$$\bar{x} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i}$$

$m_i x_i$ – moment of mass of the ith particle with respect to the origin

$M_0 = \sum_{i=1}^{n} m_i x_i$ – moment of mass of the system with respect to the origin

$M = \sum_{i=1}^{n} m_i$ – total mass of the system

$$\bar{x} = \frac{\int_0^8 x\left(\frac{3}{8}x + 2\right)dx}{\int_0^8 \left(\frac{3}{8}x + 2\right)dx}$$

$$\bar{x} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i} \qquad \bar{y} = \frac{\sum_{i=1}^{n} m_i y_i}{\sum_{i=1}^{n} m_i}$$

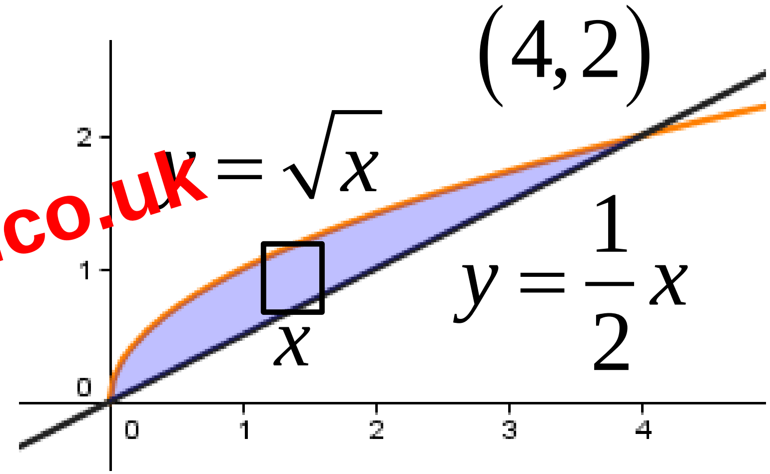
$$M_y = \sum_{i=1}^{n} m_i x_i$$ – moment of mass of the system with respect to the y – axis.

$$M_x = \sum_{i=1}^{n} m_i y_i$$ – moment of mass of the system with respect to the x – axis.

$$\bar{x} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i} \qquad \bar{y} = \frac{\sum_{i=1}^{n} m_i y_i}{\sum_{i=1}^{n} m_i}$$

$$M = \sum_{i=1}^{n} m_i \; - \; \text{total mass of the system.}$$

a. vertical strips



$$A=\int_0^4\left(\sqrt{x}-\frac{1}{2}x\right)dx$$

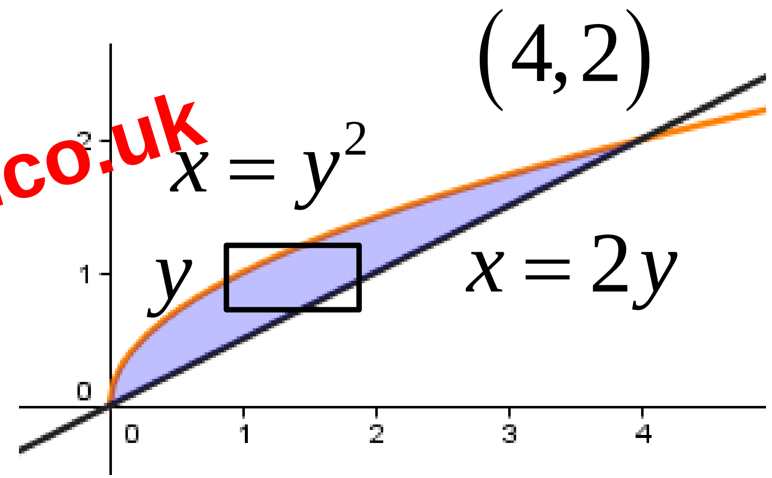
centroid at $(\bar{x},\bar{y})$ where

$$\bar{x}=\frac{M_y}{M}=\frac{\rho\int_0^4 x\left(\sqrt{x}-\frac{1}{2}x\right)dx}{\rho\int_0^4\left(\sqrt{x}-\frac{1}{2}x\right)dx}$$

