Basic Math & Pre-Algebra FOR DUMMIES®

by Mark Zegarelli

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Foolish Assumptions

If you're planning to read this book, you're likely

- A student who wants a solid understanding of the basics of math for a class or test you're taking
- An adult who wants to improve skills in arithmetic, fractions, decimals, percentages, weights and measures, geometry, algebra, and so on for when you have to use math in the real world
- Someone who wants a refresher so you can help another person understand math

My only assumption about your skill level is that you can add, subtract, multiply, and divide. So to find out whether you're ready for this book, take this simple test:

$5 + 6 = \_\_$

$10 - 7 = \_\_$

$3 \times 5 = \_\_$

$20 \div 4 = \_\_$

If you can answer these four questions, you're ready to begin.

How This Book Is Organized

This book is organized into six parts, starting you off at the very beginning of math — with topics such as counting and the number line — and taking you all the way into algebra.

Part I: Arming Yourself with the Basics of Basic Math

In Part I, I take what you already know about math and put it in perspective.

Chapter 1 gives you a brief history of what numbers are and where they came from. I discuss how number sequences arise. I show you how important sets of numbers — such as the counting numbers, the integers, and the rational numbers — all fit together on the number line. I also show you how to use the number line to perform basic arithmetic.

In this part . . .

You already know more about math than you think you know. Here, you review and gain perspective on basic math ideas such as number patterns, the number line, how place value based on the number ten turns digits into numbers, and how zero functions as a placeholder. I also reintroduce you to what I call the Big Four operations (adding, subtracting, multiplying, and dividing).

Multiplying quickly with exponents

Here's an old question that still causes surprises: Suppose you took a job that paid you just 1 penny the first day, 2 pennies the second day, 4 pennies the third day, and so on, doubling the amount every day, like this:

1 2 4 8 16 32 64 128 256 512 ...

As you can see, in the first ten days of work, you would've earned a little more than $10 (actually, $10.23 — but who's counting?). How much would you earn in 30 days? Your answer may well be, "I wouldn't take a lousy job like that in the first place." At first glance, this looks like a good answer, but here's a glimpse at your second ten days' earnings:

... 1,024 2,048 4,096 8,192 16,384 32,768 65,536 131,072 262,144 524,288 ...

By the end of the second 10 days, your total earnings would be over $10,000. And by the end of the third week, your earnings would top out around $10,000,000! How does this happen? Through the magic of exponents (also called *powers*). Each new number in the sequence is obtained by multiplying the previous number by 2:

$2^1 = 2 = 2$

$2^2 = 2 \times 2 = 4$

$2^3 = 2 \times 2 \times 2 = 8$

$2^4 = 2 \times 2 \times 2 \times 2 = 16$

As you can see, the notation 2^4 means *multiply 2 by itself 4 times.*

You can use exponents on numbers other than 2. Here's another sequence you may be familiar with:

1 10 100 1,000 10,000 100,000 1,000,000...

In this sequence, every number is 10 times greater than the number before it. You can also generate these numbers using exponents:

$10^1 = 10 = 10$

$10^2 = 10 \times 10 = 100$

$10^3 = 10 \times 10 \times 10 = 1,000$

$10^4 = 10 \times 10 \times 10 \times 10 = 10,000$

Table 2-3 3,040,070 Displayed with Two Leading Zeros

Millions			*Thousands*			*Ones*		
Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
0	0	3	0	4	0	0	7	0

The first two 0s in the number are leading zeros because they appear to the left of the 3. You can drop these 0s from the number, leaving you with 3,040,070. The remaining 0s are all to the right of the 3, so they're placeholders — be sure to write them in.

Reading long numbers

When you write a long number, you use commas to separate periods. *Periods* are simply groups of three numbers. They help make long numbers more readable. For example, here's about as long a number as you'll ever see:

234,845,021,349,230,467,304

Table 2-4 shows a larger version of the place-value chart.

Table 2-4 A Place-Value Chart Separated into Periods

Quintillions	Quadrillions	Trillions	Billions	Millions	Thousands	Ones
234	845	021	349	230	467	304

This version of the chart helps you read the number. Begin all the way to the left and read, "Two hundred thirty-four quintillion, eight hundred forty-five quadrillion, twenty-one trillion, three hundred forty-nine billion, two hundred thirty million, four hundred sixty-seven thousand, three hundred four."

REMEMBER

When you read and write whole numbers, don't say the word *and.* In math, the word *and* means you have a decimal point. That's why when you write a check, you save the word *and* for the number of cents, which is expressed as a decimal or fraction. (I discuss decimals in Chapter 11.)

Occasionally, a small change to the ones and tens digits affects the other digits. (This is a lot like when the odometer in your car rolls a bunch of 9s over to 0s.) For example:

899 → 900 1,097 → 1,100 9,995 → 10,000

Rounding numbers to the nearest hundred and beyond

To round numbers to the nearest hundred, or thousand, or beyond, focus only on two digits: the digit in the place you're rounding to and the digit to its immediate right. Change all other digits to the right of these two digits to 0s. For example, suppose you want to round 642 to the nearest hundred. Focus on the hundreds digit (6) and the digit to its immediate right (4):

642

I've underlined these two digits. Now just round these two digits as if you were rounding to the nearest ten, and change the digit to the right of them to a 0:

642 → 600

Here are a few more examples of rounding numbers to the nearest hundred:

7,891 → 7,900 15,753 → 15,800 99,961 → 100,000

When rounding numbers to the nearest thousand, underline the thousands digit and the digit to its immediate right. Round the number by focusing only on the two underlined digits and, when you're done, change all digits to the right of these to 0s:

4,984 → 5,000 78,521 → 79,000 1,099,304 → 1,099,000

Even when rounding to the nearest million, the same rules apply:

1,234,567 → 1,000,000 78,883,958 → 79,000,000

Estimating value to make problems easier

After you know how to round numbers, you can use this skill in estimating values. Estimating saves you time by allowing you to avoid complicated computations and still get an approximate answer to a problem.

REMEMBER

When you get an approximate answer, you don't use an equal sign; instead, you use this wavy symbol, which means *is approximately equal to:* ≈.

Multiplying

Multiplication is often described as a sort of shorthand for repeated addition. For example,

4×3 means *add 4 to itself 3 times:* $4 + 4 + 4 = 12$

9×6 means *add 9 to itself 6 times:* $9 + 9 + 9 + 9 + 9 + 9 = 54$

100×2 means *add 100 to itself 2 times:* $100 + 100 = 200$

Although multiplication isn't as warm and fuzzy as addition, it's a great timesaver. For example, suppose you coach a Little League baseball team and you've just won a game against the toughest team in the league. As a reward, you promised to buy three hot dogs for each of the nine players on the team. To find out how many hot dogs you need, you could add 3 together 9 times. Or you can save time by multiplying 3 times 9, which gives you 27. Therefore, you need 27 hot dogs (plus a whole lot of mustard and sauerkraut).

When you multiply two numbers, the two numbers that you're multiplying are called *factors,* and the result is the *product.*

In multiplication, the first number is also called the *multiplicand* and the second number is the *multiplier.* But almost nobody ever remembers these words.

Signs of the times

When you're first introduced to multiplication, you use the *times sign* (×). However, algebra uses the letter *x* a lot, which looks similar to the times sign, so people often choose to use other multiplication symbols for clarity. As you move onwards and upwards on your math journey, you should be aware of the conventions I discuss in the following sections.

Arriving on the dot

In math beyond arithmetic, the symbol · replaces ×. For example,

$4 \cdot 2 = 8$	means	$4 \times 2 = 8$
$6 \cdot 7 = 42$	means	$6 \times 7 = 42$
$53 \cdot 11 = 583$	means	$53 \times 11 = 583$

These steps are one complete cycle, and to complete the problem you just need to repeat them. Now ask how many times 5 goes into 36 — that is, what's $36 \div 5$? The answer is 7 (with a little left over). Write 7 just above the 6, and then multiply 7×5 to get 35; write the answer under 36:

$$
\begin{array}{r}
17 \\
5\overline{)\,860} \\
\underline{-5} \\
36 \\
\underline{35}
\end{array}
$$

Now subtract to get $36 - 35 = 1$; bring down the 0 next to the 1 to make the new number 10:

$$
\begin{array}{r}
17 \\
5\overline{)\,860} \\
\underline{-5} \\
36 \\
\underline{-35} \\
10
\end{array}
$$

Another cycle is complete, so begin the next cycle by asking how many times 5 goes into 10 — that is, $10 \div 5$. The answer this time is 2. Write down the 2 in the answer above the 0. Multiply to get $2 \times 5 = 10$, and write this answer below the 10:

$$
\begin{array}{r}
172 \\
5\overline{)\,860} \\
\underline{-5} \\
36 \\
\underline{35} \\
10 \\
\underline{10}
\end{array}
$$

Now subtract $10 - 10 = 0$. Because you have no more numbers to bring down, you're finished, and here's the answer (that is, the quotient):

$$
\begin{array}{rl}
172 & \leftarrow \text{Quotient} \\
5\overline{)\,860} & \\
\underline{-5} & \\
36 & \\
\underline{35} & \\
10 & \\
\underline{-10} & \\
0 &
\end{array}
$$

So $860 \div 5 = 172$.

Adding a negative number

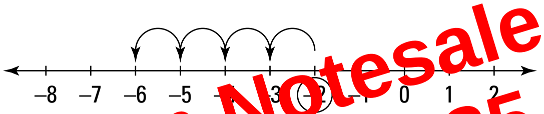
Suppose you want to solve –2 + –4. You already know to start at –2, but where do you go from there? Here's the up and down rule for adding a negative number:

REMEMBER

Adding a negative number is the same as subtracting a positive number — that is, go *down* on the number line.

By this rule, –2 + –4 is the same as –2 – 4, so

Start at –2, down 4



So –2 + (–4) = –6.

TIP

If you rewrite a subtraction problem as an addition problem — for instance, rewriting 3 – 7 as 3 + (–7) — you can use the commutative and associative properties of addition, which I discuss earlier in this chapter. Just remember to keep the negative sign attached to the number when you rearrange: (–7) + 3.

