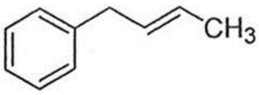
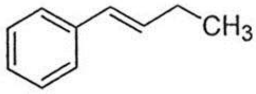
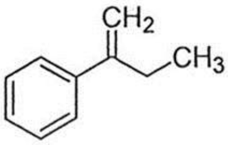
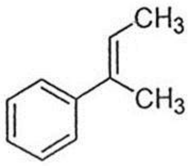
40. The major compound **Y** is

(A) 

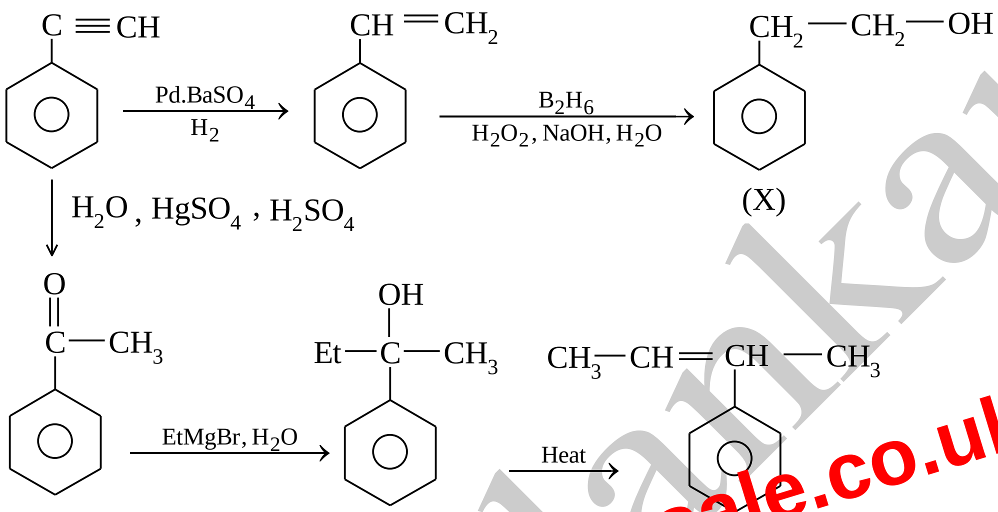
(B) 

(C) 

(D) 

40. (D)

Solution : Q. 39 & 40



PART III - MATHEMATICS

Section 1 (Maximum Marks : 32)

- This section contains **EIGHT** questions
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- Marking scheme :
  +4 If the bubble corresponding to the answer is darkened
  0 In all other cases

41. Suppose that all the terms of an arithmetic progression (A.P.) are natural numbers. If the ratio of the sum of the first seven terms to the sum of the first eleven terms is 6 : 11 and the seventh term lies in between 130 and 140, then the common difference of this A.P. is

41. [9]

Let the first term is a and common difference is d.

$$\frac{S_7}{S_{11}} = \frac{6}{11}$$

$$\frac{\frac{7}{2}\{2a+6d\}}{\frac{11}{2}\{2a+10d\}} = \frac{6}{11}$$

$$\frac{2a+6d}{2a+10d} = \frac{6}{7}$$

46. Let $f:\mathbb{R}\to\mathbb{R}$ be a continuous odd function, which vanishes exactly at one point and $f(1)=\frac{1}{2}$. Suppose that $F(x)=\int_{-1}^{x} f(t)\,dt$ for all $x\in[-1, 2]$ and $G(x)=\int_{-1}^{x} t\left|f(f(t))\right|dt$ for all $x\in[-1, 2]$. If $\lim\limits_{x\to1}\frac{F(x)}{G(x)}=\frac{1}{14}$, then the value of $f\left(\frac{1}{2}\right)$ is

46. [7]

$$\lim_{x\to1}\frac{\int_{-1}^{x} f(t)\,dt}{\int_{-1}^{x} t\,|f(t)|\,dt} = \frac{f(x)}{x\,f(f(x))} = \frac{1}{14}$$

$$\Rightarrow \frac{\frac{1}{2}}{f\left(\frac{1}{2}\right)} = \frac{1}{14} \qquad \Rightarrow f\left(\frac{1}{2}\right) = 7$$

47. Suppose that $\vec{p},\vec{q}$ and $\vec{r}$ are three non-coplanar vectors in $\mathbb{R}^3$. Let the components of a vector $\vec{s}$ along $\vec{p},\vec{q}$ and $\vec{r}$ be 4, 3 and 5, respectively. If the components of this vector $\vec{s}$ along $(-\vec{p}+\vec{q}+\vec{r}),(\vec{p}-\vec{q}+\vec{r})$ and $(-\vec{p}-\vec{q}+\vec{r})$ are x, y and z, respectively, then the value of 2x + y + z is

