Portfolio Risk and Return

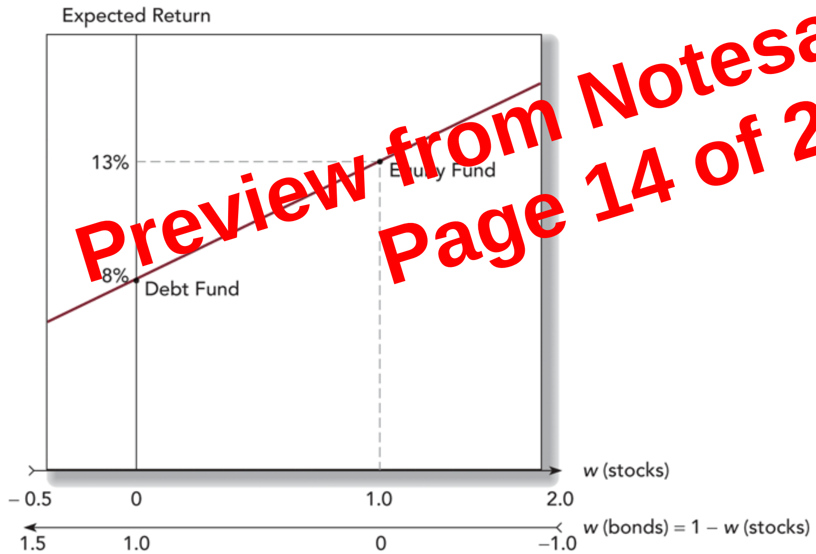

FIGURE 7.3 Portfolio expected return as a function of investment proportions

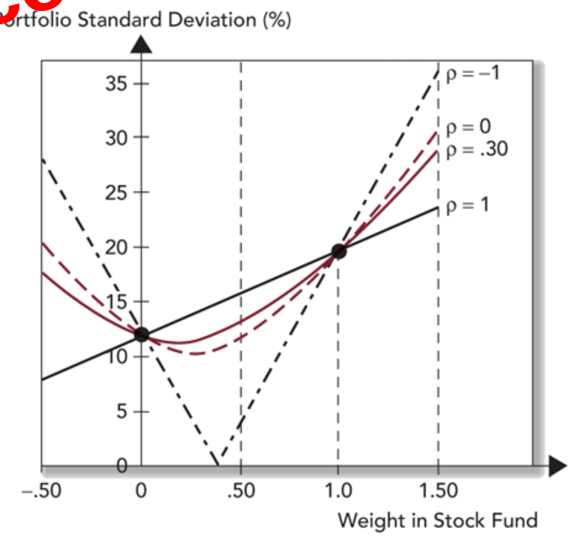

FIGURE 7.4 Portfolio standard deviation as a function of investment proportions

- Here we plot the Expected Returns and Standard Deviations in the previous table as a function of portfolio weights
- Notice that except in the case of perfect correlation, portfolio standard deviation can be made smaller than that of either of the individual component assets by choosing the "right" weights (choosing the "right" weights means minimizing the portfolio variance with respect to the weights)
- The implication of this result is that diversification is *always* a good idea

Two Risky Assets and a Risk-Free Asset

- Notice that the optimal risky portfolio does *not* depend on the risk aversion of the investor!
- Does that mean that the investor will put all his money in Portfolio P?
  - No. His risk aversion will now determine the allocation between the risk-free asset and the optimal risky portfolio.
- The investor can set his desired level of expected return or the standard deviation he is comfortable with, which will pin down the actual allocation
- Alternatively, we can assign a risk aversion parameter and a utility function to calculate the amount he should be investing in the risky asset