Subtracting a negative number

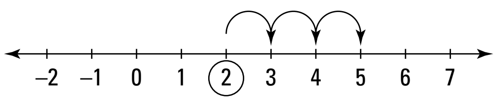
The last rule you need to know is how to subtract a negative number. For example, suppose you want to solve 2 – (–3). Here's the up and down rule:

REMEMBER

Subtracting a negative number is the same as adding a positive number — that is, *up* on the number line.

This rule tells you that 2 – (–3) is the same as 2 + 3, so

Start at 2, up 3



So 2 – (–3) = 5.

TIP

When subtracting negative numbers, you can think of the two minus signs canceling each other out to create a positive.

In many cases, however, multiplying and dividing units is okay. For example, multiplying *units of length* (such as inches, miles, or meters) results in *square units*. For example,

3 inches · 3 inches = 9 *square inches*

10 miles · 5 miles = 50 *square miles*

100 meters · 200 meters = 20,000 *square meters*

You find out more about units of length in Chapter 15. Similarly, here are some examples of when dividing units makes sense:

12 slices of pizza ÷ 4 people = 3 slices of pizza/person

140 miles ÷ 2 hours = 70 miles/hour

In these cases, you read the fraction slash (/) as *per*: slices of pizza *per* person or miles *per* hour. You find out more about multiplying and dividing by units in Chapter 15, when I show you how to convert from one unit of measurement to another.

Understanding Inequalities

Sometimes, you want to talk about when two quantities are different. These statements are called *inequalities.* In this section, I discuss four types of inequalities: ≠ (doesn't equal), < (less than), > (greater than), and ≈ (approximately equals).

Doesn't equal (≠)

The simplest inequality is ≠, which you use when two quantities are not equal. For example,

$2 + 2 \neq 5$

$3 \times 4 \neq 34$

$999{,}999 \neq 1{,}000{,}000$

You can read ≠ as "doesn't equal" or "is not equal to." Therefore, read $2 + 2 \neq 5$ as "two plus two doesn't equal five."

Beyond the Big Four: Exponents, Square Roots, and Absolute Value

In this section, I introduce you to three new operations that you need as you move on with math: exponents, square roots, and absolute value. As with the Big Four operations, these three operations take numbers and tweak them in various ways.

To tell the truth, these three operations have fewer everyday applications than the Big Four. But you'll be seeing a lot more of them as you progress in your study of math. Fortunately, they aren't difficult, so this is a good time to become familiar with them.

Understanding exponents

Exponents (also called *powers*) are shorthand for repeated multiplication. For example, 2^3 means to multiply 2 by itself 3 times. To do that, use the following notation:

$$2^3 = 2 \cdot 2 \cdot 2 = 8$$

In this example, 2 is the *base number* and 3 is the *exponent*. You can read 2^3 as "2 to the third power" or "2 to the power of 3" (or even "2 cubed," which has to do with the formula for finding the value of a cube — see Chapter 16 for details).

Here's another example:

10^5 means to multiply 10 by itself 5 times

That works out like this:

$$10^5 = 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 100{,}000$$

This time, 10 is the base number and 5 is the exponent. Read 10^5 as "10 to the fifth power" or "10 to the power of 5."

TIP

When the base number is 10, figuring out any exponent is easy. Just write down a 1 and that many 0s after it:

$10^2 = 100$	(1 with two 0s)
$10^7 = 10{,}000{,}000$	(1 with seven 0s)
$10^{20} = 100{,}000{,}000{,}000{,}000{,}000{,}000$	(1 with twenty 0s)

Exponents with a base number of 10 are very important in scientific notation, which I cover in Chapter 14.

The most common exponent is the number 2. When you take any whole number to the power of 2, the result is a square number. (For more information on square numbers, see Chapter 1.) For this reason, taking a number to the power of 2 is called *squaring* that number. You can read 3^2 as "three squared," 4^2 as "four squared," and so forth. Here are some squared numbers:

$3^2 = 3 \cdot 3 = 9$

$4^2 = 4 \cdot 4 = 16$

$5^2 = 5 \cdot 5 = 25$

REMEMBER

Any number raised to the 0 power equals 1. So 1^0, 37^0, and $999{,}999^0$ are equivalent, or equal.

Discovering your roots

Earlier in this chapter, in "Knowing Properties of the Big Four Operations," I show you how addition and subtraction are inverse operations. I also show you how multiplication and division are inverse operations. In a similar way, roots are the inverse operation of exponents.

The most common root is the square root. A *square root* undoes an exponent of 2. For example,

$3^2 = 3 \cdot 3 = 9$, so $\sqrt{9} = 3$

$4^2 = 4 \cdot 4 = 16$, so $\sqrt{16} = 4$

$5^2 = 5 \cdot 5 = 25$, so $\sqrt{25} = 5$

You can read the symbol $\sqrt{\ }$ either as "the square root of" or as "radical." So, read $\sqrt{9}$ as either "the square root of 9" or "radical 9."

As you can see, when you take the square root of any square number, the result is the number that you multiplied by itself to get that square number in the first place. For example, to find $\sqrt{100}$, you ask the question, "What number when multiplied by itself equals 100?" The answer in this case is 10, because

$10 \cdot 10 = 100$, so $\sqrt{100} = 10$

You probably won't use square roots too much until you get to algebra, but at that point they become very handy.

First, evaluate the exponent:

$$= 3 + 25 - 6$$

At this point, the expression contains only addition and subtraction, so you can evaluate it from left to right in two steps:

$$= 28 - 6$$

$$= 22$$

So $3 + 5^2 - 6 = 22$.

Understanding order of precedence in expressions with parentheses

In math, parentheses — ( ) — are often used to group together parts of an expression. When it comes to evaluating expressions, here's what you need to know about parentheses:

REMEMBER

To evaluate expressions that contain parentheses,

1. **Evaluate the contents of parentheses, from the inside out.**
2. **Evaluate the rest of the expression.**

Big Four expressions with parentheses

Similarly, suppose you want to evaluate $(1 + 15 \div 5) + (3 - 6) \cdot 5$. This expression contains two sets of parentheses, so evaluate these from left to right. Notice that the first set of parentheses contains a mixed-operator expression, so evaluate this in two steps starting with the division:

$$= (1 + 3) + (3 - 6) \cdot 5$$

$$= 4 + (3 - 6) \cdot 5$$

Now evaluate the contents of the second set of parentheses:

$$= 4 + -3 \cdot 5$$

Now you have a mixed-operator expression, so evaluate the multiplication $(-3 \cdot 5)$ first:

$$= 4 + -15$$

With the parentheses removed, you're ready to evaluate the exponent:

$= 8$

Once in a rare while, the exponent itself contains parentheses. As always, evaluate what's in the parentheses first. For example,

$21^{(19 + \underline{3 \cdot -6})}$

This time, the smaller expression inside the parentheses is a mixed-operator expression. I underlined the part that you need to evaluate first:

$= 21^{(19 + -18)}$

Now you can finish off what's inside the parentheses:

$= 21^{1}$

At this point, all that's left is a very simple exponent:

$= 21$

So $21^{(19 + 3 \cdot -6)} = 21$.

Note: Technically, you don't need to put parentheses around the exponent. If you see an expression in the exponent, treat it as though it had parentheses around it. In other words, $21^{19 + 3 \cdot -6}$ means the same thing as $21^{(19 + 3 \cdot -6)}$.

Expressions with nested parentheses

Occasionally, an expression has *nested parentheses:* one or more sets of parentheses inside another set. Here, I give you the rule for handling nested parentheses.

REMEMBER

When evaluating an expression with nested parentheses, evaluate what's inside the *innermost* set of parentheses first and work your way toward the *outermost* parentheses.

For example, suppose you want to evaluate the following expression:

$2 + (9 - (\underline{7 - 3}))$

I underlined the contents of the innermost set of parentheses, so evaluate these contents first:

$= 2 + (9 - 4)$

However, when the digital root of a number is anything other than 3, 6, or 9, the number isn't divisible by 3. Here's an example:

706: 7 + 0 + 6 = 13; 1 + 3 = 4

Because the digital root of 706 is 4, 706 *isn't* divisible by 9. If you try to divide 706 by 9, you get 78 r 4.

Divisible by 11

Two-digit numbers that are divisible by 11 are hard to miss because they simply repeat the same digit twice. Here are all the numbers less than 100 that are divisible by 11:

11 22 33 44 55 66 77 88 99

For numbers between 100 and 200, use this rule: Every three-digit number whose first and third digits add up to its second digit is divisible by 11. For example, suppose you want to decide whether the number 154 is divisible by 11. Just add up the first and third digits:

1 + 4 = 5

Because these two numbers add up to the second digit, 5, the number 154 is divisible by 11. If you divide, you get 154 ÷ 11 = 14, a whole number.

Now suppose you want to figure out whether 136 is divisible by 11. Add the first and third digits:

1 + 6 = 7

Because the first and third digits add up to 7 instead of 3, the number 136 isn't divisible by 11. You can find that 136 ÷ 11 = 12 r 4.

For numbers of any length, the rule is slightly more complicated, but it's still often easier than doing long division. A number is divisible by 11 when its *alternate* digits

- Add up to the same number *or*
- Add up to two numbers that, when one is subtracted from the other, result in a number that's divisible by 11

For example, suppose you want to discover whether the number 15,983 is divisible by 11. To start out, underline alternate digits (every other digit):

$\underline{1}5,\underline{9}8\underline{3}$

Similarly, 3 is prime because when you divide by any number but 1 or 3, you get a remainder. So the only way to multiply two numbers together and get 3 as a product is the following:

$$1 \cdot 3 = 3$$

On the other hand, 4 is a composite number because it's divisible by three numbers: 1, 2, and 4. In this case, you have two ways to multiply two counting numbers and get a product of 4:

$$1 \cdot 4 = 4$$

$$2 \cdot 2 = 4$$

But 5 is a prime number, because it's divisible only by 1 and 5. Here's the only way to multiply two counting numbers together and get 5 as a product:

$$1 \cdot 5 = 5$$

And 6 is a composite number because it's divisible by 1, 2, 3, and 6. Here are two ways to multiply two counting numbers and get a product of 6:

$$1 \cdot 6 = 6$$

$$2 \cdot 3 = 6$$

REMEMBER

Every counting number except 1 is either prime or composite. The reason 1 is neither is that it's divisible by only *one* number, which is 1.

