9) **Equally likely outcomes:**

Outcomes are said to be equally likely if none of them can be expected in preference to the other ~~by~~ while performing the random experiment under consideration.

In experiments like tossing of a coin, rolling of a die etc., if the outcomes may be considered to be equally likely, then the point on the die is said to be fair, symmetric, unbiased, balanced, true etc.

10) **Classical definition of probability (due to Laplace)**

Suppose

1) The sample space Ω w.r.t. a given random experiment is finite. [Let N be the total number of mutually exclusive and exhaustive outcomes in Ω]

2) All the outcomes are equally likely.

Given an event A, let $n(A)$ be the number of outcomes favourable to A. Then by classical definition, the probability of A is given by $P(A) = \dfrac{n(A)}{N}$

Important Rules:

1) Associative Rule

a) $(A \cup B) \cup C = A \cup (B \cup C)$

b) $(A \cap B) \cap C = A \cap (B \cap C)$

2) Commutative Rule

a) $A \cup B = B \cup A$

b) $A \cap B = B \cap A$

3) Distributive Rule

(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

4) De Morgan's Rule

(a) $(A \cup B)^c = A^c \cap B^c$

(b) $(A \cap B)^c = A^c \cup B^c$

Combinatorial Analysis

1) Let us first consider the following problem: Suppose a communication system consists of n antennas which are to be arranged in a linear fashion. The system remains functional as long as no two consecutive antennas are defective. If m out of n antennas are defective, what is the probability that the system will remain functional?

Let us take $n=4$, $m=2$. We shall denote by '1' are non-defective antenna and by '0' a defective one. The possible arrangements may be listed as shown below:

1100
1010
1001
0011
0101
0110

Thus, there are in all six possible arrangements of which for three arrangements the system remains functional.

no. of ways in which a committee of r members may be formed out of n members is ${}^nC_r$. Again a chairperson may be selected out of these r members in r ways. Hence, the ~~total~~ no. of committees consisting of r members may be formed is $r\,{}^nC_r$. Consequently, the total no. of committees consisting of atleast one member that can be formed is

$$\sum_{r=1}^{n} r\binom{n}{r} = \sum_{r=0}^{n} r\binom{n}{r}$$

Again, out of the n members of the ~~locat~~ locality, a chairperson may be selected in n ways, then for each of the other members, there are two options — he/she may belong to a committee or not. Thus the total no. of ways in which the other ~~n~~ $n-1$ members may belong to a committee is $\underbrace{2\times2\times\cdots\times2}_{(n-1)\text{ times}} = 2^{n-1}$. Thus the total no. of committees may be formed is $n\,2^{n-1}$.

$$\therefore \sum_{r=0}^{n} r\binom{n}{r} = n\,2^{n-1}$$