① ⇒ In Centre-of-mass frame of reference in which the centre-of-mass remains at rest and hence momentum of both the particles before and after collision is zero.

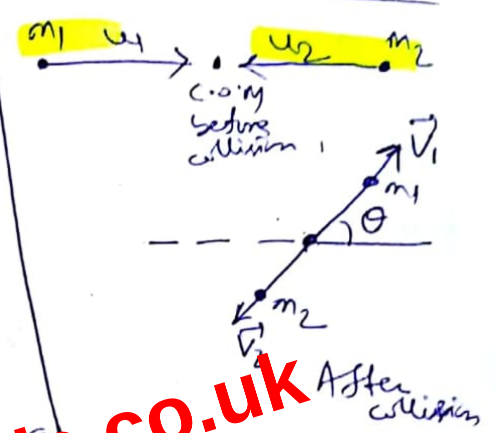
⇒ Since the velocity of the centre of mass in the laboratory frame of reference is $\left[\vec{V} = \dfrac{m_1\vec{u_1}}{(m_1+m_2)}\right]$ in accordance with the eq. it follows that C.O.M. frame moves with velocity $\vec{V}_{c.om}$ relative to lab $(\vec{V} = \vec{V}_{lab})$.

The initial velocities of two particles in the C.O.M. frame becomes.

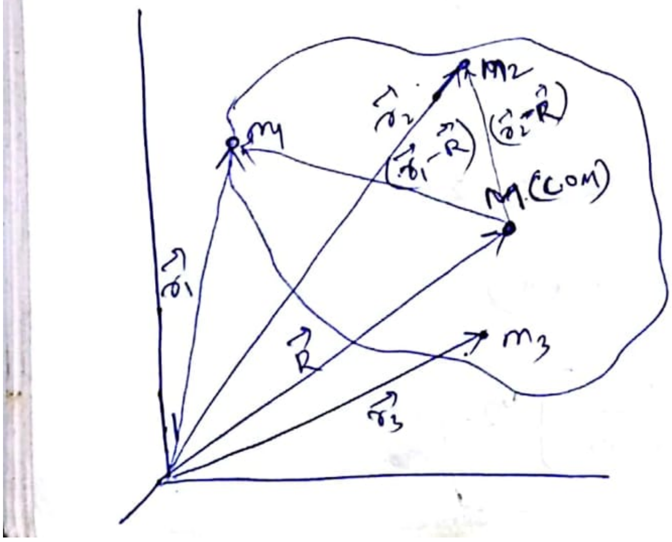
$$\vec{u_1'} = \vec{u_1} - \vec{V}_{COM} = \vec{u_1} - \frac{m_1\vec{u_1}}{m_1+m_2}$$

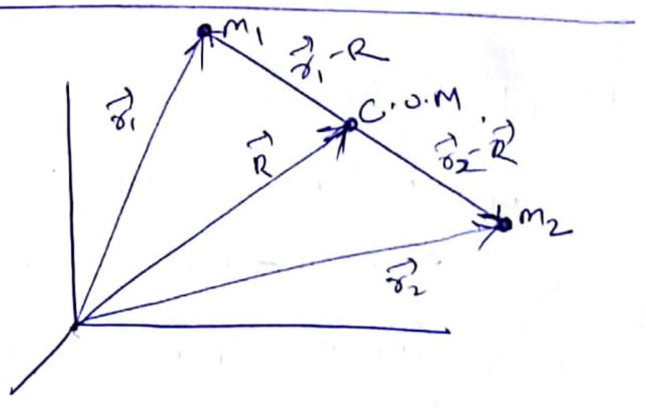
$$\boxed{\vec{u_1'} = \frac{m_2\vec{u_1}}{m_1+m_2}} \longrightarrow (6)$$

and $\boxed{\vec{u_2'} = \vec{u_2} - \vec{V}_{com} = 0 - \vec{V}_{com} = \dfrac{-m_1\vec{u_1}}{(m_1+m_2)}}$

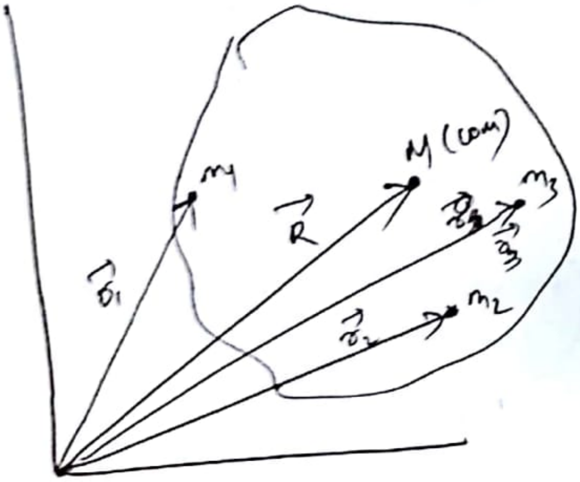


After the collision, the two particles stick together, the combined mass being (m_1+m_2) but they must be at rest since their momentum in this frame must remain zero relative to the frame, however the velocity of the combined mass in the C.O.M. frame must naturally be the same as that of the frame itself v.i.z. $V_{com} = \left(\dfrac{m_1\vec{u_1}}{m_1+m_2}\right)$ which, it will be readily seen, is the same as the velocity $\vec{V}$ of the combined mass in lab frame.





Centre of Mass :-



$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2}$$

$$= \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{M}$$

$$\Rightarrow M\vec{R} = m_1\vec{r}_1 + m_2\vec{r}_2$$

$$\vec{R} = \frac{\sum m_i\vec{r}_i}{\sum m_i} = \frac{\sum m_i\vec{r}_i}{M}.$$

If $\vec{r}_k = x_k\hat{i} + y_k\hat{j} + z_k\hat{k}$

and $\vec{R} = X\hat{i} + Y\hat{j} + Z\hat{k}$

Then $X = \frac{1}{M}\sum_{k=1}^{n} m_k x_k$; $Y = \frac{1}{M}\sum_{k=1}^{n} m_k y_k$; $Z = \frac{1}{M}\sum_{k=1}^{n} m_k z_k$

$$\vec{R} = \frac{1}{M}\int \vec{r}\,dm = \frac{\int \vec{r}\,dm}{\int dm}$$

Motion of velocity of C.O.M :-

$M\vec{R} = \sum m_i\vec{r}_i \Rightarrow M\frac{d\vec{R}}{dt} = m_1\frac{d\vec{r}_1}{dt} + m_2\frac{d\vec{r}_2}{dt} + \cdots + m_n\frac{d\vec{r}_n}{dt}$

$$\Rightarrow \boxed{M\vec{V} = m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots + m_n\vec{v}_n} \longrightarrow (1)$$

$$\Rightarrow \boxed{\vec{V} = \frac{\sum m_i\vec{v}_i}{M}} \longrightarrow (2)$$

From (1) it is clear that vector sum of the linear momenta of the individual particles i.e. the total linear momentum of the system is equal to the product of the total mass of the system and the velocity of Centre of Mass.

In the absence of any external force $M\vec{V} = \vec{P} =$ constant

$\Rightarrow \vec{V} =$ constant.