Here's a list of the prime numbers that are less than 30:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29

TIP

Remember the first four prime numbers: 2, 3, 5, and 7. Every composite number less than 100 is divisible by at least one of these numbers. This fact makes it easy to test whether a number under 100 is prime: Simply test it for divisibility by 2, 3, 5, and 7. If it's divisible by any of these numbers, it's composite — if not, it's prime.

For example, suppose you want to find out whether the number 79 is prime or composite without actually doing the division. Here's how you think it out, using the tricks I show you earlier in "Knowing the Divisibility Tricks":

- 79 is an odd number, so it isn't divisible by 2.
- 79 has a digital root of 8 (because 7 + 9 = 16; 1 + 6 = 7), so it isn't divisible by 3.

- 79 doesn't end in 5 or 0, so it isn't divisible by 5.
- Even though there's no trick for divisibility by 7, you know that 77 is divisible by 7. So 79 ÷ 7 would leave remainder of 2, which tells you that 79 isn't divisible by 7.

Because 79 is less than 100 and isn't divisible by 2, 3, 5, or 7, you know that 79 is a prime number.

Now test whether 93 is prime or composite:

- 93 is an odd number, so it isn't divisible by 2.
- 93 has a digital root of 3 (because 9 + 3 = 12 and 1 + 2 = 3), so 93 is divisible by 3.

You don't need to look further. Because 93 is divisible by 3, you know it's composite.

The resulting prime factorization for 84 is as follows:

$$84 = 2 \cdot 7 \cdot 2 \cdot 3$$

If you like, though, you can rearrange the factors from lowest to highest:

$$84 = 2 \cdot 2 \cdot 3 \cdot 7$$

By far, the most difficult situation occurs when you're trying to find the prime factors of a prime number but don't know it. For example, suppose you want to find the prime factorization for the number 71. This time, you don't recognize the number from the multiplication tables and it isn't divisible by 2 or 5. What next?

REMEMBER

If a number that's less than 100 (actually, less than 121) isn't divisible by 2, 3, 5, or 7, it's a prime number.

Testing for divisibility by 3 by finding the digital root of 71 (that is, by adding the digits) is easy. As I explain in Chapter 7, numbers divisible by 3 have digital roots of 3, 6, or 9.

$$7 + 1 = 8$$

Because the digital root of 71 is 8, 71 isn't divisible by 3. Divide to test whether 71 is divisible by 7:

$$71 \div 7 = 10 \text{ r } 1$$

So now you know that 71 isn't divisible by 2, 3, 5, or 7. Therefore, 71 is a prime number, so you're done.

Finding prime factorizations for numbers greater than 100

Most of the time, you don't have to worry about finding the prime factorizations of numbers greater than 100. Just in case, though, here's what you need to know.

As I mention in the preceding section, factor out the 5s and 2s first. A special case is when the number you're factoring ends in one or more 0s. In this case, you can factor out a 10 for every 0. For example, Figure 8-7 shows the first step.

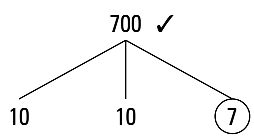


Figure 8-7: The first step in factoring 700.

To find the GCF of a set of numbers, list all the factors of each number, as I show you in “Generating a number’s factors.” The greatest factor appearing on every list is the GCF. For example, to find the GCF of 6 and 15, first list all the factors of each number.

Factors of 6: 1, 2, 3, 6

Factors of 15: 1, 3, 5, 15

Because 3 is the greatest factor that appears on both lists, 3 is the GCF of 6 and 15.

As another example, suppose you want to find the GCF of 9, 20, and 25. Start by listing the factors of each:

Factors of 9: 1, 3, 9

Factors of 20: 1, 2, 4, 5, 10, 20

Factors of 25: 1, 5, 25

In this case, the only factor that appears on all three lists is 1, so 1 is the GCF of 9, 20, and 25.

Using prime factorization to find the GCF

You can use prime factorization to find the GCF of a set of numbers. This often works better for large numbers, where generating lists of all factors can be time-consuming.

Here’s how to find the GCF of a set of numbers using prime factorization:

1. **List the prime factors of each number (see the earlier “Prime factors” section).**
2. **Circle every common prime factor — that is, every prime factor that’s a factor of every number in the set.**
3. **Multiply all the circled numbers.**

   The result is the GCF.

For example, suppose you want to find the GCF of 28, 42, and 70. Step 1 says to list the prime factors of each number. Step 2 says to circle every prime factor that’s common to all three numbers (as shown in Figure 8-9).

Figure 8-9: Finding the GCF of 28, 42, and 70.

$28 = 2 \cdot 2 \cdot 7$

$42 = 2 \cdot 3 \cdot 7$

$70 = 2 \cdot 5 \cdot 7$

Next, list multiples of 3, listing ten of them (because $2 \cdot 5 = 10$):

Multiples of 3: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, ...

The only numbers repeated on both lists are 15 and 30. In this case, you can save yourself the trouble of making the last list because 30 is obviously a multiple of 2, and 15 isn't. So 30 is the LCM of 2, 3, and 5.

Using prime factorization to find the LCM

A second method for finding the LCM of a set of numbers is to use the prime factorizations of those numbers. Here's how:

1. **List the prime factors of each number.**

   I show you how to find the prime factors of a number earlier in this chapter, in "Prime factors."

   Suppose you want to find the LCM of 18 and 24. List the prime factors of each number:

   $18 = 2 \cdot 3 \cdot 3$

   $24 = 2 \cdot 2 \cdot 2 \cdot 3$

2. **For each prime number listed, underline the most repeated occurrence of this number in any prime factorization.**

   The number 2 appears once in the prime factorization of 18 but three times in that of 24, so underline the three 2s:

   $18 = 2 \cdot 3 \cdot 3$

   $24 = \underline{2} \cdot \underline{2} \cdot \underline{2} \cdot 3$

   Similarly, the number 3 appears twice in the prime factorization of 18 but only once in that of 24, so underline the two 3s:

   $18 = 2 \cdot \underline{3} \cdot \underline{3}$

   $24 = \underline{2} \cdot \underline{2} \cdot \underline{2} \cdot 3$

3. **Multiply all the underlined numbers.**

   Here's the product:

   $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 = 72$

   So the LCM of 18 and 24 is 72. This checks out because

   $18 \cdot 4 = 72$

   $24 \cdot 3 = 72$

Chapter 9

Fooling with Fractions

In This Chapter

- Looking at basic fractions
- Knowing the numerator from the denominator
- Understanding proper fractions, improper fractions, and mixed numbers
- Increasing and reducing the terms of fractions
- Converting between improper fractions and mixed numbers
- Using cross-multiplication to compare fractions

Suppose that today is your birthday and your friends are throwing you a surprise party. After opening all your presents, you finish blowing out the candles on your cake, but now you have a problem: Eight of you want some cake, but you have only *one cake.* Several solutions are proposed:

- You could all go into the kitchen and bake seven more cakes.
- Instead of eating cake, everyone could eat celery sticks.
- Because it's your birthday, you could eat the *whole* cake and everyone else could eat celery sticks. (That was your idea.)
- You could cut the cake into eight equal slices so that everyone can enjoy it.

After careful consideration, you choose the last option. With that decision, you've opened the door into the exciting world of fractions. Fractions represent parts of a thing that can be cut into pieces. In this chapter, I give you some basic information about fractions that you need to know, including the three basic types of fractions: *proper fractions*, *improper fractions*, and *mixed numbers*.

I move on to increasing and reducing the terms of fractions, which you need when you begin applying the Big Four operations to fractions in Chapter 10. I also show you how to convert between improper fractions and mixed numbers. Finally, I show you how to compare fractions using cross-multiplication. By the time you're done with this chapter, you'll see how fractions really can be a piece of cake!

In this example, the number 3 is the numerator, and the number 4 is the denominator. Similarly, look at this fraction:

$$\frac{55}{89}$$

The number 55 is the numerator, and the number 89 is the denominator.

Flipping for reciprocals

When you flip a fraction over, you get its *reciprocal.* For example, the following numbers are reciprocals:

$$\frac{2}{3} \quad \text{and} \quad \frac{3}{2}$$

$$\frac{11}{14} \quad \text{and} \quad \frac{14}{11}$$

$\frac{19}{19}$ is its own reciprocal

Using ones and zeros

When the denominator (bottom number) of a fraction is 1, the fraction is equal to the numerator by itself. Or conversely, you can turn any whole number into a fraction by drawing a line and placing the number 1 under it. For example,

$$\frac{2}{1} = 2 \qquad \frac{9}{1} = 9 \qquad \frac{157}{1} = 157$$

REMEMBER

When the numerator and denominator match, the fraction equals 1. That's because if you cut a cake into eight pieces and you keep all eight of them, you have the entire cake. Here are some fractions that equal 1:

$$\frac{8}{8} = 1 \qquad \frac{11}{11} = 1 \qquad \frac{365}{365} = 1$$

When the numerator of a fraction is 0, the fraction is equal to 0. For example,

$$\frac{0}{1} = 0 \qquad \frac{0}{12} = 0 \qquad \frac{0}{113} = 0$$

2. **Repeat Step 1 until the numerator or denominator (or both) is no longer divisible by 2.**

   In the resulting fraction, both numbers are still even, so repeat the first step again:

   $$\frac{12}{30} = \frac{6}{15}$$

3. **Repeat Step 1 using the number 3, and then 5, and then 7, continuing testing prime numbers until you're sure that the numerator and denominator have no common factors.**

   Now, the numerator and the denominator are both divisible by 3 (see Chapter 7 for easy ways to tell if one number is divisible by another), so divide both by 3:

   $$\frac{6}{15} = \frac{2}{5}$$

   Neither the numerator nor the denominator is divisible by 3, so this step is complete. At this point, you can move on to test for divisibility by 5, 7, and so on, but you really don't need to. The numerator is 2, and it obviously isn't divisible by any larger number, so you know that the fraction $^{24}/_{60}$ reduces to $^{2}/_{5}$.

