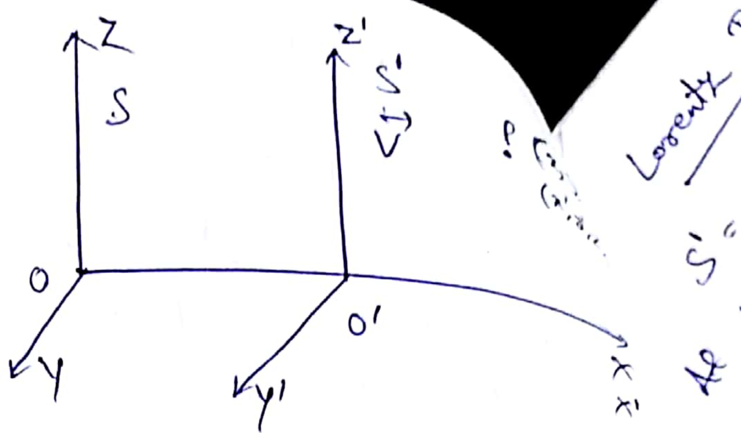
Gallilean Transformation :-

$$\begin{bmatrix} x' = x - vt \\ y' = y \\ z' = z \\ t' = t \end{bmatrix} \longrightarrow (1)$$

This is called Gallilean Transformation



Inverse Gallilean Transformation is

$x = x' + Vt$, $y = y'$, $z = z'$ and $t = t$.

Invariance Properties :-

Velocity :- Since $x' = x - Vt$ $\Rightarrow \frac{dx'}{dt} = \frac{dx}{dt} - V$

$\Rightarrow u_{x'} = u_x - V$ $\left[\because u = \frac{d}{dt} \right]$

Similarly $u_{y'} = u_y$

$u_{z'} = u_z$.

Thus velocity is variant under Gallilean Transformation.

Acceleration :-

$u_{x'} = u_x - V$

$\Rightarrow \frac{d\, u_{x'}}{dt} = \frac{d\, u_x}{dt}$.

$\Rightarrow a_{x'} = a_x - 0$.

$\Rightarrow a_{x'} = a_x$

So Acceleration is invariant

Force $\Rightarrow$ Invariant, Momentum = Invariant

Kinetic Energy - Invariant.

→ The velocity of enterprise relative to the Fleople is derived from eqn (27) i.e. $u'_x = \dfrac{u_x - V}{1 - \frac{V}{c^2} u_x}$

$$\Rightarrow u'_x = \frac{0.9c - (-0.9c)}{1 - \left[\frac{(-0.9c)(0.9c)}{c^2}\right]} = \frac{1.8c}{1.81}$$

$$\Rightarrow \boxed{u'_x = 0.99c}$$

The relative speed is less than c.

⇒ The relativistic transformation of velocities assumes that we can not exceed the velocity of light by changing reference frames.

⇒ If $u_x = c$, the velocity in the moving system is then

$$u'_x = \frac{c - V}{\left(1 - \frac{Vc}{c^2}\right)} = c$$

is independent of V. This agrees with the postulates made before Lorentz Transformations; The velocity of light is the same for all observers whatever their relative speed may be and no particle can attain a speed greater than the speed of light.

1) $\sqrt{1 - \frac{v^2}{c^2}} > 0$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \geq 1$$

$$1 \geq \sqrt{1 - \frac{v^2}{c^2}}$$

$$\sqrt{1 - \frac{v^2}{c^2}} < 1$$

Now as θ be made infinitely smaller and smaller, u would achieve greater and greater values and that it may exceed c depending up on value of θ

~~Note~~ Such examples of velocities are only of ~~φ~~ Geometrical ~~imp~~ ~~sig~~ significance and can play no role in physical world.

