Also, the events A_i's and B_i's are independent.

Now, $P(A \text{ wins}) = P[A_1 \cup (A_1^c \cap B_2^c \cap A_3) \cup (A_1^c \cap B_2^c \cap A_3^c \cap B_4^c \cap A_5) \cup \ldots]$

$$= P(A_1) + P(A_1^c) \cdot P(B_2^c) P(A_3) + P(A_1^c) P(B_2^c) P(A_3^c) P(B_4^c) P(A_5) + \ldots$$

$$= p + (1-p)(1-p)p + (1-p)^4 p + \cdots$$

$$= p\left[1 + (1-p)^2 + (1-p)^4 + \cdots\right]$$

$$= p \frac{1}{1-(1-p)^2}$$

$$= \frac{p}{2p - p^2}$$

$$= \frac{1}{2-p}$$

$\therefore 2 - p < 2$

$\Rightarrow \frac{1}{2-p} > \frac{1}{2}$

$\therefore P(A \text{ wins}) > \frac{1}{2}$

13)(b) From 10 +ve and 6 −ve numbers, 3 numbers are chosen at random without repetition. What is the prob. that their product is a negative number?

— 3 nos. can be chosen out of $(10+6) = 16$ nos. in $\binom{16}{3}$ ways.
Those 3 nos. can either be all negative or one negative and two positive.

$$\therefore \frac{p_{i+1}}{\beta^{i+1}} = \frac{\alpha}{\beta^{i+1}} + \frac{p_i}{\beta^i}$$

We similarly have,

$$\frac{p_i}{\beta^i} = \frac{\alpha}{\beta^i} + \frac{p_{i-1}}{\beta^{i-1}}$$

$$\Rightarrow \frac{p_{i-1}}{\beta^{i-1}} = \frac{\alpha}{\beta^{i-1}} + \frac{p_{i-2}}{\beta^{i-2}}$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$\frac{p_2}{\beta^2} = \frac{\alpha}{\beta^2} + \frac{p_1}{\beta}$$

Adding all the above eq^{n}s we get

$$\frac{p_{i+1}}{\beta^{i+1}} = \frac{\alpha}{\beta^{i+1}}\left\{1+\beta+\beta^2+\dots+\beta^{i-1}\right\} + \frac{p_1}{\beta}$$

$$\Rightarrow p_{i+1} = \alpha\left(\frac{1-\beta^i}{1-\beta}\right) + p_1\beta^i$$

$$= \frac{a}{a+b+1} \times \frac{a+b+1}{a+b}\left(1-\beta^i\right) + \frac{a}{a+b}\beta^i$$

$$= \frac{a}{a+b}$$

Hence in general, $p_n = \frac{a}{a+b}$

2) Let A_i $(i=1,2,\dots,r)$ be mutually exclusive and exhaustive events

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defined on a prob. space (Ω, A, P) such that $P(A_i) > 0 \ \forall i$. Further, let B & C be 2 events such that $P(A_i \cap B) > 0 \ \forall i$.
Show that

$$P(C|B) = \frac{\sum_{i=1}^{r} P(A_i)P(B|A_i)P(C|A_i \cap B)}{\sum_{i=1}^{r} P(A_i)P(B|A_i)}$$

[The above result is known as Extended Bayes Theorem]

From theorem of total probability,

$$P(B) = \sum_{i=1}^{r} P(B|A_i)P(A_i) \quad \cdots (2)$$

From theorem of compound probability,

$$P(C \cap B) = P\left(B \cap C \cap \left[\bigcup_{i=1}^{r} A_i\right]\right)$$

$$= P\left(\bigcup_{i=1}^{r} [B \cap C \cap A_i]\right)$$

$$= \sum_{i=1}^{r} P(A_i \cap B \cap C)$$

$$= \sum_{i=1}^{r} P(A_i)P(B|A_i)P(C|A_i \cap B)$$