Chapter No 4 Quadratic Equations

Quadratic Equation

An equation containing one or more terms in which the variable is raised to maximum positive power two. In general;

$$ax^2 + bx + c = 0 \quad \text{where } a \neq 0$$

is called Quadratic Equation in variable x.

3 Methods.

To solve Quadratic Equation there are three different methods named as;

1. *Factorization method.*
2. *Completing Square method.*
3. *Quadratic Formula method.*

Example 1

Solve by Factorization $x^2 - 7x + 10 = 0$

$$x^2 - 2x - 5x + 10 = 0$$
$$x(x-2) - 5(x-2) = 0$$
$$(x-2)(x-5) = 0$$

$10x^2$

$-2x \quad -5x$

$x - 2 = 0 \quad , \quad x - 5 = 0$

$\Rightarrow x = 2 \quad , \Rightarrow x = 5$

$\{2, 5\}$

Example 2

Solve $x^2 + 4x - 437 = 0$ by Completing Sq.

$$x^2 + 4x = 437$$

Adding $\left(\frac{4}{2}\right)^2 = (2)^2$ on both sides.

$$x^2 + 4x + (2)^2 = 437 + (2)^2$$
$$(x+2)^2 = 437 + 4$$
$$(x+2)^2 = 441$$
$$x + 2 = \pm 21 \qquad \because \sqrt{441} = 21$$

$x + 2 = 21 \quad , \quad x + 2 = -21$

$x = 21 - 2 \quad , \quad x = -21 - 2$

$x = 19 \quad , \quad x = -23$

$\{19, -23\}$

Example 3

Solve $6x^2 + x - 15 = 0$ by Q. Formula

Comparing $6x^2 + x - 15 = 0$

with $ax^2 + bx + c = 0$

we have $a = 6,\ b = 1,\ c = -15$

By using Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(6)(-15)}}{2(6)}$$

$$x = \frac{-1 \pm \sqrt{1 + 360}}{12} \Rightarrow x = \frac{-1 \pm \sqrt{361}}{12}$$

$$x = \frac{-1 \pm 19}{12} \qquad \because \sqrt{361} = 19$$

$$x = \frac{-1 + 19}{12} \quad , \quad x = \frac{-1 - 19}{12}$$

$$x = \frac{18}{12} \quad , \quad x = \frac{-20}{12}$$

$$x = \frac{3}{2} \quad , \quad x = -\frac{5}{3} \qquad \left\{\frac{3}{2}, -\frac{5}{3}\right\}$$

Example 4

Solve $8x^2 - 14x - 15 = 0$ by Quadratic For

Comparing $8x^2 - 14x - 15 = 0$

with $ax^2 + bx + c = 0$

We have $a = 8,\ b = -14,\ c = -15$

By using $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-(-14) \pm \sqrt{(-14)^2 - 4(8)(-15)}}{2(8)}$$

$$x = \frac{14 \pm \sqrt{196 + 480}}{16} \Rightarrow x = \frac{14 \pm \sqrt{676}}{16}$$

$$x = \frac{14 \pm 26}{16} \qquad \because \sqrt{676} = 26$$

$$x = \frac{14 + 26}{16} \quad , \quad x = \frac{14 - 26}{16}$$

$$x = \frac{40}{16} \quad , \quad x = \frac{-12}{16}$$

$$x = \frac{5}{2} \quad , \quad x = -\frac{3}{4} \qquad \left\{\frac{5}{2}, -\frac{3}{4}\right\}$$