Unit 1

Derivatives of and Integrals Yielding Transcendental Functions

Theorem

$$\int \frac{1}{\sqrt{a^2 - u^2}}\,du = \text{Arcsin}\frac{u}{a} + C$$

$$\int \frac{1}{a^2 + u^2}\,du = \frac{1}{a}\text{Arctan}\frac{u}{a} + C$$

$$\int \frac{1}{u\sqrt{u^2 - a^2}}\,du = \frac{1}{a}\text{Arcsec}\frac{u}{a} + C$$

If $y = \log_a x$

$$\log_a x = \frac{\log_m x}{\log_m a}$$

$$y = \frac{\ln x}{\ln a}$$

$$\frac{dy}{dx} = \frac{1}{\ln a} \cdot \frac{1}{x} = \frac{1}{x \ln a}$$

Theorem

If u is a differentiable function of x, then

$$D_x\left(\log_a u\right) = \frac{1}{u \ln a} \cdot D_x u$$

Theorem

$$\int \tan u \, du = \ln\left|\sec u\right| + C$$

$$\int \cot u \, du = \ln\left|\sin u\right| + C$$

$$\int \sec u \, du = \ln\left|\sec u + \tan u\right| + C$$

$$\int \csc u \, du = \ln\left|\csc u - \cot u\right| + C$$

Required Exercises

Answer Exercises in TC7.

Exercises 5.2 (5-36) on page 449-450

Exercises 5.3 (15-44) on page 457

Check your answers on A-155.

Example 1.3.1

Differentiate the function given by $y=\dfrac{(2x-1)^5(x^2+1)^4}{\sqrt{x^3+1}}$.

$$y=\frac{(2x-1)^5(x^2+1)^4}{(x^3+1)^{1/2}}$$

$$|y|=\left|\frac{(2x-1)^5(x^2+1)^4}{\sqrt{x^3+1}}\right|$$

$$|y|=\frac{|2x-1|^5\,|x^2+1|^4}{|x^3+1|^{1/2}}$$

Example 1.4.4

Evaluate $\int 2^x \csc\left(2^x\right) dx$.

$$\int 2^x \csc\left(2^x\right) dx$$

$$= \frac{1}{\ln 2} \int \csc u \, du$$

$$= \frac{1}{\ln 2} \cdot \ln\left|\csc u - \cot u\right| + C$$

$$= \frac{\ln\left|\csc\left(2^x\right) - \cot\left(2^x\right)\right|}{\ln 2} + C$$

Let $u = 2^x$

$$du = 2^x \ln 2 \, dx$$

$$\frac{du}{\ln 2} = 2^x dx$$

$$P(x) = (\cosh x, \sinh x)$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$\frac{\cosh^2 x}{\cosh^2 x} - \frac{\sinh^2 x}{\cosh^2 x} = \frac{1}{\cosh^2 x}$$

$$1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$y = \sinh x$$

$$y = \frac{e^x - e^{-x}}{2}$$

$$\frac{dy}{dx} = \frac{1}{2} \cdot \left(e^x - e^{-x}(-1)\right)$$

$$= \frac{e^x + e^{-x}}{2}$$

$$= \cosh x$$

$$y = \cosh x$$

$$y = \frac{e^x + e^{-x}}{2}$$

$$\frac{dy}{dx} = \frac{1}{2} \cdot \left(e^x + e^{-x}(-1)\right)$$

$$= \frac{e^x - e^{-x}}{2}$$

$$= \sinh x$$

Theorem

If u is a differentiable function of x then

$$D_x(\sinh u) = \cosh u \cdot D_x u$$

$$D_x(\cosh u) = \sinh u \cdot D_x u$$

$$D_x(\tanh u) = \operatorname{sech}^2 u \cdot D_x u$$

$$D_x(\operatorname{csch} u) = -\operatorname{csch} u \coth u \cdot D_x u$$

$$D_x(\operatorname{sech} u) = -\operatorname{sech} u \tanh u \cdot D_x u$$

$$D_x(\coth u) = -\operatorname{csch}^2 u \cdot D_x u$$

Required Exercises

Answer Exercises in TC7.