47. [9]

$\vec{s} = 4\vec{p}+3\vec{q}+5\vec{r}$ … (i)

$\vec{s} = (-\vec{p}+\vec{q}+\vec{r})x+(\vec{p}-\vec{q}+\vec{r})y+(-\vec{p}-\vec{q}+\vec{r})z$

$\vec{s} = (-x + y - z)\,\vec{p} + (x - y - z)\,\vec{q} + (x + y + z)\,\vec{r}$ … (ii)

$-x + y - z = 4$ … (iii)

$x - y - z = 3$ … (iv)

$x + y + z = 5$ … (v)

$x = 4,\ \ y = \frac{9}{2},\ z = -\frac{7}{2}$

$2x + y + z = 9$

48. For any integer k, let $a_k=\cos\left(\frac{k\pi}{7}\right)+i\sin\left(\frac{k\pi}{7}\right)$, where $i=\sqrt{-1}$. The value of the expression $\dfrac{\sum_{k=1}^{12}\left|\alpha_{k+1}-\alpha_k\right|}{\sum_{k=1}^{3}\left|\alpha_{4k-1}-\alpha_{4k-2}\right|}$ is

48. [4]

$$\frac{\sum_{K=1}^{12}\left|e^{\frac{i(K+1)\pi}{7}}-e^{\frac{iK\pi}{7}}\right|}{\sum_{K=1}^{3}\left|e^{\frac{i(4K-1)\pi}{7}}-e^{\frac{i(4K-2)\pi}{7}}\right|} = \frac{\sum_{K=1}^{12}\left|e^{\frac{iK\pi}{7}}\right|\cdot\left|e^{\frac{i\pi}{7}}-1\right|}{\sum_{K=1}^{3}\left|e^{\frac{i(4K-1)\pi}{7}}\right|\cdot\left|e^{\frac{i\pi}{7}}-1\right|} = \frac{\sum_{K=1}^{12}\left|e^{\frac{i\pi}{7}}-1\right|}{\sum_{K=1}^{3}\left|e^{\frac{i\pi}{7}}-1\right|} = \frac{12\left|e^{\frac{i\pi}{7}}-1\right|}{3\left|e^{\frac{i\pi}{7}}-1\right|} = 4$$

Also, $m = \frac{1}{3}y_1 = \frac{1}{3}\sqrt{x_1^2 - 1}$

$$\frac{dm}{dx_1} = \frac{x_1}{3\sqrt{x_1^2 - 1}}$$

56. The option(s) with the values of a and L that satisfy the following equation is(are)

$$\frac{\int_0^{4\pi} e^t(\sin^6 at + \cos^4 at)\,dt}{\int_0^{\pi} e^t(\sin^6 at + \cos^4 at)\,dt} = L?$$

(A) $a = 2,\ L = \frac{e^{4\pi} - 1}{e^{\pi} - 1}$ (B) $a = 2,\ L = \frac{e^{4\pi} + 1}{e^{\pi} + 1}$

(C) $a = 4,\ L = \frac{e^{4\pi} - 1}{e^{\pi} - 1}$ (D) $a = 4,\ L = \frac{e^{4\pi} + 1}{e^{\pi} + 1}$

56. (A), (C)

$$\frac{\int_0^{4\pi} e^t(\sin^6(at) + \cos^4(at))\,dt}{\int_0^{\pi} e^t(\sin^6(at) + \cos^4(at))dt}$$

Let $f(t) = e^t\,(\sin^6(at) + \cos^6(at))$

$$F(k\pi + t) = e^{k\pi + t}(\sin^6(a(k\pi + t)) + \cos^6(a(k\pi + t)))$$

$$= e^{k\pi} f(t) \quad \ldots \text{(for even values of a)}$$

$$\therefore \frac{\int_0^{4\pi} f(t)\,dt}{\int_0^{\pi} f(t)\,dt} = \frac{(1 + e^{\pi} + e^{2\pi} + e^{3\pi})\int_0^{\pi} f(t)\,dt}{\int_0^{\pi} f(t)\,dt} = \frac{e^{4\pi} - 1}{e^{\pi} - 1}$$

Section 3 (Maximum Marks : 16)

- This section contains **TWO** paragraph.
- Based on each paragraph, there will be **TWO** questions
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- Marking scheme :

  +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened

  0 If none of the bubbles is darkened

  –2 In all other cases

PARAGRAPH 1

Let $F : \mathbb{R} \to \mathbb{R}$ be a thrice differentiable function. Suppose that $F(1) = 0$, $F(3) = -4$ and $F'(x) < 0$ for all $x \in (1/2, 3)$. Let $f(x) = xF(x)$ for all $x \in \mathbb{R}$.