Converting Between Improper Fractions and Mixed Numbers

In "Knowing the fraction facts of life," I tell you that any fraction whose numerator is greater than its denominator is an *improper fraction.* Improper fractions are very useful and easy to work with, but for some reason, people just don't like them. (The word *improper* should've tipped you off.) Teachers especially don't like them, and they really don't like an improper fraction to appear as the answer to a problem. However, they love mixed numbers. One reason they love them is that estimating the approximate size of a mixed number is easy.

For example, if I tell you to put $^{31}/_{3}$ of a gallon of gasoline in my car, you'd probably find it hard to estimate roughly how much that is: 5 gallons, 10 gallons, 20 gallons?

But if I tell you to get $10\,^{1}/_{3}$ gallons of gasoline, you know immediately that this amount is a little more than 10 but less than 11 gallons. Although $10\,^{1}/_{3}$ is the same as $^{31}/_{3}$, knowing the mixed number is a lot more helpful in practice. For this reason, you often have to convert improper fractions to mixed numbers.

1. **Cross-multiply the two fractions and subtract the second number from the first to get the numerator of the answer.**

   For example, suppose you want to subtract $\frac{6}{7} - \frac{2}{5}$. To get the numerator, cross-multiply the two fractions and then subtract the second number from the first number (see Chapter 9 for info on cross-multiplication):

   $$\frac{6}{7} - \frac{2}{5}$$

   $$(6 \cdot 5) - (2 \cdot 7) = 30 - 14 = 16$$

   After you cross-multiply, be sure to subtract in the correct order. (The first number is the numerator of the first fraction times the denominator of the second.)

2. **Multiply the two denominators together to get the denominator of the answer.**

   $$7 \cdot 5 = 35$$

3. **Putting the numerator over the denominator gives you your answer.**

   $$\frac{16}{35}$$

Here's another example to work with:

$$\frac{9}{10} - \frac{5}{6}$$

This time, I put all the steps together:

$$\frac{9}{10} - \frac{5}{6} = \frac{9 \cdot 6 - 5 \cdot 10}{10 \cdot 6}$$

With the problem set up like this, you just have to simplify the result:

$$= \frac{54 - 50}{60} = \frac{4}{60}$$

In this case, you can reduce the fraction:

$$\frac{4}{60} = \frac{1}{15}$$

Cutting it short with a quick trick

The easy way I show you in the preceding section works best when the numerators and denominators are small. When they're larger, you may be able to take a shortcut.

Before you subtract fractions with different denominators, check the denominators to see whether one is a multiple of the other (for more on multiples, see Chapter 8). If it is, you can use the quick trick:

In pairs: Adding two mixed numbers

Adding mixed numbers looks a lot like adding whole numbers: You stack them up one on top of the other, draw a line, and add. For this reason, some students feel more comfortable adding mixed numbers than adding fractions. Here's how to add two mixed numbers:

1. **Add the fractional parts using any method you like, and if necessary, change this sum to a mixed number and reduce it.**
2. **If the answer you found in Step 1 is an improper fraction, change it to a mixed number, write down the fractional part, and *carry* the whole number part to the whole number column.**
3. **Add the whole number parts (including any number carried).**

Your answer may also need to be reduced to lowest terms (see Chapter 9). In the examples that follow, I show you everything you need to know.

Summing up mixed numbers when the denominators are the same

As with any problem involving fractions, adding is always easier when the denominators are the same. For example, suppose you want to add $3\frac{1}{3} + 5\frac{1}{3}$. Doing mixed number problems is often easier if you place one number above the other:

$$\begin{array}{r} 3\,\frac{1}{3} \\ +\ 5\,\frac{1}{3} \\ \hline \end{array}$$

As you can see, this arrangement is similar to how you add whole numbers, but it includes an extra column for fractions. Here's how you add these two mixed numbers step by step:

1. **Add the fractions.**

   $$\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

2. **Switch improper fractions to mixed numbers; write down your answer.**

   Because $\frac{2}{3}$ is a proper fraction, you don't have to change it.

3. **Add the whole number parts.**

   $$3 + 5 = 8$$

Here's how your problem looks in column form:

$$\begin{array}{r} 3\,\frac{1}{3} \\ +\ 5\,\frac{1}{3} \\ \hline 8\,\frac{2}{3} \end{array}$$

commonly, you need to round a decimal either to a whole number or to one or two decimal places.

To round a decimal to a whole number, focus on the ones digit and the tenths digit. Round the decimal either up or down to the *nearest* whole number, dropping the decimal point:

7.1 -> 7 32.9 -> 33 184.3 -> 184

When the tenths digit is 5, round the decimal *up*:

83.5 -> 84 296.5 -> 297 1,788.5 -> 1,789

If the decimal has other decimal digits, just drop them:

18.47 -> 18 21.618 -> 22 3.1415927 -> 3

Occasionally, a small change to the ones digit affects the other digits. (This may remind you of when the odometer in your car rolls a bunch of 9s over to 0s.)

99.9 -> 100 999.5 -> 1,000 99,999.712 -> 100,000

The same basic idea applies to rounding a decimal to any number of places. For example, to round a decimal to one decimal place, focus on the first and second decimal places (that is, the tenths and hundredths places):

76.543 -> 76.5 100.6822 -> 100.7 10.10101 -> 10.1

To round a decimal to two decimal places, focus on the second and third decimal places (that is, the hundredths and thousandths places):

444.4444 -> 444.44 26.55555 -> 26.56 99.997 -> 100.00

Performing the Big Four with Decimals

Everything you already know about adding, subtracting, multiplying, and dividing whole numbers (see Chapter 3) carries over when you work with decimals. In fact, in each case, there's really only one key difference: how to handle that pesky little decimal point. In this section, I show you how to perform the Big Four math operations with decimals.

The most common use of adding and subtracting decimals is when you're working with money — for example, balancing your checkbook. Later in this book, you find that multiplying and dividing by decimals is useful for calculating percentages (see Chapter 12), using scientific notation (see Chapter 14), and measuring with the metric system (see Chapter 15).

Adding decimals

Adding decimals is almost as easy as adding whole numbers. As long as you set up the problem correctly, you're in good shape. To add decimals, follow these steps:

1. **Line up the decimal points.**
2. **Add as usual from right to left, column by column.**
3. **Place the decimal point in the answer in line with the other decimal points in the problem.**

For example, suppose you want to add the numbers 14.5 and 1.89. Line up the decimal points neatly as follows:

$$\begin{array}{r} 14.5 \\ \underline{+\ 1.89} \end{array}$$

Begin adding from the right-hand column. Treat the blank space after 14.5 as a 0 — you can write this in as a trailing 0 (see earlier in this chapter to see why adding zeros to the end of a decimal doesn't change its value). Adding this column gives you 0 + 9 = 9:

$$\begin{array}{r} 14.50 \\ \underline{+\ 1.89} \\ 9 \end{array}$$

Continuing to the left, 5 + 8 = 13, so put down the 3 and carry the 1:

$$\begin{array}{r} 1 \\ 14.50 \\ \underline{+\ 1.89} \\ 39 \end{array}$$

But at least you can take comfort in the fact that when you know how to do long division (which I cover in Chapter 3), figuring out how to divide decimals is easy. The main difference comes at the beginning, before you start dividing.

Here's how to divide decimals:

1. **Turn the *divisor* (the number you're dividing by) into a whole number by moving the decimal point all the way to the right; at the same time, move the decimal point in the *dividend* (the number you're dividing) the same number of places to the right.**

   For example, suppose you want to divide 10.274 by 0.11. Write the problem as usual:

   $$0.11\overline{)10.274}$$

   Turn 0.11 into a whole number by moving the decimal point in 0.11 two places to the right, giving you 11. At the same time, move the decimal point in 10.274 two places to the right, giving you 1,027.4:

   $$11.\overline{)1027.4}$$

2. **Place a decimal point in the *quotient* (the answer) directly above where the decimal point now appears in the dividend.**

   Here's what this step looks like:

   $$\begin{array}{r} . \\ 11.\overline{)1027.4} \end{array}$$

3. **Divide as usual, being careful to line up the quotient properly so that the decimal point falls into place.**

   To start out, notice that 11 is too large to go into either 1 or 10. However, 11 does go into 102 (9 times). So write the first digit of the quotient just above the 2 and continue:

   $$\begin{array}{r} 9. \\ 11.\overline{)1027.4} \\ \underline{99} \\ 37 \end{array}$$

   I left off after bringing down the next number, 7. This time, 11 goes into 37 three times. The important thing is to place the next digit in the answer just above the 7:

   $$\begin{array}{r} 93. \\ 11.\overline{)1027.4} \\ \underline{99} \\ 37 \\ \underline{33} \\ 44 \end{array}$$

The last stop: Terminating decimals

Sometimes, when you divide the numerator of a fraction by the denominator, the division eventually works out evenly. The result is a *terminating decimal.*

For example, suppose you want to change the fraction $\frac{2}{5}$ to a decimal. Here's your first step:

$$5\overline{)2}$$

One glance at this problem, and it looks like you're doomed from the start because 5 doesn't go into 2. But watch what happens when I add a few trailing zeros. Notice that I also place another decimal point in the answer just above the first decimal point. This step is important — you can read more about it in "Dividing decimals":

$$\begin{array}{r} . \\ 5\overline{)2.000} \end{array}$$

Now you can divide because although 5 doesn't go into 2, 5 does go into 20 four times:

$$\begin{array}{r} 0.4 \\ 5\overline{)2.000} \\ \underline{20} \\ 0 \end{array}$$

You're done! As it turns out, you only needed one trailing zero, so you can ignore the rest:

$$\frac{2}{5} = 0.4$$

Because the division worked out evenly, the answer is an example of a *terminating decimal.*