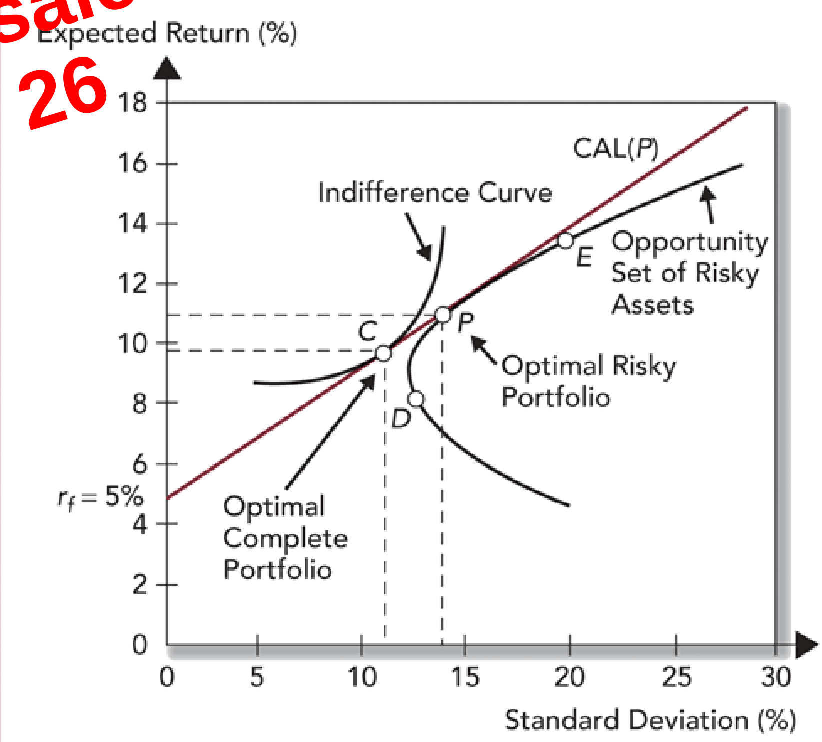

FIGURE 7.8 Determination of the optimal complete portfolio

Two Risky Assets and a Risk-Free Asset

- **By using Excel's Solver (or the formula given in the book) we solved for the optimal weights of the risky portfolio, P:**

  $w_E = 0.6$ and $w_D = 0.4$

  $E(r_p) = 11\%$, $\sigma_p = 14.2\%$

- **Take the investor's risk aversion coefficient A as 4, the risk-free rate as 5%, and calculate his optimal portfolio allocation**

  $y^* = [E(r_p) - r_f]/A\sigma^2_p$

  $y^* = (0.11 – 0.05)/(4^*0.142^2)$

  $y^* = 0.7439$

- **Putting those two solutions together:**

  Portfolio P consists of 40% bonds hence the fraction of the complete portfolio that will be invested in bonds is 0.40*0.7439 = 0.2976.

  Likewise, the fraction invested in stocks is 0.60*0.7439 = 0.4463

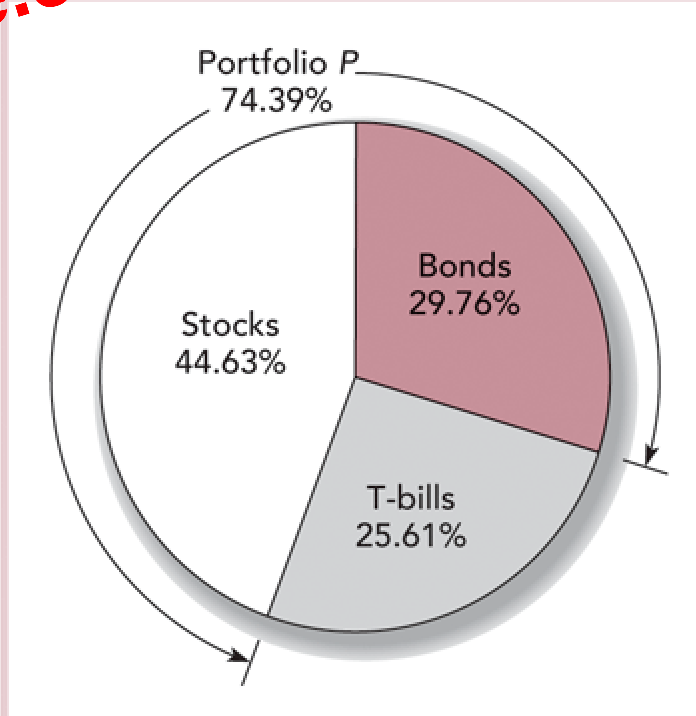

FIGURE 7.9 The proportions of the optimal complete portfolio

The Power of Diversification

- We now have the tools to quantify the benefits of diversification that we showed graphically at the beginning of this lecture. Assume the simplest case: all stocks have the same standard deviation and *zero correlation* with each other.
  - Most people facing this situation might think that it should not matter which stock or stocks you choose; after all they are all identical in terms of risk and do not affect each other, right?
  - Wrong. In a big way.
- Let's put an equal weight (1/n) on each stock (just to simplify the math)
- Because all covariances are zero, the variance of the portfolio is the sum of the variances of the individual stocks

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + \ldots + w_n^2\sigma_n^2$$

$$\sigma_p^2 = \frac{1}{n^2}\sum_{i=1}^{n}\sigma^2$$

$$\sigma_p^2 = \frac{\sigma^2}{n}$$

- What happens as n increases? Goes to infinity?