57. The Correct statement(s) is(are)

(A) $f'(1) < 0$

(B) $f(2) < 0$

(C) $f'(x) \neq 0$ for any $x \in (1, 3)$

(D) $f'(x) = 0$ for some $x \in (1, 3)$

57. (A), (B), (C)

$f(x) = x F(x)$

$f'(x) = F(x) + x F'(x)$

$\therefore \ f'(1) = F(1) + F'(1) = F'(1) < 0 \qquad (\because F'(x) < 0 \ \forall \ x \in (1/2, 3)$

$f(2) = 2F(2) < 0 \qquad (\because F(2) < 0)$

$f'(x) = F(x) + x F'(x) < 0 \quad \forall \ x \in (1, 3)$

$(\because F(x) < 0$ and $x F'(x) < 0$ for all $x \in (1, 3))$

$\therefore \ f'(x) \neq 0$ for any $x \in (1, 3)$

Hence (A), (B), (C) are correct.

58. If $\int_1^3 x^2 F'(x)\,dx = -12$ and $\int_1^3 x^3 F''(x)\,dx = 40$, then the correct expression(s) is(are)

(A) $9f'(3) + f'(1) - 32 = 0$

(B) $\int_1^3 f(x)\,dx = 12$

(C) $9f'(3) - f'(1) + 32 = 0$

(D) $\int_1^3 f(x)\,dx = -12$

58. (C), (D)

$$\int_1^3 f(x)\,dx = \int_1^3 x F(x)\,dx = \left(F(x)\cdot\frac{x^2}{2}\right)_1^3 - \int_1^3 \frac{x^2}{2}\cdot F'(x)\,dx$$

$$= \frac{9}{2}(-4) - 0 - \frac{1}{2}(-12) = -12$$

$$\int_1^3 x^3 F''(x) = 40 \Rightarrow \left(x^3 F'(x)\right)_1^3 - \int_1^3 3x^2 F'(x) = 40$$

$$\Rightarrow 27F'(3) - F'(1) = 4 \ (1)$$

Also, $f'(3) = F(3) + 3F'(3)$

$f'(1) = F'(1)$

(C) $\Rightarrow 9f'(3) - f'(1) + 32$

$= 9F(3) + 27F'(3) - F'(1) + 32$

$= 27F'(3) - F'(1) - 4 = 0 \qquad (\because F(3) = -4)$

PARAGRAPH 2

Let n_1 and n_2 be the number of red and black balls, respectively, in box I. Let n_3 and n_4 be the number of red and black balls, respectively, in box II.

59. One of the two boxes, box I and box II, was selected at random and a ball was drawn randomly out of this box. The ball was found to be red. If the probability that this red ball was drawn from box II is $\frac{1}{3}$, then the correct option(s) with the possible values of n_1, n_2, n_3 and n_4 is(are)

(A) $n_1 = 3, n_2 = 3, n_3 = 5, n_4 = 15$
(B) $n_1 = 3, n_2 = 6, n_3 = 10, n_4 = 50$
(C) $n_1 = 8, n_2 = 6, n_3 = 5, n_4 = 20$
(D) $n_1 = 6, n_2 = 12, n_3 = 5, n_4 = 20$

59. (A), (B)

$P(B_1) = 1/2$ $\longrightarrow$ $P(R/B_1) = \dfrac{n_1}{n_1 + n_2}$

$P(B_2) = 1/2$ $\longrightarrow$ $P(R/B_2) = \dfrac{n_3}{n_3 + n_4}$

$$P\left(\frac{B_2}{R}\right) = \frac{1}{3}$$

$$\frac{\frac{1}{2}\times\left(\frac{n_3}{n_3+n_4}\right)}{\frac{1}{2}\left(\frac{n_3}{n_3+n_4}\right)+\frac{1}{2}\left(\frac{n_1}{n_1+n_2}\right)} = \frac{1}{3}$$

60. A ball is drawn at random from box I and transferred to box II. If the probability of drawing a red ball from box I, after this transfer, is $\frac{1}{3}$, then the correct option(s) with the possible values of n_1 and n_2 is(are)

(A) $n_1 = 4$ and $n_2 = 6$
(B) $n_1 = 2$ and $n_2 = 3$
(C) $n_1 = 10$ and $n_2 = 20$
(D) $n_1 = 3$ and $n_2 = 6$

60. (C), (D)

$$P(E) = \frac{1}{3}$$

$$\left(\frac{n_1}{n_1+n_2}\right)\left(\frac{n_1-1}{n_1+n_2-1}\right)+\left(\frac{n_2}{n_1+n_2}\right)\left(\frac{n_1}{n_1+n_2-1}\right) = \frac{1}{3}$$

Among the given options, (C) and (D) satisfies the equation.

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