In relativistic addition of velocities, the particle is moving with velocity $\vec{u}$ relative to frames S and S' respectively

then $\vec{u} = \hat{i} u_x + \hat{j} u_y + \hat{k} u_z$

and $\vec{u}' = \hat{i} u'_x + \hat{j} u'_y + \hat{k} u'_z$

and $u_x = \frac{dx}{dt}$

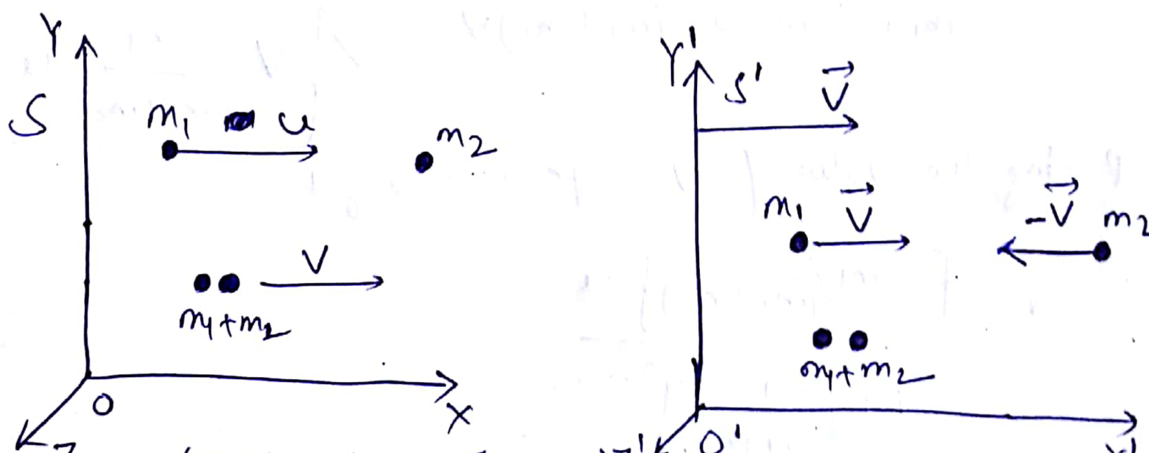
$u'_x = \frac{dx'}{dt'}$

(3) Prof. E.C.G. Sudershan has found the existence of particles that moves faster than light. These particles has imaginary mass and as known as TACHYONS. The possibility of existence of Tachyons is not denied by STR which ~~opens~~ is a great interest of study. However, we may still state "No material particle can have the speed greater than the speed of Light".

Variation of Mass with Velocity :-

The value of the mass of a particle, when it is ~~at~~ at rest in an inertial frame, is the same for all the inertial frames; but the moving mass is measured to be greater than the rest mass.

Proof :-



(Variation of Mass with velocity)

To prove this, let us consider an inertial frame S' is associated with the moving mass. Let the S' frame be moving with velocity $\vec{V}$ along positive x-direction of another inertial frame S. Now consider that further in the S' frame, there are two masses m_1 and m_2 which move in opposite directions with ~~velocity~~ velocities $\vec{V}$ and $-\vec{V}$ relative to the origin in S' frame. Suppose they collide inelastically and come to rest in S' frame. When viewed from the S-frame, m_2 will appear to be initially at rest while m_1 will have some velocity (say u) before collision. For the S-frame after collision, the ~~[illegible]~~ two masses will attain a common velocity V as shown in fig.

We relate the velocity of mass m_1 in the ~~S~~ S-frame with it's velocity in the S'-frame using the velocity transformation eqn

$$u_x = \frac{u'_x + V}{1 + u'_x V/c^2} \longrightarrow (1)$$

Let us put $u_x = u$ and $u'_x = V$ as the motion is along x-direction only.

$$\therefore\ u = \frac{V+V}{1+\frac{V^2}{c^2}} = \frac{2V}{1+(V^2/c^2)} \longrightarrow (2)$$

Since the law of conservation of momentum holds in all the inertial frame, we have in S-frame

$$m_1 u + m_2 \cdot 0 = (m_1+m_2)V \Rightarrow \boxed{V = \frac{m_1}{m_1+m_2} \cdot u} \longrightarrow (3)$$

Putting the value of V in eqn (2) we get

$$u = \frac{\left[2m_1/(m_1+m_2)\right] u}{1+\left[\frac{m_1}{m_1+m_2}\right]^2 \cdot \frac{u^2}{c^2}}$$

$$\Rightarrow c^2 - u^2 = \frac{c^2(c^2+u_x'v)^2 - c^4\left[u'^2+v^2+2u_x'v - \frac{v^2}{c^2}(u'^2-u_x'^2)\right]}{(c^2+u_x'v)^2}$$

$$= \frac{c^4(c^2-v^2) - c^4u'^2 + c^2v^2u'^2}{(c^2+u_x'v)^2}$$

$$\Rightarrow \boxed{c^2-u^2 = \frac{c^2(c^2-u'^2)(c^2-v^2)}{(c^2+u_x'v)^2}} \longrightarrow (5)$$

