$$
$$y = \frac{e^x + e^{-x}}{2}$$

$$\frac{dy}{dx} = \frac{1}{2} \cdot (e^x + e^{-x}(-1))$$

$$= \frac{e^x - e^{-x}}{2}$$

$$= \sinh x$$

# Theorem

If  $u$  is a differentiable function of  $x$  then

$$D_x(\sinh u) = \cosh u \cdot D_x u$$

$$D_x(\cosh u) = \sinh u \cdot D_x u$$

$$D_x(\tanh u) = \operatorname{sech}^2 u \cdot D_x u$$

$$D_x(\operatorname{csch} u) = -\operatorname{csch} u \coth u \cdot D_x u$$

$$D_x(\operatorname{sech} u) = -\operatorname{sech} u \tanh u \cdot D_x u$$

$$D_x(\coth u) = -\operatorname{csch}^2 u \cdot D_x u$$

# Required Exercises

Preview from Notesale.co.uk  
Page 87 of 105

Answer Exercises in TC7.

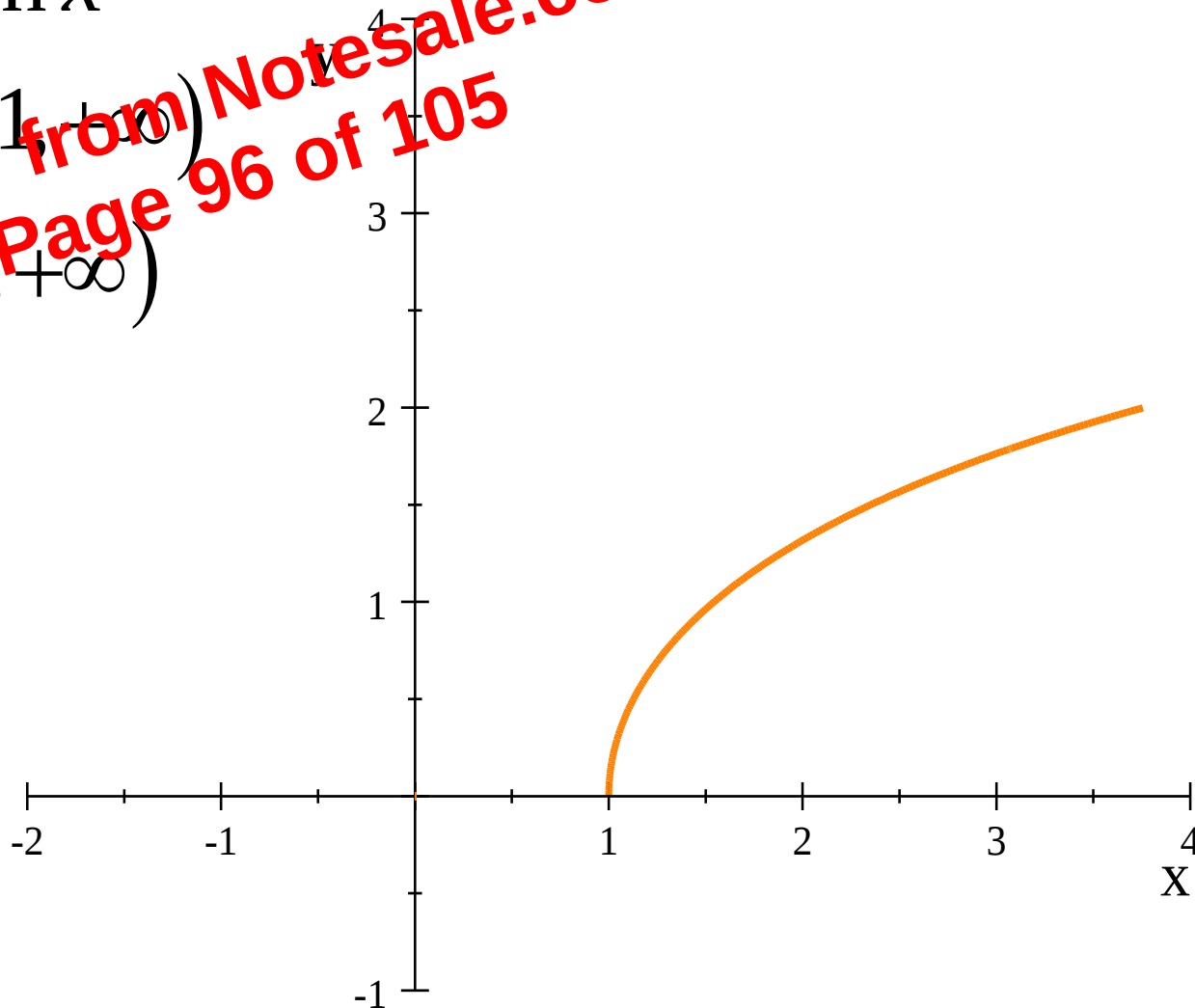
Exercises 5.9 (1-32) on page 524

Check your answers on A-158.

$$y = \operatorname{Arg} \cosh x$$

$$\text{Domain} = [1, +\infty)$$

$$\text{Range} = [0, +\infty)$$



# Theorem

If  $u$  is a differentiable function of  $x$  then

$$D_x (\text{Argsinh } u) = \frac{1}{\sqrt{1+u^2}} D_x u$$

$$D_x (\text{Argcosh } u) = \frac{1}{\sqrt{u^2-1}} D_x u$$

$$D_x (\text{Argtanh } u) = \frac{1}{1-u^2} D_x u$$

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Page 101 of 105

$$D_x(\operatorname{Argcsch} u) = \frac{1}{|u|\sqrt{1+u^2}} D_x u$$

$$D_x(\operatorname{Argsech} u) = -\frac{1}{u\sqrt{1-u^2}} D_x u$$

$$D_x(\operatorname{Argcoth} u) = \frac{1}{1-u^2} D_x u$$

Preview from Notesale.co.uk  
Page 105 of 105

# End of Unit 1.6