As another example, suppose you want to find out how to represent $\frac{7}{16}$ as a decimal. As earlier, I attach three trailing zeros:

$$\begin{array}{r} 0.437 \\ 16\overline{)7.000} \\ \underline{64} \\ 60 \\ \underline{48} \\ 120 \\ \underline{112} \\ 8 \end{array}$$

Figuring out simple percent problems

TIP

A lot of percent problems turn out to be easy when you give them a little thought. In many cases, just remember the connection between percents and fractions and you're halfway home:

- **Finding 100% of a number:** Remember that 100% means the whole thing, so 100% of any number is simply the number itself:

  100% of 5 is 5

  100% of 91 is 91

  100% of 732 is 732

- **Finding 50% of a number:** Remember that 50% means half, so to find 50% of a number, just divide it by 2:

  50% of 20 is 10

  50% of 88 is 44

  50% of 7 is $\frac{7}{2}$ (or $3\frac{1}{2}$ or 3.5)

- **Finding 25% of a number:** Remember that 25% equals ¼, so to find 25% of a number, divide it by 4:

  25% of 40 is 10

  25% of 88 is 22

  25% of 15 is $\frac{15}{4}$ (or $3\frac{3}{4}$ or 3.75)

- **Finding 20% of a number:** Finding 20% of a number is handy if you like the service you've received in a restaurant, because a good tip is 20% of the check. Because 20% equals ⅕, you can find 20% of a number by dividing it by 5. But I can show you an easier way:

  To find 20% of a number, move the decimal point one place to the left and double the result:

  20% of 80 = $8 \cdot 2 = 16$

  20% of 300 = $30 \cdot 2 = 60$

  20% of 41 = $4.1 \cdot 2 = 8.2$

- **Finding 10% of a number:** Finding 10% of any number is the same as finding ⅒ of that number. To do this, just move the decimal point one place to the left:

  10% of 30 is 3

  10% of 41 is 4.1

  10% of 7 is 0.7

requires you to add and subtract decimals, which I discuss in Chapter 11. Even though the decimals may look intimidating, this problem is fairly simple to set up:

> Antonia bought 4.53 pounds of beef and 3.1 pounds of lamb. Lance bought 5.24 pounds of chicken and 0.7 pounds of pork. Which of them bought more meat, and how much more?

To solve this problem, you first find out how much each person bought:

Antonia = 4.53 + 3.1 = 7.63

Lance = 5.24 + 0.7 = 5.94

You can already see that Antonia bought more than Lance. To find how much more, subtract:

7.63 – 5.94 = 1.69

So Antonia bought 1.69 pounds more than Lance.

Splitting the vote: Percents

When percents represent answers in polls, votes in an election, or portions of a budget, the total often has to add up to 100%. In real life, you may see such info organized as a pie chart (which I discuss in Chapter 17). Solving problems about this kind of information often involves nothing more than adding and subtracting percents. Here's an example:

> In a recent mayoral election, five candidates were on the ballot. Faber won 39% of the vote, Gustafson won 31%, Ivanovich won 18%, Dixon won 7%, Obermayer won 3%, and the remaining votes went to write-in candidates. What percentage of voters wrote in their selection?

The candidates were in a single election, so all the votes have to total 100%. The first step here is just to add up the five percentages. Then subtract that value from 100%:

39% + 31% + 18% + 7% + 3% = 98%

100% – 98% = 2%

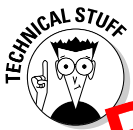
Because 98% of voters voted for one of the five candidates, the remaining 2% wrote in their selections.

Problems about Multiplying Fractions

In word problems, the word *of* almost always means *multiplication*. So whenever you see the word *of* following a fraction, decimal, or percent, you can usually replace it with a times sign.

When you think about it, *of* means multiplication even when you're not talking about fractions. For example, when you point to an item in a store and say, "I'll take three *of* those," in a sense you're saying, "I'll take that one *multiplied by* three."

The following examples give you practice turning word problems that include the word *of* into multiplication problems that you can solve with fraction multiplication.



When you divide up a single thing — such as one pizza or one death-by-chocolate cake — the word *of* still means to multiply; you're technically multiplying each fraction by 1. For example, the fraction that represents half of a pizza — that is, ½ of 1 pizza — is ½ · 1 = ½. Because anything times 1 is itself, you don't have to write the 1 at all — you can just add the fractions, as I do earlier in "Sharing a pizza: Fractions."

Renegade grocery shopping: Buying less than they tell you to

After you understand that the word *of* means multiplication, you have a powerful tool for solving word problems. For instance, you can figure out how much you'll spend if you don't buy food in the quantities listed on the signs. Here's an example:

> If beef costs $4 a pound, how much does ⅝ of a pound cost?

Here's what you get if you simply change the *of* to a multiplication sign:

$$\frac{5}{8} \cdot 1 \text{ pound of beef}$$

That's how much beef you're buying. However, you want to know the cost. Because the problem tells you that 1 pound = $4, you can replace *1 pound of beef* with *$4:*

$$= \frac{5}{8} \cdot \$4$$

Now you have an expression you can evaluate. Use the rules of multiplying fractions from Chapter 10 and solve:

After you know this trick, representing a lot of large numbers as powers of ten is easy — just count the 0s! For example, the number 1 trillion — 1,000,000,000,000 — is a 1 with twelve 0s after it, so

$$1{,}000{,}000{,}000{,}000 = 10^{12}$$

This trick may not seem like a big deal, but the higher the numbers get, the more space you save by using exponents. For example, a really big number is a *googol,* which is 1 followed by a hundred 0s. You can write this:

10,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000
000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000

As you can see, a number of this size is practically unmanageable. You can save yourself some trouble and write 10^{100}.

A 10 raised to a negative number is also a power of ten.

You can also represent decimals using *negative exponents*. For example,

$10^{-1} = 0.1$ $\quad$ $10^{-2} = 0.01$ $\quad$ $10^{-3} = 0.001$ $\quad$ $10^{-4} = 0.0001$

Although the idea of negative exponents may seem strange, it makes sense when you think about it alongside what you know about positive exponents. For example, to find the value of 10^7, start with 1 and make it *larger* by moving the decimal point 7 spaces to the *right*:

$$10^7 = 10{,}000{,}000$$

Similarly, to find the value of 10^{-7}, start with 1 and make it *smaller* by moving the decimal point 7 spaces to the *left*:

$$10^{-7} = 0.0000001$$

Negative powers of 10 always have *one fewer 0* between the 1 and the decimal point than the power indicates. In this example, notice that 10^{-7} has six 0s between them.

As with very large numbers, using exponents to represent very small decimals makes practical sense. For example,

$$10^{-23} = 0.00000000000000000000001$$

As you can see, this decimal is easy to work with in its exponential form but almost impossible to read otherwise.

Writing in scientific notation

Here's how to write any number in scientific notation:

1. **Write the number as a decimal (if it isn't one already).**

   Suppose you want to change the number 360,000,000 to scientific notation. First, write it as a decimal:

   360,000,000.0

2. **Move the decimal point just enough places to change this number to a new number that's between 1 and 10.**

   Move the decimal point to the right or left so that only one nonzero digit comes before the decimal point. Drop any leading or trailing zeros as necessary.

   Using 360,000,000.0, only the 3 should come before the decimal point. So move the decimal point eight places to the left, drop the trailing zeros, and get 3.6.

   360,000,000.0 becomes 3.6

3. **Multiply the new number by 10 raised to the number of places you moved the decimal point in Step 2.**

   You moved the decimal point eight places, so multiply the new number by 10^8:

   $3.6 \cdot 10^8$

4. **If you moved the decimal point to the right in Step 2, put a minus sign on the exponent.**

   You moved the decimal point to the left, so you don't have to take any action here. Thus, 360,000,000 in scientific notation is $3.6 \cdot 10^8$.

Changing a decimal to scientific notation basically follows the same process. For example, suppose you want to change the number 0.00006113 to scientific notation:

1. **Write 0.00006113 as a decimal (this step's easy, because it's already a decimal):**

   0.00006113

2. **To change 0.00006113 to a new number between 1 and 10, move the decimal point five places to the right and drop the leading zeros:**

   6.113

1. **Multiply 6.02 by 9 to find the decimal part of the answer:**

   $6.02 \cdot 9 = 54.18$

2. **Multiply 10^{23} by 10^{-28} by adding the exponents (check Chapter 4 if you need info on adding negative numbers):**

   $10^{23} \cdot 10^{-28} = 10^{23 + -28} = 10^{-5}$

3. **Write the answer as the product of the two numbers:**

   $54.18 \cdot 10^{-5}$

4. **Because 54.18 is greater than 10, move the decimal point one place to the left and add 1 to the exponent:**

   $5.418 \cdot 10^{-4}$

   Note: In decimal form, this number equals 0.0005418.

Scientific notation really pays off when you're multiplying very large and very small numbers. If you'd tried to multiply the numbers in the preceding example the usual way, here's what you would've been up against:

$$602{,}000{,}000{,}000{,}000{,}000{,}000{,}000 \cdot 0.0000000000000000000000000009$$

As you can see, scientific notation makes the job a lot easier.

Estimating longer distances and speed

Here's how to convert kilometers to miles: 1 kilometer ≈ 0.62 miles. For a ballpark estimate, you can remember that 1 kilometer is about ½ a mile. By the same token, 1 kilometer per hour is about ½ mile per hour.

This guideline tells you that if you live 2 miles from the nearest supermarket, then you live about 4 kilometers from there. A marathon of 26 miles is about 52 kilometers. And if you run on a treadmill at 6 miles per hour, then you can run at about 12 kilometers per hour. By the same token, a 10 kilometer race is about 5 miles. If the Tour de France is about 4,000 kilometers, then it's about 2,000 miles. And if light travels about 300,000 kilometers per second, then it travels about 150,000 miles per second.

Approximating volume: 1 liter is about 1 quart (¼ gallon)

Here's how to convert liters to gallons: 1 liter ≈ 0.26 gallons. A good estimate here is that 1 liter is about 1 quart (that is, there are about 4 liters to the gallon).