Dividing the above eqn by c^2 and taking square root, we have

$$\sqrt{1-\frac{u^2}{c^2}} = \frac{\sqrt{1-\frac{u'^2}{c^2}}\sqrt{1-\frac{v^2}{c^2}}}{\left(1+\frac{vu_x'}{c^2}\right)}$$

on inverting we get

$$\Rightarrow \boxed{\frac{1}{\sqrt{1-u^2/c^2}} = \frac{1+(u_x'v/c^2)}{\sqrt{1-(u'^2/c^2)}\sqrt{1-v^2/c^2}}} \longrightarrow (6).$$

using Eqn (6) in eqn (1-a) for P_x we get

$$P_x = m_0 u_x \frac{1+(u_x'v/c^2)}{\sqrt{1-\frac{u'^2}{c^2}}\sqrt{1-\frac{v^2}{c^2}}} \longrightarrow (7)$$

substituting the value of u_x from eqn (3) we get

$$P_x = m_0 \frac{u_x'+v}{\left(1+\frac{u_x'v}{c^2}\right)} \cdot \frac{1+\frac{u_x'v}{c^2}}{\sqrt{1-\frac{u'^2}{c^2}}\sqrt{1-\frac{v^2}{c^2}}}$$

$$= \frac{m_0u_x'}{\sqrt{1-\frac{u'^2}{c^2}}\sqrt{1-\frac{v^2}{c^2}}} + \frac{m_0v}{\sqrt{1-\frac{u'^2}{c^2}}\sqrt{1-\frac{v^2}{c^2}}}$$

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Similarly inverse transformation can be written by putting $-v$ for $+v$ as

$$p_x' = \frac{p_x - v(E/c^2)}{\sqrt{1-\frac{v^2}{c^2}}} \; ; \; p_y' = p_y \; ; \; p_z' = p_z$$

and $$E' = \frac{E - vp_x}{\sqrt{1-v^2/c^2}}$$

Eqn (12 - a, b, c, d)

If we compare eqn (12 - a, b, c) with eqns

$$\left(x = \gamma(x' + vt') \; ; \; y' = y \; ; \; z' = z \text{ and } t = \gamma\left(t' + \frac{vx'}{c^2}\right) \right)$$

where $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$

we find that transformation for momentum and energy in eqn (12) can be obtained ~~for~~ from Lorentz transformations by replacing x, y, z and t by p_x, p_y, p_z and E/c^2 respectively.

This is to be compared the eqn of light signal

$$x^2 + y^2 + z^2 = c^2t^2 \quad \text{or} \quad x'^2 + y'^2 + z'^2 = c^2t'^2 .$$

is equivalent to the eqn for momentum of a photon.

$$p^2 = p_x^2 + p_y^2 + p_z^2$$

$$\left(\frac{h\nu}{c}\right)^2 = p_x^2 + p_y^2 + p_z^2$$

$$\Rightarrow \frac{E^2}{c^2} = p_x^2 + p_y^2 + p_z^2 = \left(\frac{E^2}{c^4}\right) c^2$$

and for S' $$p_x'^2 + p_y'^2 + p_z'^2 = c^2\left(\frac{E'^2}{c^4}\right)$$

(13 - a, b, c, d)

~~Red Shift~~

Red shift :- when the source is receding away from the observer, we put $\underline{\theta = 180^\circ}$. Then eqn (8) and (9) becomes

$$n = \frac{n'\sqrt{1-\frac{v^2}{c^2}}}{1+\frac{v}{c}} = n'\sqrt{\frac{c-v}{c+v}} \longrightarrow (12)$$

and

$$\lambda = \frac{\lambda'\left(1+\frac{v}{c}\right)}{\sqrt{1-\frac{v^2}{c^2}}} = \lambda'\sqrt{\frac{c+v}{c-v}} \rightarrow (13)$$

Here $\underline{n < n'}$ or $\underline{\lambda > \lambda'}$.

we find that when the source is receding, the frequency of the emitted pulses appears to decrease for viewer and the wavelength appears to increase. This is called RED SHIFT, since the red colour is on the longer-wavelength end of the visible spectrum. The red shift on the E.M wave radiation issued from the receding galaxies had been observed by Hubble and we got a proof of the expanding universe on it's basis.