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## Unit 2

# Techniques of Integration

# Example 2.0

Evaluate the integral.

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$$\int \cos(2x)dx = \frac{1}{2}\sin(2x) + C$$

$$2. \int \sin(3x)dx = -\frac{1}{3}\cos(3x) + C$$

$$3. \int 2\sec^2(5x)dx = \frac{2}{5}\tan(5x) + C$$

$$4. \int \csc(7x)\cot(7x)dx = -\frac{1}{7}\csc(7x) + C$$

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# End of Unit 2.0

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$$\int xe^x dx = xe^x - e^x + C$$

Therefore,

$$\int x^2 e^x dx = x^2 e^x - 2xe^x + 2e^x + C_1$$

4.

$$\int e^x \cos x dx$$

$$= \int u dv$$

$$= uv - \int v du$$

$$= e^x \sin x - \int e^x \sin x dx$$

Let  $u = e^x \quad dv = \cos x dx$   
 $du = e^x dx \quad v = \sin x$

# Required Exercises

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Answer Exercises in TC7.

Exercises 7.1 (1-32) on page 582

Check your answers on A-159.

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$$\begin{aligned} &= - \int (1 - u^2)^2 u^2 du \\ &= - \int (1 - 2u^2 + u^4) u^2 du \\ &= - \int (u^2 - 2u^4 + u^6) du \\ &= - \left( \frac{u^3}{3} - \frac{2u^5}{5} + \frac{u^7}{7} \right) + C \\ &= - \frac{\cos^3 x}{3} + \frac{2\cos^5 x}{5} - \frac{\cos^7 x}{7} + C \end{aligned}$$

## Example 2.2.3

Evaluate the integrals.

$$\begin{aligned}1. \quad & \int \tan^2(2x) \sec^4(2x) dx \\&= \int \tan^2(2x) \sec^2(2x) \sec^2(2x) dx \\&= \int \tan^2(2x) (1 + \tan^2(2x)) \sec^2(2x) dx \\&= \frac{1}{2} \int u^2 (1 + u^2) du \\&= \frac{1}{2} \int (u^2 + u^4) du \\&= \frac{1}{2} \left( \frac{u^3}{3} + \frac{u^5}{5} \right) + C\end{aligned}$$

Let  $u = \tan(2x)$

$$du = 2 \sec^2(2x) dx$$
$$\frac{du}{2} = \sec^2(2x) dx$$

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# Preliminary Exercise

Express the following in terms of  
 $\sin \theta$  and  $\cos \theta$ .

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$$1. \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$2. \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$3. \sec \theta = \frac{1}{\cos \theta}$$

$$4. \csc \theta = \frac{1}{\sin \theta}$$

Express the following in terms of  
 $\tan \theta$  and  $\sec \theta$ .

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$$1. \cot \theta = \frac{1}{\tan \theta}$$

$$2. \cos \theta = \frac{1}{\sec \theta}$$

$$3. \sin \theta = \frac{\tan \theta}{\sec \theta}$$

$$4. \csc \theta = \frac{\sec \theta}{\tan \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$= \frac{1}{25} \int \cos \theta d\theta$$

$$= \frac{1}{25} \sin \theta + C$$

$$= \frac{1}{25} \cdot \frac{\tan \theta}{\sec \theta} + C$$

$$= \frac{1}{25} \cdot \frac{x}{\sqrt{x^2 + 25}} + C$$

$$\boxed{\begin{aligned}x &= 5 \tan \theta \\ \sqrt{x^2 + 25} &= 5 \sec \theta\end{aligned}}$$

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# When to Use Partial Fractions

Consider  $\int \frac{P}{Q} dx$

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Conditions:

1.  $P$  and  $Q$  are polynomials.
2.  $\deg P < \deg Q$ . Otherwise, divide  $P$  by  $Q$ .
3.  $P$  and  $Q$  have no common factors.

Otherwise, cancel common factors.

# Case 1: Distinct Linear Factors

If  $Q$  has distinct linear factors,

$$Q = (a_1x + b_1)(a_2x + b_2) \cdots (a_nx + b_n)$$

write  $\frac{P}{Q} = \frac{A_1}{a_1x + b_1} + \frac{A_2}{a_2x + b_2} + \cdots + \frac{A_n}{a_nx + b_n}$

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$$\frac{4x+3}{2x^2+3x+1} = \frac{A}{x+1} + \frac{B}{2x+1}$$

$$A = 1 \text{ and } B = 2$$

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$$\text{Therefore, } \frac{4x+3}{2x^2+3x+1} = \frac{1}{x+1} + \frac{2}{2x+1}.$$

$$\begin{aligned}\int \frac{4x+3}{2x^2+3x+1} dx &= \int \left( \frac{1}{x+1} + \frac{2}{2x+1} \right) dx \\ &= \ln|x+1| + 2 \cdot \frac{1}{2} \ln|2x+1| + C\end{aligned}$$

$$u = \sqrt{x^2 - 2}$$

$$= -2 \int \frac{(u^3 + 2u) du}{u - 5}$$

$$= -2 \int \left( u^2 + 5u + 27 + \frac{135}{u - 5} \right) du$$

$$= -2 \left( \frac{u^3}{3} + \frac{5u^2}{2} + 27u + 135 \ln|u - 5| \right) + C$$

$$= \frac{-2}{3} (x^2 - 2)^{3/2} - 5(x^2 - 2) - 54\sqrt{x^2 - 2}$$

$$- 270 \ln \left| \sqrt{x^2 - 2} - 5 \right| + C$$

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# End of Unit 2.5

$$\begin{aligned}
 2. \quad & \int \frac{dx}{\sin x + \tan x} \\
 &= \int \frac{dx}{\sin x + \frac{\sin x}{\cos x}} \\
 &= \int \frac{\frac{2dz}{1+z^2}}{\frac{2z}{1+z^2} + \frac{1+z^2}{1-z^2}} \\
 &\quad \frac{1+z^2}{1-z^2}
 \end{aligned}$$

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Let  $z = \tan\left(\frac{x}{2}\right)$

$$dx = \frac{2dz}{1+z^2}$$

$$\sin x = \frac{2z}{1+z^2}$$

$$\cos x = \frac{1-z^2}{1+z^2}$$