Using this estimate, a gallon of milk is 4 quarts, so it's about 4 liters. If you put 10 gallons of gasoline in your tank, this is about 40 liters. In the other direction, if you buy a 2-liter bottle of cola, you have about 2 quarts. If you buy an aquarium with a 100-liter capacity, it holds about 25 gallons of water. And if a pool holds 8,000 liters of water, it holds about 2,000 gallons.

Estimating weight: 1 kilogram is about 2 pounds

Here's how to convert kilograms to pounds: 1 kilogram ≈ 2.20 pounds. For estimating, figure that 1 kilogram is equal to about 2 pounds.

By this estimate, a 5-kilogram bag of potatoes weighs about 10 pounds. If you can bench press 70 kilograms, then you can bench press about 140 pounds. And because a liter of water weighs exactly 1 kilogram, you know that a quart of water weighs about 2 pounds. Similarly, if a baby weighs 8 pounds at birth, he or she weighs about 4 kilograms. If you weigh 150 pounds, then you weigh about 75 kilograms. And if your New Year's resolution is to lose 20 pounds, then you want to lose about 10 kilograms.

Estimating temperature

The most common reason for estimating temperature in Celsius is in connection with the weather. The formula for converting from Celsius to Fahrenheit is kind of messy:

$$\text{Fahrenheit} = \text{Celsius} \cdot \tfrac{9}{5} + 32$$

Instead, use the handy chart in Table 15-3.

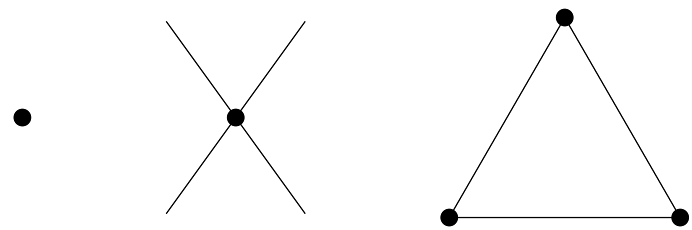
Getting on the Plane: Points, Lines, Angles, and Shapes

Plane geometry is the study of figures on a two-dimensional surface — that is, on a plane. You can think of the plane as a piece of paper with no thickness at all. Technically, a plane doesn't end at the edge of the paper — it continues forever.

In this section, I introduce you to four important concepts in plane geometry: points, lines, angles, and shapes (such as squares, circles, triangles, and so forth).

Making some points

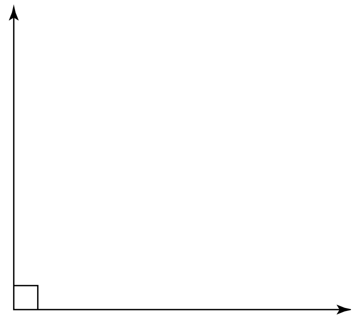
A *point* is a location on a plane. It has no size or shape. Although in reality a point is too small to be seen, you can represent it visually in a drawing by using a dot.



When two lines intersect, as shown above, they share a single point. Additionally, each corner of a polygon is a point. (See below for more on lines and polygons.)

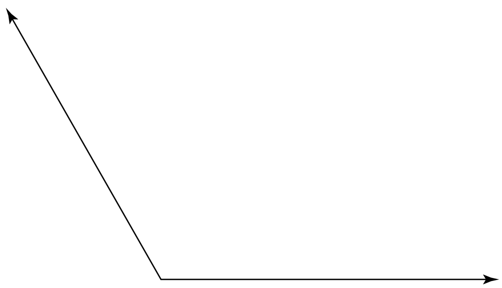
Knowing your lines

A *line* — also called a *straight line* — is pretty much what it sounds like; it marks the shortest distance between two points, but it extends infinitely in both directions. It has length but no width, making it a one-dimensional (1-D) figure.



Angles that have fewer than 90° — that is, angles that are sharper than a right angle — are called *acute angles,* like this one:

Angles that measure greater than 90° — that is, angles that aren't as sharp as a right angle — are called *obtuse angles,* as seen here:



When an angle is exactly 180°, it forms a straight line and is called a *straight angle*.

Shaping things up

A shape is any closed geometrical figure that has an *inside* and an *outside*. Circles, squares, triangles, and larger polygons are all examples of shapes.

Note: Because $2 \cdot r$ is the same as the diameter, you also can write the formula as $C = \pi \cdot d$.

The symbol π is called *pi* (pronounced *pie*). It's just a number whose approximate value is as follows (the decimal part of pi goes on forever, so you can't get an exact value for pi):

$$\pi \approx 3.14$$

So given a circle with a radius of 5 mm, you can figure out the approximate circumference:

$$C \approx 2 \cdot 3.14 \cdot 5 \text{ mm} = 31.4 \text{ mm}$$

The formula for the area *(A)* of a circle also uses π:

$$A = \pi \cdot r^2$$

Here's how you use this formula to find the approximate area of a circle with a radius of 5 mm:

$$A \approx 3.14 \cdot (5 \text{ mm})^2 = 3.14 \cdot 25 \text{ mm}^2 = 78.5 \text{ mm}^2$$

Measuring triangles

In this section, I discuss how to measure the perimeter and area of all triangles. Then, I show you a special feature of right triangles that allows you to measure them more easily.

Finding the perimeter and area of a triangle

Mathematicians have no special formula for finding the perimeter of a triangle — they just add up the lengths of the sides.

To find the area of a triangle, you need to know the length of one side — the *base* (*b* for short) — and the *height (h)*. Note that the height forms a right angle with the base. Figure 16-13 shows a triangle with a base of 5 cm and a height of 2 cm:

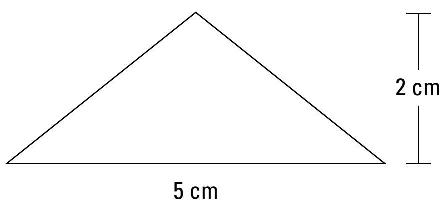


Figure 16-13: The base and height of a triangle.

Reading a bar graph is easy after you get used to it. Here are a few types of questions someone could ask about the bar graph in Figure 17-1:

- **Individual values:** *How many new clients does Jay have?* Find the bar representing Jay's clients and notice that he has 23 new clients.
- **Differences in value:** *How many more clients does Rita have compared with Dwayne's?* Notice that Rita has 20 new clients and Dwayne has 18, so she has 2 more than he does.
- **Totals:** *Together, how many clients do the three women have?* Notice that the three women — Edna, Iris, and Rita — have 25, 16, and 20 new clients, respectively, so they have 61 new clients altogether.

Pie chart

A *pie chart,* which looks like a divided circle, shows you how a whole object is cut up into parts. Pie charts are most often used to represent percentages. For example, Figure 17-2 is a pie chart representing Eileen's monthly expenses.

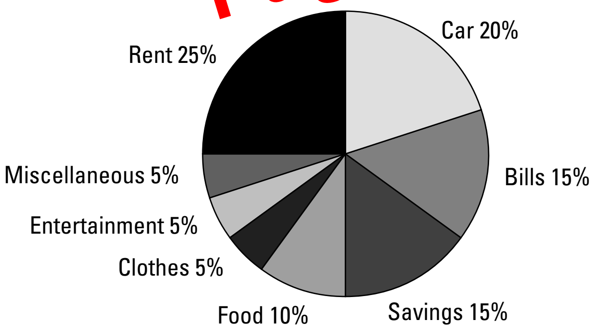


Figure 17-2: Eileen's monthly expenses.

You can tell at a glance that Eileen's largest expense is rent and that her second largest is her car. Unlike the bar graph, the pie chart shows numbers that are dependent upon each other. For example, if Eileen's rent increases to 30% of her monthly income, she'll have to decrease her spending in at least one other area.

Here are a few typical questions you may be asked about a pie chart:

- **Individual percentages:** *What percentage of her monthly expenses does Eileen spend on food?* Find the slice that represents what Eileen spends on food, and notice that she spends 10% of her income there.

For more-complex conversion problems, a good tool is the conversion chain. A *conversion chain* links together a sequence of unit conversions.

Setting up a short chain

Here's a problem that shows you how to set up a short conversion chain to make a conversion you won't find a specific equation for:

> Vendors at the Fragola County Strawberry Festival sold 7 tons of strawberries in a single weekend. How many 1-ounce servings of strawberries is that?

You don't have an equation to convert tons directly to ounces. But you do have one to convert tons to pounds and another to convert pounds to ounces. You can use these equations to build a bridge from one unit to another.

tons → pounds → ounces

So here are the two equations that you'll want to use:

1 ton = 2,000 lbs.

1 lb. = 16 oz.

To convert tons to pounds, note that these fractions equal 1, because the numerator (top number) equals the denominator (bottom number):

$$\frac{1 \text{ ton}}{2{,}000 \text{ lbs.}} \quad \text{or} \quad \frac{2{,}000 \text{ lbs.}}{1 \text{ ton}}$$

To convert pounds to ounces, note that these fractions equal 1:

$$\frac{1 \text{ lb.}}{16 \text{ oz.}} \quad \text{or} \quad \frac{16 \text{ oz.}}{1 \text{ lb.}}$$

You could do this conversion in two steps. But when you know the basic idea, you set up a conversion chain instead. To help make this idea clear, take a look at how to get from tons to ounces:

tons → pounds → ounces

So here's how to set up a conversion chain to turn 7 tons into pounds and then into ounces. Because you already have tons on top, you want the tons-and-pounds fraction that puts *ton* on the bottom. And because that fraction puts *pounds* on the top, use the pounds-and-ounces fraction that puts *pound* on the bottom:

$$\frac{7 \text{ tons}}{1} \cdot \frac{2{,}000 \text{ lbs.}}{1 \text{ ton}} \cdot \frac{16 \text{ oz.}}{1 \text{ lb.}}$$

As you can see, I drew a circle for the fountain and labeled its diameter as 32 feet. The outside edge of the path around the fountain is also a circle, and its circumference is 120 feet. The problem is asking for the width of the path. The picture shows you that the path's width is the distance from the inner circle to the outer circle.