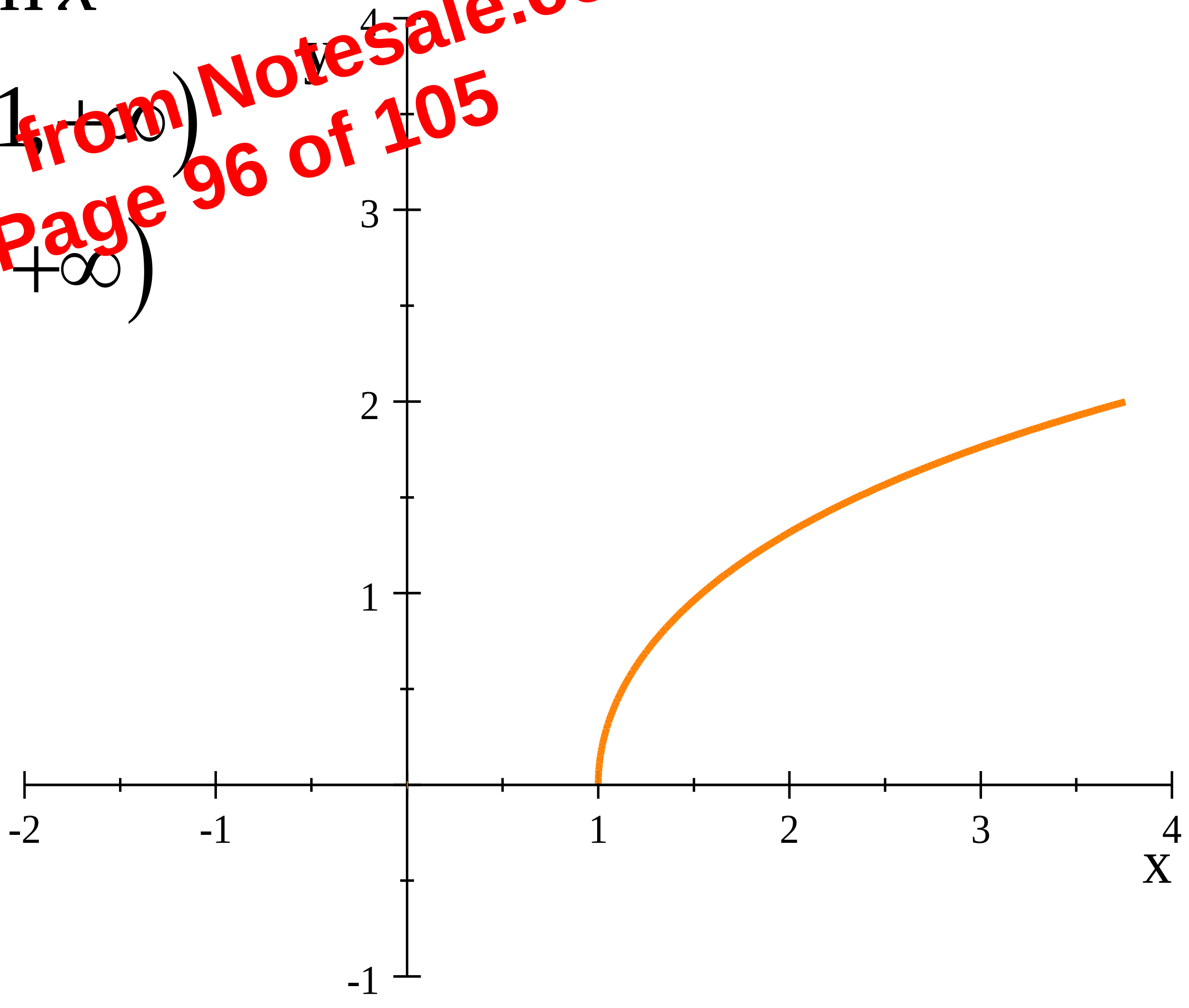
Exercises 5.9 (1-32) on page 524

Check your answers on A-158.

$$y = \operatorname{Arg}\cosh x$$

$$\text{Domain} = [1, +\infty)$$

$$\text{Range} = [0, +\infty)$$



Theorem

If u is a differentiable function of x then

$$D_x\left(\operatorname{Arg}\sinh u\right)=\frac{1}{\sqrt{1+u^2}}D_x u$$

$$D_x\left(\operatorname{Arg}\cosh u\right)=\frac{1}{\sqrt{u^2-1}}D_x u$$

$$D_x\left(\operatorname{Arg}\tanh u\right)=\frac{1}{1-u^2}D_x u$$

$$D_x\left(\operatorname{Arg}\operatorname{csch} u\right) = -\frac{1}{|u|\sqrt{1+u^2}} D_x u$$

$$D_x\left(\operatorname{Arg}\operatorname{sech} u\right) = -\frac{1}{u\sqrt{1-u^2}} D_x u$$

$$D_x\left(\operatorname{Arg}\operatorname{coth} u\right) = \frac{1}{1-u^2} D_x u$$

End of Unit 1.6