$$\bar{y}=\frac{M_x}{M}=\frac{\rho\int_0^4 \frac{1}{2}\left(\sqrt{x}+\frac{1}{2}x\right)\left(\sqrt{x}-\frac{1}{2}x\right)dx}{\rho\int_0^4\left(\sqrt{x}-\frac{1}{2}x\right)dx}$$

b. horizontal strips

$$A=\int_0^2\left(2y-y^2\right)dy$$



centroid at $\left(\bar{x},\bar{y}\right)$ where

$$\bar{x}=\frac{M_y}{M}=\frac{\rho\int_0^2\frac{1}{2}\left(2y+y^2\right)\left(2y-y^2\right)dy}{\rho\int_0^2\left(2y-y^2\right)dy}$$

$$\bar{y}=\frac{M_x}{M}=\frac{\rho\int_0^2 y\left(2y-y^2\right)dy}{\rho\int_0^2\left(2y-y^2\right)dy}$$

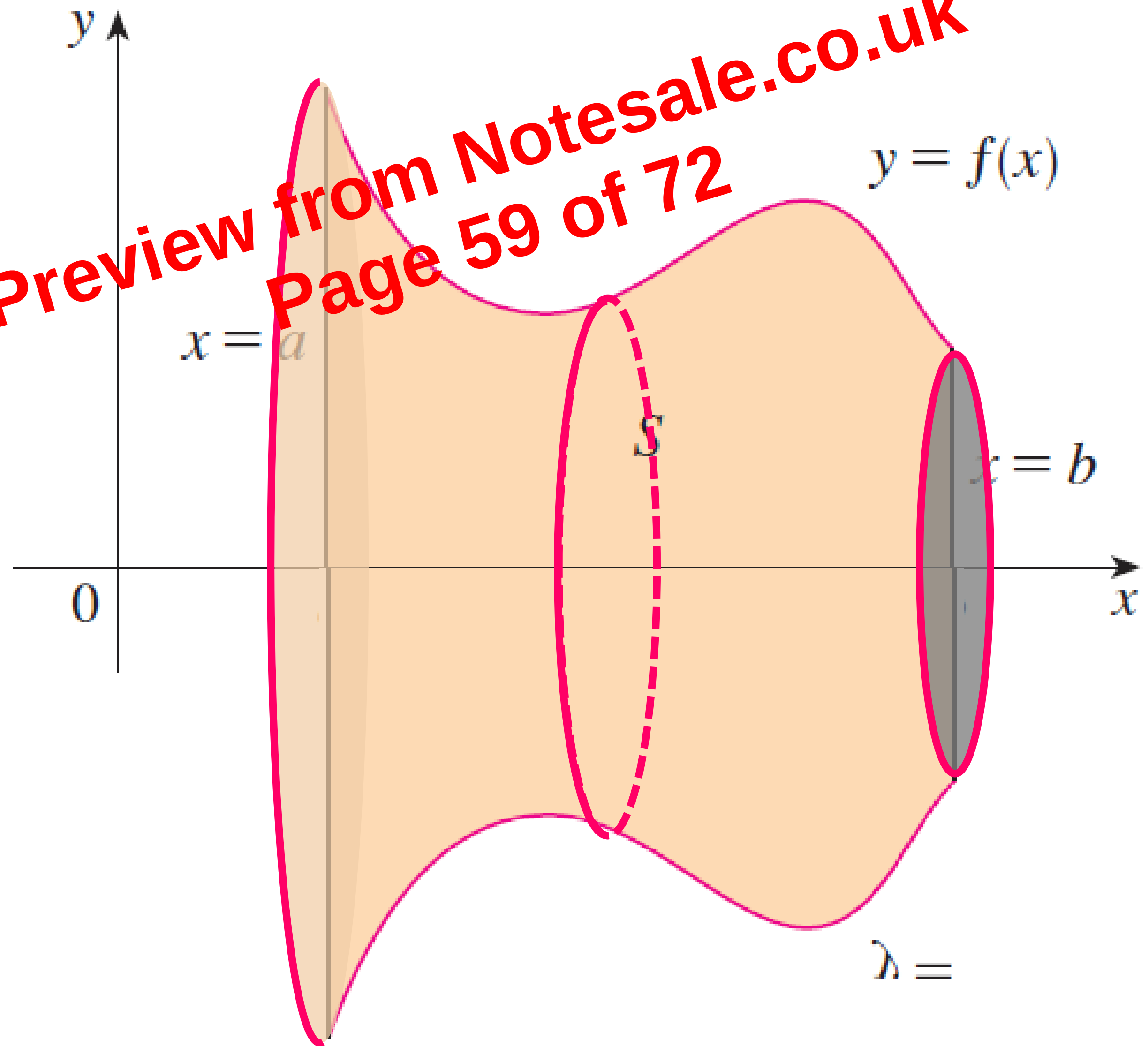
Unit 3.4.3

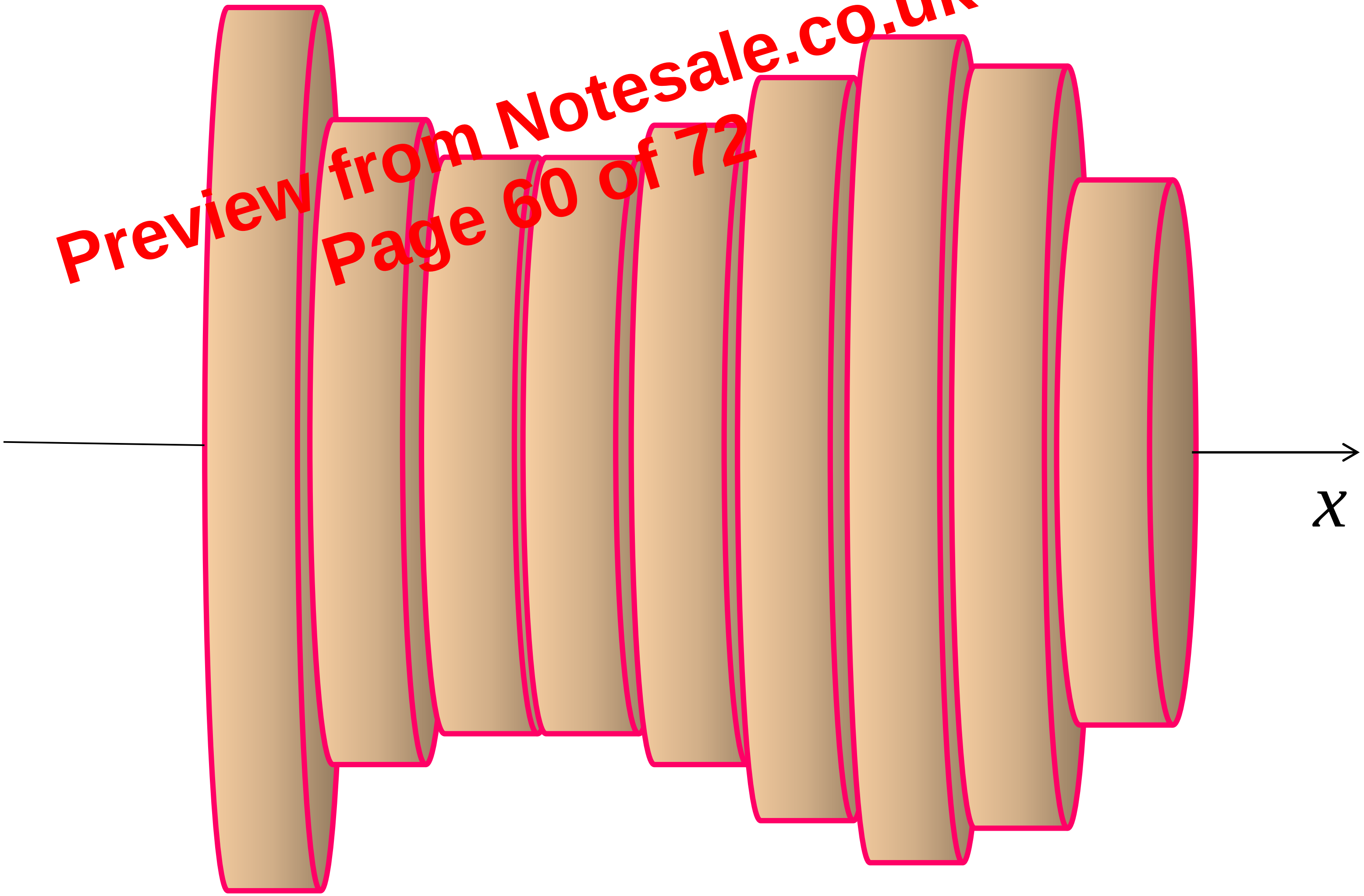
Center of Mass of a Solid of Revolution

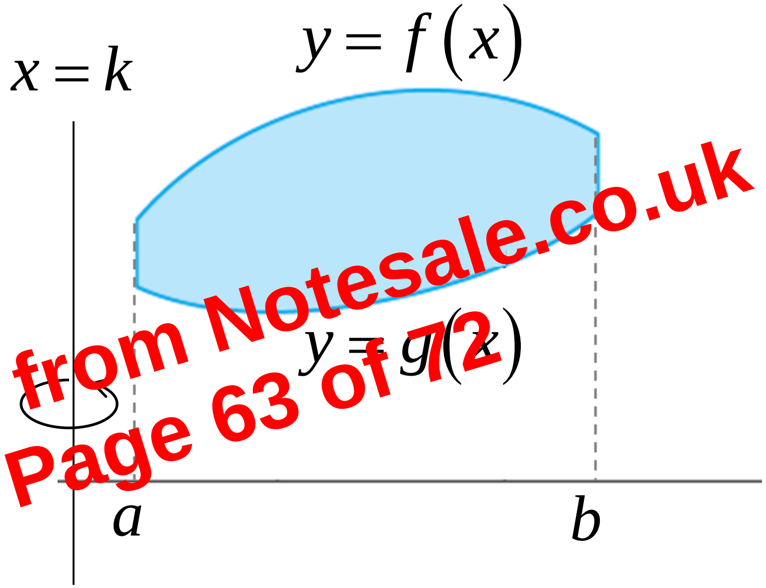
The center of mass of the system is at $\left(\bar{x}, \bar{y}, \bar{z}\right)$

where

$$\bar{x} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i} \qquad \bar{y} = \frac{\sum_{i=1}^{n} m_i y_i}{\sum_{i=1}^{n} m_i} \qquad \bar{z} = \frac{\sum_{i=1}^{n} m_i z_i}{\sum_{i=1}^{n} m_i}$$


y
y = f(x)
x = a
S
x = b
0
x


x



$$\text{Volume} = \int_a^b V(x)\,dx$$

The centroid of the solid is at $(\bar{x}, \bar{y}, \bar{z})$ where

$$\bar{x} = k \qquad \bar{y} = \frac{\rho\int_a^b \frac{1}{2}\left(f(x)+g(x)\right)V(x)\,dx}{\rho\int_a^b V(x)\,dx} \qquad \bar{z} = 0$$

Example 3.4.3.2

Set up the expressions in terms of x that give the centroid of the solid generated when the shaded region is revolved about $x = 3$.