The *radius* of a circle is the distance from its center to the circle itself (see Chapter 16). So if you know the radius of each circle, you can find the width of the path by subtracting:

Width of path = radius of outer circle – radius of inner circle

This word equation is the key to the problem. Look at the diagram and make sure you understand it before continuing.

You already know that the diameter of the inner circle is 32 feet, so you can find its radius using this formula from Chapter 16, where d is the diameter and r is the radius:

$d = 2 \cdot r$

Plugging in the diameter of the inner circle, you get the following:

32 ft. = 2 · radius of inner circle

You can probably solve this problem in your head:

Radius of inner circle = 16 ft.

You also know the circumference of the outer circle, so you can find its radius using this formula from Chapter 16, where C is the circumference and r is the radius:

$C = 2 \cdot \pi \cdot r$

Plugging in the circumference and 3.1 for π gives you

120 ft. = 2 · 3.1 · radius of outer circle

This equation can be simplified a little as follows:

120 ft. = 6.2 · radius of outer circle

Chapter 19

Figuring Your Chances: Statistics and Probability

In This Chapter

- Knowing how statistics works with both qualitative and quantitative data
- Finding out how to calculate a percentage and the mode of a sample
- Calculating the mean and median
- Finding the probability of an event

Statistics and probability are two of the most important and widely used applications of math. They're applicable to virtually every aspect of the real world — business, biology, city planning, politics, meteorology, and many more areas of study. Even physics, once thought to be devoid of uncertainty, now relies on probability.

In this chapter, I give you a basic understanding of these two mathematical ideas. First, I introduce you to statistics and the important distinction between qualitative and quantitative data. I show you how to work with both types of data to find meaningful answers. Then I give you the basics of probability. I show you how the probability that an event will occur is always a fraction from 0 to 1. After that, I demonstrate how to build this fraction by counting both favorable outcomes and possible outcomes. Finally, I put these ideas to work by showing you how to calculate the probability of tossing coins and rolling dice.

Gathering Data Mathematically: Basic Statistics

Statistics is the science of gathering and drawing conclusions from *data,* which is information that's measured objectively in an unbiased, reproducible way.

This time, the middle value is 18, so 18 is the median score.

If you have an even number of values in the data set, put the numbers in order and find the mean of the *two middle numbers* in the list (see the preceding section for details on the mean). For instance, consider the following:

2 3 5 7 9 11

The two center numbers are 5 and 7. Add them together to get 12 and then divide by 2 to get their mean. The median in this list is 6.

Now recall the company president who makes $19,010,000 a year and his 99 employees who each earn $10,000. Here's how this data looks:

10,000 10,000 10,000 ... 10,000 19,010,000

As you can see, if you were to write out all 100 salaries, the center numbers would obviously both be 10,000. The median salary is $10,000, and this result is much more reflective of what you'd probably earn if you were to work at this company.

Looking at Likelihoods: Basic Probability

Probability is the mathematics of deciding how likely an event is to occur. For example,

- What's the likelihood that the lottery ticket I bought will win?
- What's the likelihood that my new car will need repairs before the warranty runs out?
- What's the likelihood that more than 100 inches of snow will fall in Manchester, New Hampshire, this winter?

Probability has a wide variety of applications in insurance, weather prediction, biological sciences, and even physics.

The study of probability started hundreds of years ago when a group of French noblemen began to suspect that math could help them turn a profit, or at least not lose so heavily, in the gambling halls that they frequented.

You can read all about the details of probability in *Probability For Dummies* (Wiley). In this section, I give you a little taste of this fascinating subject.

Figuring the probability

The *probability* that an event will occur is a fraction whose numerator (top number) and denominator (bottom number) are as follows (for more on fractions, flip to Chapter 9):

$$\frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$$

In this case, a *favorable outcome* is simply an outcome in which the event you're examining *does* happen. In contrast, a *possible outcome* is an outcome that *can* happen.

For example, suppose you want to know the probability that a tossed coin will land heads up. Notice that there are two possible outcomes (heads or tails), but only one of these outcomes is favorable — the outcome in which heads comes up. To find the probability of this event, make a fraction as follows:

$$\frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{1}{2}$$

So the probability that the coin will land heads up is ½.

So what's the probability that when you roll a die, the number 3 will land face up? To figure this one out, notice that there are *six* possible outcomes (1, 2, 3, 4, 5, or 6), but in only *one* of these does 3 land face up. To find the probability of this outcome, make a fraction as follows:

$$\frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{1}{6}$$

So the probability that the number 3 will land face up is ⅙.

And what's the probability that if you pick a card at random from a deck, it'll be an ace? To figure this out, notice that there are *52* possible outcomes (one for each card in the deck), but in only *four* of these do you pick an ace. So

$$\frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{4}{52}$$

So the probability that you'll pick an ace is $\frac{4}{52}$, which reduces to $\frac{1}{13}$ (see Chapter 9 for more on reducing fractions).

Probability is always a fraction or decimal from 0 to 1. When the probability of an outcome is 0, the outcome is *impossible*. When the probability of an outcome is 1, the outcome is *certain*.

Chapter 21

Enter Mr. X: Algebra and Algebraic Expressions

In This Chapter

- Meeting Mr. *X* head-on
- Understanding how a variable such as *x* stands for a number
- Using substitution to evaluate an algebraic expression
- Identifying and rearranging the terms in any algebraic expression
- Simplifying algebraic expressions

You never forget your first love, your first car, or your first *x*. Unfortunately for some folks, remembering their first *x* in algebra is similar to remembering their first love who stood them up at the prom or their first car that broke down someplace in Mexico.

The most well-known fact about algebra is that it uses letters — like *x* — to represent numbers. So, if you have a traumatic *x*-related tale, all I can say is that the future will be brighter than the past.

What good is algebra? That's a common question, and it deserves a decent answer. Algebra is used for solving problems that are just too difficult for ordinary arithmetic. And because number crunching is so much a part of the modern world, algebra is everywhere (even if you don't see it): architecture, engineering, medicine, statistics, computers, business, chemistry, physics, biology, and of course higher math. Anywhere that numbers are useful, algebra is there. That's why virtually every college and university insists that you leave (or enter) with at least a passing familiarity with algebra.

Variable Part	*Examples of Similar Terms*		
x	$4x$	$12x$	$99.9x$
x^2	$6x^2$	$-20x^2$	$\frac{8}{3}x^2$
y	y	$1{,}000y$	πy
xy	$-7xy$	$800xy$	$\frac{22}{7}xy$
x^3y^3	$3x^3y^3$	$-111x^3y^3$	$3.14x^3y^3$

As you can see, in each example, the variable part in all three similar terms is the same. Only the coefficient changes, and it can be any real number: positive or negative, whole number, fraction, decimal, or even an irrational number such as π. (For more on real numbers, see Chapter 25.)

Considering algebraic terms and the Big Four

In this section, I get you up to speed on how to apply the Big Four to algebraic expressions. For now, just think of working with algebraic expressions as a set of tools that you're collecting, for use when you get on the job. You find how useful these tools are in Chapter 22, when you begin solving algebraic equations.

Adding terms

Add similar terms by adding their coefficients and keeping the same variable part.

For example, suppose you have the expression $2x + 3x$. Remember that $2x$ is just shorthand for $x + x$, and $3x$ means simply $x + x + x$. So when you add them up, you get the following:

$$= x + x + x + x + x = 5x$$

As you can see, when the variable parts of two terms are the same, you add these terms by adding their coefficients: $2x + 3x = (2 + 3)x$. The idea here is roughly similar to the idea that 2 apples + 3 apples = 5 apples.

You *cannot* add non-similar terms. Here are some cases in which the variables or their exponents are different:

$$2x + 3y$$

$$2yz + 3y$$

$$2x^2 + 3x$$

Multiplying terms

Unlike adding and subtracting, you can multiply non-similar terms. Multiply *any* two terms by multiplying their coefficients and combining — that is, by collecting or gathering up — all the variables in each term into a single term, as I show you below.

For example, suppose you want to multiply $5x(3y)$. To get the coefficient, multiply $5 \cdot 3$. To get the algebraic part, combine the variables x and y:

$$= 5(3)xy = 15xy$$

Now suppose you want to multiply $2x(7x)$. Again, multiply the coefficients and collect the variables into a single term:

$$= 7(2)xx = 14xx$$

Remember that x^2 is shorthand for xx, so you can write the answer more efficiently:

$$= 14x^2$$

Here's another example. Multiply all three coefficients together and gather up the variables:

$$2x^2(3y)(4xy)$$
$$= 2(3)(4)x^2xyy$$
$$= 24x^3y^2$$

As you can see, the exponent 3 that's associated with x is just the count of how many x's appear in the problem. The same is true of the exponent 2 associated with y.

A fast way to multiply variables with exponents is to add the exponents together. For example:

$$(x^4y^3)(x^2y^5)(x^6y) = x^{12}y^9$$

In this example, I added the exponents of the x's $(4 + 2 + 6 = 12)$ to get the exponent of x in the solution. Similarly, I added the exponents of the y's $(3 + 5 + 1 = 9$ — don't forget that $y = y^1$!) to get the exponent of y in the solution.

Now you can begin canceling variables. I do this in two steps as before:

$$= \frac{3xxyzzz}{4xxyyz}$$

At this point, just cross out any occurrence of a variable that appears in both the numerator and denominator:

$$= \frac{3zz}{4y}$$

$$= \frac{3z^2}{4y}$$

WARNING!

You can't cancel out variables or coefficients if either the numerator or denominator has more than one term in it.

Simplifying Algebraic Expressions

As algebraic expressions grow more complex, simplifying them can make them easier to work with. Simplifying an expression means (quite simply!) making it smaller and easier to manage. You see how important simplifying expressions becomes when you begin solving algebraic equations.

For now, think of this section as a kind of algebra toolkit. Here, I show you *how* to use these tools. In Chapter 22, I show you *when* to use them.

Combining similar terms

When two algebraic terms are similar (when their variables match), you can add or subtract them (see the earlier "Considering algebraic terms and the Big Four" section). This feature comes in handy when you're trying to simplify an expression. For example, suppose you're working with following expression:

$$4x - 3y + 2x + y - x + 2y$$

As it stands, this expression has six terms. But three terms have the variable x and the other three have the variable y. Begin by rearranging the expression so that all similar terms are grouped together:

$$= 4x + 2x - x - 3y + y + 2y$$

To remove parentheses without a sign, multiply the term outside the parentheses by every term inside the parentheses; then remove the parentheses. When you follow those steps, you're using the *distributive property*.

Here's an example:

$$2(3x - 5y + 4)$$

In this case, multiply 2 by each of the three terms inside the parentheses:

$$= 2(3x) + 2(-5y) + 2(4)$$

For the moment, this expression looks more complex than the original one, but now you can get rid of all three sets of parentheses by multiplying:

$$= 6x - 10y + 8$$

Multiplying by every term inside the parentheses is simply distribution of multiplication over addition — also called the *distributive property* — which I discuss in Chapter 4.

As another example, suppose you have the following expression:

$$-2x(-3x + y + 6) + 2xy - 5x^2$$

Begin by multiplying $-2x$ by the three terms inside the parentheses:

$$= -2x(-3x) - 2x(y) - 2x(6) + 2xy - 5x^2$$

The expression looks worse than when you started, but you can get rid of all the parentheses by multiplying:

$$= 6x^2 - 2xy - 12x + 2xy - 5x^2$$

Now you can combine similar terms. I do this in two steps, first rearranging and then combining:

$$= 6x^2 - 5x^2 - 2xy + 2xy - 12x$$

$$= x^2 - 12x$$

Parentheses by FOILing

Sometimes, expressions have two sets of parentheses next to each other without a sign between them. In that case, you need to multiply *every term* inside the first set by *every term* inside the second.

TIP

When you have two terms inside each set of parentheses, you can use a process called FOILing. The word *FOIL* is an acronym to help you make sure you multiply the correct terms. It stands for *F*irst, *O*utside, *I*nside, and *L*ast. Here's how the process works:

1. **Start out by multiplying the two *First* terms in the parentheses.**

   Suppose you want to simplify the expression $(2x - 2)(3x - 6)$. The first term in the first set of parentheses is $2x$, and $3x$ is the first term in the second set of parentheses. Therefore, multiply $2x$ by $3x$:

   $$(\underline{2x} - 2)(\underline{3x} - 6) \quad 2x(3x) = 6x^2$$

2. **Then multiply the two *Outside* terms.**

   The two outside terms, $2x$ and -6, are on the ends:

   $$(\underline{2x} - 2)(3x \underline{- 6}) \quad 2x(-6) = -12x$$

3. **Next, multiply the two *Inside* terms.**

   The two terms in the middle are -2 and $3x$:

   $$(2x \underline{- 2})(\underline{3x} - 6) \quad -2(3x) = -6x$$

4. **Finally, multiply the two *Last* terms.**

   The last term in the first set of parentheses is -2, and -6 is last term in the second set:

   $$(2x \underline{- 2})(3x \underline{- 6}) \quad -2(-6) = 12$$

Add these four results together to get the simplified expression:

$$6x^2 - 12x - 6x + 12$$

In this case, you can simplify this expression still further by combining the similar terms $-12x$ and $-6x$:

$$= 6x^2 - 18x + 12$$

Notice that during this process, you multiply every term inside one set of parentheses by every term inside the other set. FOILing just helps you keep track and make sure you've multiplied everything.

TECHNICAL STUFF

FOILing is really just an application of the distributive property, which I discuss in the section before this one. In other words, $(2x - 2)(3x - 6)$ is really the same thing as $2x(3x - 6) + -2(3x - 6)$ when distributed. Then, distributing again gives you $6x^2 - 6x - 12x + 12$.

Chapter 22

Unmasking Mr. X: Algebraic Equations

In This Chapter

- Using variables (such as x) in equations
- Knowing some quick ways to solve for x in simple equations
- Understanding the *balance-scale method* for solving equations
- Rearranging terms in an algebraic equation
- Isolating algebraic terms on one side of an equation
- Removing parentheses from an equation
- Cross multiplying to remove fractions

When it comes to algebra, solving equations is the main event.

Solving an algebraic equation means finding out what number the variable (usually x) stands for. Not surprisingly, this process is called *solving for x,* and when you know how to do it, your confidence — not to mention your grades — in your algebra class will soar through the roof.

That's what this chapter is all about. First, I show you a few informal methods to solve for x when an equation isn't too difficult. Then, I show you how to solve more difficult equations by thinking of them as a balance scale.

The balance-scale method is really the heart of algebra (yes, algebra has a heart after all!). After you understand this simple idea, you're ready to solve more complicated equations using all the tools I show you in Chapter 21, such as simplifying expressions and removing parentheses. You find out how to extend these skills to algebraic equations. Finally, I show you how cross multiplying (see Chapter 9) can make solving algebraic equations with fractions a piece of cake.

By the end of this chapter, you should have a solid grasp of a bunch of ways to solve equations for the elusive and mysterious x.

Using the balance scale to isolate x

The simple idea of balance is at the heart of algebra, and it enables you to find out what x is in many equations. When you solve an algebraic equation, the goal is to *isolate x* — that is, to get x alone on one side of the equation and some number on the other side. In algebraic equations of middling difficulty, this is a three-step process:

1. Get all constants (non-x terms) on one side of the equation.
2. Get all x-terms on the other side of the equation.
3. Divide to isolate x.

For example, take a look at the following problem:

$$11x - 13 = 9x + 3$$

As you follow the steps, notice how I keep the equation balanced at each step:

1. Get all of the constants on one side of the equation by adding 13 to both sides of the equation:

$$\begin{array}{rcl} 11x - 13 & = & 9x + 3 \\ +13 & & +13 \\ \hline 11x & = & 9x + 16 \end{array}$$

   Because you've obeyed the rules of the balance scale, you know that this new equation is also correct. And now, the only non-x term (16) is on the right side of the equation.

2. Get all of the x-terms on the other side by subtracting $9x$ from both sides of the equation:

$$\begin{array}{rcl} 11x & = & 9x + 16 \\ -9x & & -9x \\ \hline 2x & = & 16 \end{array}$$

   Again, the balance is preserved, so the new equation is correct.

3. Divide by 2 to isolate x:

$$\frac{2x}{2} = \frac{16}{2}$$
$$x = 8$$

The Real Number Line

The number line has been around for a very long time, and it's one of the first visual aids that teachers use to teach kids about numbers. Every point on the number line stands for a number. Well, okay, that sounds pretty obvious, but strange to say, this concept wasn't *fully* understood for thousands of years.

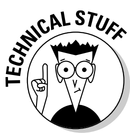
The Greek philosopher Zeno of Elea posed this problem, called *Zeno's Paradox:* In order to walk across the room, you have to first walk half the distance ($\frac{1}{2}$) across the room. Then you have to go half the remaining distance ($\frac{1}{4}$). After that, you have to go half the distance that still remains ($\frac{1}{8}$). This pattern continues forever:

$$\frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{8} \quad \frac{1}{16} \quad \frac{1}{32} \quad \frac{1}{64} \quad \frac{1}{128} \quad \frac{1}{256} \quad \ldots$$

So you can *never* get to the other side of the room.

Obviously, in the real world, you can and do walk across rooms all the time. But from the standpoint of math, Zeno's Paradox and other similar paradoxes remained unanswered for about 2,000 years.

The basic problem was this: All the fractions listed in the preceding sequence are between 0 and 1 on the number line. And there are an infinite number of them. But how can you have an *infinite* number of numbers in a *finite* space?



Mathematicians of the 19th century — Augustin Cauchy, Richard Dedekind, Karl Weierstrass, and Georg Cantor foremost among them — solved this paradox. The result was *real analysis,* the advanced mathematics of the real number line.

The Imaginary Number *i*

The *imaginary numbers* are a set of numbers not found on the real number line. If that idea sounds unbelievable — where else would they be? — don't worry: For thousands of years, mathematicians didn't believe in them, either. But real-world applications in electronics, particle physics, and many other areas of science have turned skeptics into believers. So if your summer plans include wiring your secret underground lab or building a flux capacitor for your time machine — or maybe just studying to get a degree in electrical engineering — you'll find that imaginary numbers are too useful to be ignored.

See Chapter 25 for info on imaginary and complex numbers.

Moving through Transcendental Numbers

A *transcendental number,* in contrast to an algebraic number (see the preceding section), is *never* the solution of a polynomial equation. Like the irrational numbers, transcendental numbers are also a sort of catchall: Every number on the number line that isn't algebraic is transcendental.

The best known transcendental number is π, whose approximate value is 3.1415926535… . Its uses begin in geometry but extend to virtually all areas of mathematics. (See Chapters 16 and 24 for more on π.)

Other important transcendental numbers come about when you study *trigonometry,* the math of right triangles. Sines, cosines, tangents, and other trigonometric functions are often transcendental numbers.

Another important transcendental number is *e,* whose approximate value is 2.7182818285…. The number *e* is the base of the natural logarithm, which you probably won't use until you get to pre-calculus or calculus. People use *e* to do problems on compound interest, population growth, radioactive decay, and the like.

Getting Grounded in Real Numbers

The set of *real numbers* is the set of all rational and irrational numbers (see the earlier sections). The real numbers comprise every point on the number line.

Like the rational numbers (see "Knowing the Rationale behind Rational Numbers," earlier in this chapter), the set of real numbers is closed under the Big Four operations. That is, if you take any two real numbers and add, subtract, multiply, or divide them, the result is always another real number.

Trying to Imagine Imaginary Numbers

An *imaginary number* is any real number multiplied by $\sqrt{-1}$.

To understand what's so strange about imaginary numbers, it helps to know a bit about square roots. The *square root* of any number is a second number that, when multiplied by itself, gives you the first number. For example, the square root of 9 is 3 because $3 \cdot 3 = 9$. And the square root of 9 is also –3 because $-3 \cdot -3 = 9$. (See Chapter 4 for more on square roots and multiplying negative numbers.)

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